ENGINEERING TRANSACTIONS • *Engng. Trans.* • **64**, 4, 541–546, 2016
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM) • Poznan University of Technology

A Novel Heuristic Algorithm for Minimum Compliance Topology Optimization

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The implementation of efficient and versatile methods to the generation of optimal topologies for engineering structural elements is one of the most important issues stimulating progress within the structural topology optimization area. Over the years, optimization problems have been typically solved by the use of classical gradient-based mathematical programming algorithms. Nowadays, these traditional techniques are more often replaced by other algorithms, usually by the ones based on heuristic rules. Heuristic optimization techniques are gaining widespread popularity among researchers because they are easy to implement numerically, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code. In this paper, a novel heuristic algorithm for a minimum compliance topology optimization is proposed. Its effectiveness is illustrated by the results of numerical generation of optimal topologies for selected plane structures.

Key words: topology optimization, heuristic algorithm.

1. Motivation

The topology optimization of structures is a continuously developing research area. Since the publication of [2] in the late 1980s, numerous approaches to generating optimal topologies, based both on the optimality criteria and evolutionary methods, were presented in the literature. A general overview as well as a broad discussion on topology optimization concepts is provided by many research papers and books, e.g., [3, 5–7]. At the same time, hundreds of papers present numerous solutions, including classic Michell's examples as well as complicated spatial engineering structures, implementing specific methods ranging from gradient-based approaches to evolutionary structural optimization, biologically inspired algorithms, the material cloud method, and the level set method. One of the most important issues stimulating this progress nowadays is the implementation of efficient and versatile methods to the generation of optimal topologies for engineering structural elements. Among them, there are many heuristic algorithms. Heuristic optimization techniques are gaining widespread popularity among researchers (see, e.g., [9]), because they are easy to implement numerically, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code.

2. Problem

In topology optimization, one searches for a distribution of material within a design domain that is optimal in some sense. The design process consists of the redistribution of material parts that are not necessary from an objective point of view are selectively removed.

Many topology optimization problems regard the minimization of structure compliance U for applied loads and supports. The formulation of such problem within the frame of this paper is as follows:

$$\begin{aligned} &\text{minimize} \qquad U(\boldsymbol{d}) = \sum_{i=1}^{N} d_i^p \boldsymbol{u}_i^T \boldsymbol{k}_i \boldsymbol{u}_i, \\ &\text{subject to} \qquad 0 \leqslant d_{\min} \leqslant d_i \leqslant 1, \end{aligned} \tag{2.1}$$

where $\boldsymbol{u}_i$ and $\boldsymbol{k}_i$ are the element displacement vector and stiffness matrix, respectively. The power law approach defining solid isotropic material with penalization (SIMP) if often adapted with design variables being the relative densities of material (e.g., [3]). The elastic modulus E_i of each element is modelled as a function of relative density d_i using the power law: $E_i = d_i^p E_0$. In this formula, E_0 is the elastic modulus of a solid material, and the power p is usually equal to 3, penalizes intermediate densities and drives the design to a material/void structure. The total volume constraint $V = \kappa V_0$, if present, is set globally and imposed after each iteration. The quantity κ stands for a prescribed volume fraction and V_0 is a design domain volume.

3. Algorithm

The idea of an original heuristic concept proposed in this paper is as follows. Based on the results of structural analysis, the values of local compliances are evaluated for N elements/design elements. Next, compliances are sorted in ascending order and $N_{\min}$ elements of the smallest and $N_{\max}$ of the largest compliance values are selected. Finally, value of C is assigned to each design element i $(i = 1, 2, \ldots, N)$ according to the following relationship:

$$
(3.1)\qquad C(i) = \begin{cases} -1, & \text{if} \quad i < N_{\min}, \\ \dfrac{2}{N_{\max} - N_{\min}} i - \dfrac{N_{\max} + N_{\min}}{N_{\max} - N_{\min}}, & \text{if} \quad N_{\min} \leqslant i \leqslant N_{\max}, \\ 1, & \text{if} \quad i > N_{\max}. \end{cases}
$$

The illustration of $C(i)$ is presented in Fig. 1.

