Nonlocal Approach to the CAFE Solution of Creep Crack Growth Problem

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Solving a creep crack growth problem using the *CAFE* methodology encounters the problems typical of other local methods. This article presents a nonlocal grid method applied in order to regularize these problems. The subject of the analysis is a rectangular plate with a central hole and an internal crack. The results obtained for different mesh sizes have subsequently been compared.

Key words: nonlocal method, grid method, CAFE, creep damage.

1. Mesh-dependence of the local approach

The problem of crack growth in creep condition has been found to be a very promising application in continuum damage mechanics (CDM). By introducing a damage parameter as a new state variable, it is possible to define the damage zone as a region with the damage parameter equal to its critical value. This damage zone may, in some circumstances, be treated as a model of crack, and its spreading as a crack growth. Using typical Kachanov-Rabotnov equations for modeling damage development, a reasonable solution of complex problems related to fracture initiation and propagation may be obtained (e.g., [1]).

However, such an approach known as local approach to fracture encounters many problems. These problems are related to the Kachanov-Rabotnov equations themselves and the application of the finite element method (FEM) to solve them. The Kachanov equation:

$$(1.1) \qquad \frac{d\omega}{dt} = C\frac{\sigma^m}{(1-\omega)^m},$$

where ω – damage parameter, σ – stress, and C, m – material constants, introduces the rate of damage tending to infinity for the damage parameter approaching its critical value of 1. Therefore, this makes the crack width to be zero.

In the FEM solution, which is mesh dependent, the crack width is always of the minimum element size, and unlimited stress redistribution leads to strong strain localization (see also [2]). Not only size, but also type and shape of the elements are essential, as the path of crack development depends on them (see also [3]).

Additional consequence is related to the time-to-failure dependence. Several stages may be distinguished in the creep damage process. The first stage ends at time t_1, which is defined as the time when the first macroscopic damage occurs. In the creep crack growth problem, this time is identical to the time when the crack starts. In numerical analysis, this time is approximated by the time of achieving the critical value of the damage parameter at the first integration point. The second stage ends at time t_2, with the occurrence of the characteristic cross-section failure. In the crack growth problem, this is equivalent to the moment when the crack reaches its critical length.

The time-to-failure analysis is also affected by mesh dependence. According to MURAKAMI [4], time t_1 changes with the element size due to stress singularity at the crack tip, following the rule presented below:

$$(1.2) \qquad\qquad t_1 \propto \left(\Delta e \right)^{m/(n+1)},$$

where Δe – element size, n – creep index, m – index in the damage development equation (1.1). Due to stress redistribution, which occurs in the creep deformation process and which reduces stress singularity at the crack tip, time t_2 is much less mesh dependent than t_1.

2. Methods of regularization, nonlocal damage theory

There are several known methods of mesh-dependency regularization (see, e.g., [4, 5]). One of the simplest methods is limiting the mesh size, where it is treated as a material parameter. The development of this method has led to the formulation of *the nonlocal approach to fracture*, in which the solution does not depend exclusively on the values of state variables at the local point, but also on its neighborhood. The grid method developed in, e.g., [6–8] is an example of this approach.

The size of the grid is closely related to the material in question. HALL and HAYHURST [6] suggested that the most important material parameter for brittle fractures in polycrystalline metals is the grain size, and the size of the grid should

be equal to six diameters of the grain. On the other hand, Bilby *et al.* [7] – for ductile fractures – determined the grid size according to the spacing of the most significant second phase particles, as these particles are responsible for the initiation of ductile cracks. The variables responsible for damage growth are averaged inside the cells of this grid and the nonlocal damage is calculated on the basis of these averaged values. The damage parameter is subsequently spread over the relevant integration points. Such a grid is called *material grid* here, to distinguish it from the finite element (FE) and other grids.

3. Nonlocal approach to CAFE methodology

3.1. CAFE model of damage development

The cellular automata (CA) model developed by the author and presented in [9] does not use the Kachanov equation (1.1) for damage growth. Instead, it applies the discrete CA rule to it, modeling non-uniform, random material behavior. The randomness of the model is related mainly to the grain structure of the material simulated by a probabilistic CA rule. Two types of damage development mechanisms are modeled: by deformation and by diffusion. The first mechanism is responsible for the development of intragranular microvoids and the other one for the development of intergranular microcracks.

Some of the problems related to infinite increase of damage rate are irrelevant in this model, as the rate of damage does not tend to infinity for the damage parameter close to its critical value. However, there are other problems resulting from stress redistribution and damage localization, which are similar to those characteristic of other local methods. Additionally, the randomness incorporated in the CA model makes the solution ambiguous and this, in turn, affects the analysis of the solution convergence, making it more difficult.

The CA methodology is strictly connected with the finite element grid, as separate CA processes are run at every integration point. The output from CA is the damage value parameter, which is used in a constitutive equation solved by the FEM. The resulting strain, in turn, constitutes an input for the CA process. Both CA and FE jointly contribute to the CAFE model (see also [10, 11]). The size of finite element, related to the size of representative volume element (RVE), is treated as a material characteristic.

3.2. Nonlocal approach

Following the nonlocal approach methodology, the present model separates the finite element grid from the material grid, which is a new element in the author's research in comparison to his earlier works. The size of the material

grid cell is still related to the size of RVE, but the FE grid can be refined for better approximation of strain and stress fields.

The strain, which provides input for the CA process, is now calculated as an average value over the volume of material grid cell, according to the following equation (see also [8]):

$$(3.1) \qquad \overline{\varepsilon}_{kl} = \frac{1}{V_{\text{cell}}} \sum_{i=1}^{n} w_i \Delta V_i \varepsilon_{kl},$$

where $V_{\text{cell}} = \sum_{i=1}^{n} w_i \Delta V_i$, ε_{kl} – strain tensor elements, n – number of integration points within a material grid cell, i – index of such point, ΔV_i – volume associated with integration point, and w_i – weighting function. The strain affects the CA model in two ways: first, by deformation of CA cells reflecting the overall RVE deformation and, second, by controlling the void volume fraction parameter (see also [13]):

$$(3.2) \qquad f_v = f_{v0} + (1 - f_{v0}) H \varepsilon^{\gamma},$$

where f_v – current void volume fraction, f_{v0} – initial void volume fraction, ε – total linear strain in tension direction, and H, γ – material parameters. A more detailed description of the CA model can be found in [9].

The damage value obtained in the CA process is then spread over the integration points of the finite element mesh connected with a given material grid cell.

Subsequently, the boundary value problem is solved by the FEM. The model by CHRZANOWSKI [12] was used here with the purpose of accommodating primary, secondary and tertiary creep, since it couples the damage parameter with the hardening theory:

$$(3.3) \qquad \dot{\varepsilon}_{ij}^c = \frac{3}{2} B_c \frac{\sigma_{\text{eff}}^{n_c-1}}{(1 - \omega)^n (\varepsilon_{\text{eff}}^c)^2} s_{ij},$$

where ε_{ij}^c – tensor of creep strain, $\varepsilon_{\text{eff}}^c$ – effective creep strain, σ_{eff} – Huber-von Mises effective stress, s_{ij} – stress deviator, B_c, n_c – material constants.

4. Simulations and results

The efficiency of the applied regularization method has been examined on a rectangular copper plate with a circular hole in the center. As the model can be applied both to undamaged and originally damaged structures, two samples were considered: one with and one without the initial crack (see Fig. 1). The
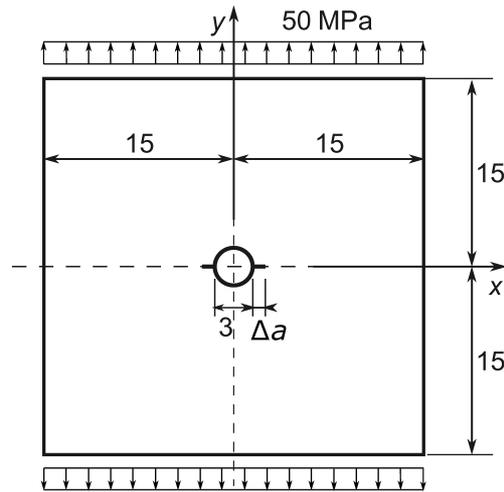
Fig. 1. Dimensions of the examined sample
(in mm) and the applied loading.

initial crack lengths were assumed to be 0.5 and 1.5 mm. The plate was subjected
to uniaxial tension of 50 MPa at temperature 723 K, equivalent to 0.54 T_m (T_m
– melting point).

The material parameters used in the simulation were as follows: elastic pa-
rameters – $E = 82.7$ GPa, $\nu = 0.33$, creep deformation parameters (Eq. (3.3))
– $B_c = 5.6E{-}22$ $(\text{MPa})^{-n}\text{s}^{-1}$, $n_c = 9.76$, void volume development parameters
(Eq. (3.2)) – $f_{v0} = 0.005$, $H = 0.782$, and $\gamma = 1.869$. The CA damage model
parameters were calibrated by experimental results on copper with mean grain
size of 30 µm (see also [9]).

Due to the double symmetry of the plate, the simulations were performed
only in one of its quarters. Three different finite element meshes were examined.
They were characterized by the size of the element along the central line $\Delta e =$
0.1, 0.25 or 0.5 mm (see Fig. 2).
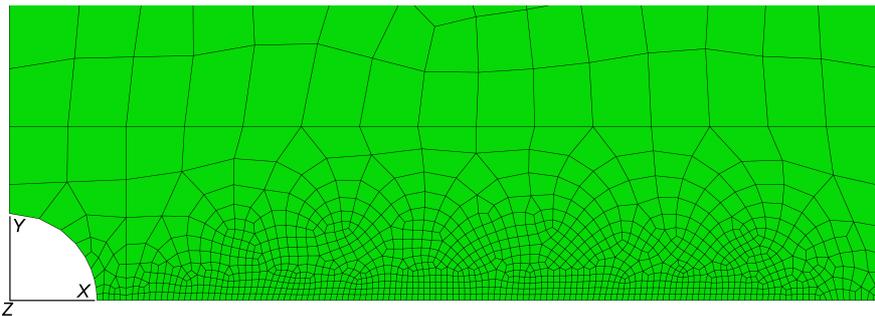
Fig. 2. Fragment of the finite element mesh for minimum element size $\Delta e = 0.1$ mm.

The material grid was kept constant and its size was 1 mm. It is too large in comparison with the mean grain size of the modeled material, but the main goal of this study is to verify the correctness and efficiency of the proposed method, rather than the accuracy of the solution. The weighting parameter w_i in Eq. (3.1) was equal to 1 for all integration points.

The dependencies of times t_1 on the mesh size obtained in the simulations were compared, and the results are shown in Fig. 3. For the local damage model, Eq. (1.2) is valid. For the material data used in the simulation, the exponent in this equation is about 0.6. In nonlocal simulations, as may be seen in Fig. 3, this value is one order of magnitude smaller, which indicates that the solution is much less mesh dependent.
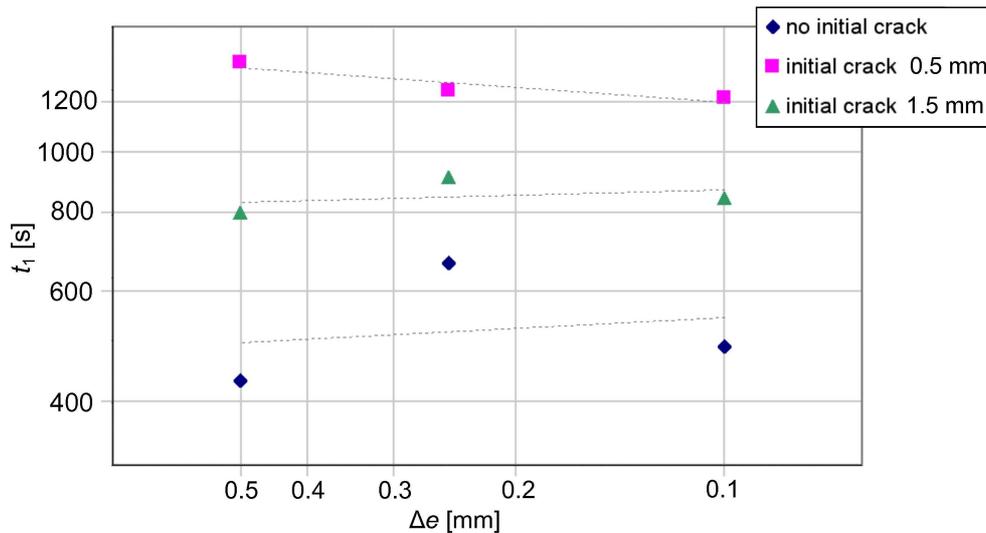
FIG. 3. Time to the occurrence of the first damage t_1, dependent on the minimum element size.

Moreover, analysis of the crack initiation locus and the directions of its development show that the nonlocal method regularizes the solution, and the use of a more dense mesh enhances its accuracy.

5. CONCLUSIONS

The presented results show that the nonlocal approach may be successfully applied not only to the standard FEM, but also to a nonstandard one. The separation of the finite element mesh from the material grid enables improving the CAFE method. The size of the finite element cell is determined by the

required model accuracy, and the size of the material cell may be treated as a material property. This enables description of the processes of crack initiation and propagation, taking into account the polycrystalline material structure.

One of the aims of the CA model of damage development was to introduce the material inhomogeneity into the examined specimen. In such situation, the ambiguity of the model response may be seen as having two different sources: the first related to the intended model randomness and the second – resulting from the mesh and finite element dependency. The nonlocal approach enables elimination of spurious solutions and examination of the influence of the initial material inhomogeneity on the process of creep crack growth.

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