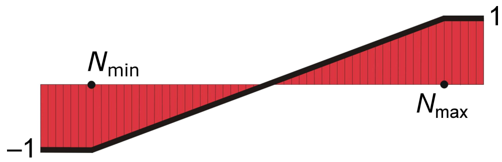

FIG. 1. $C(i)$ distribution for sorted compliances.

The next stage is the selection of the neighborhood for an element. From the proposed method's point of view, there are no special requirements regarding this selection. In Fig. 2, two easy choices of the neighborhood for a square mesh are presented.

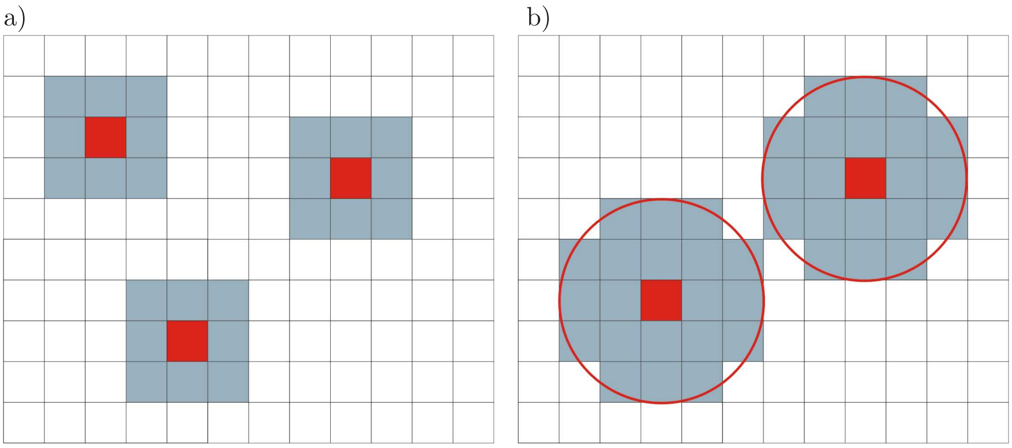

FIG. 2. Neighbourhood for square lattice: Moore type (a) and radial type (b).

Having C values assigned to all elements according to (3.1) and the neighborhood selected, the local update rule applied to design element d_i can be constructed. In what follows, taking into account a particular element i and M elements forming its neighborhood, the proposed update rule takes the form:

$$
(3.2)\qquad d_i^{(t+1)} = d_i^{(t)} + \Delta d_i, \qquad \Delta d_i = \frac{1}{M+1}\left[C(i) + \sum_{k=1}^{M} C(k)\right] m.
$$

The quantity m stands for an admissible change of design variable value, and (t) is a current iteration number.

4. Results

The selected optimal topologies obtained within the framework of this paper illustrate the proposed concept. The first example is a rectangular structure supported along the left edge and loaded by a set of three forces as presented in Fig. 3. For 320×160 elements, load $P = 50$ N, $a = 160$ mm, material data: $E = 10$ GPa, $\nu = 0.3$, volume fraction 0.5, a minimal compliance of 84.54 Nmm has been obtained and the final topology is shown in Fig. 3.

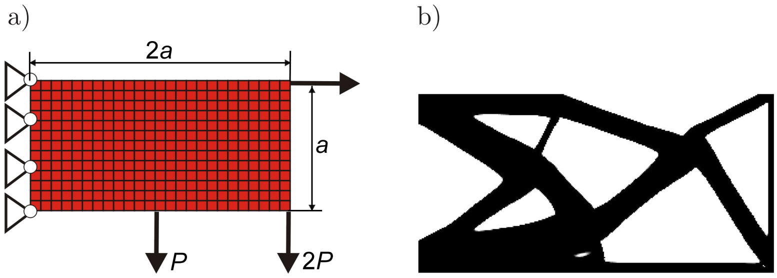

Fig. 3. a) Initial structure, b) and final topology.

The second example is a rectangular cantilever shown in Fig. 4, for which: 160×40 elements, load $P = 100$ N, $a = 40$ mm, material data: $E = 10$ GPa, $\nu = 0.3$, and volume fraction of 0.5 have been applied. The final topology of compliance 362.81 Nmm, presented in Fig. 4, has been found.

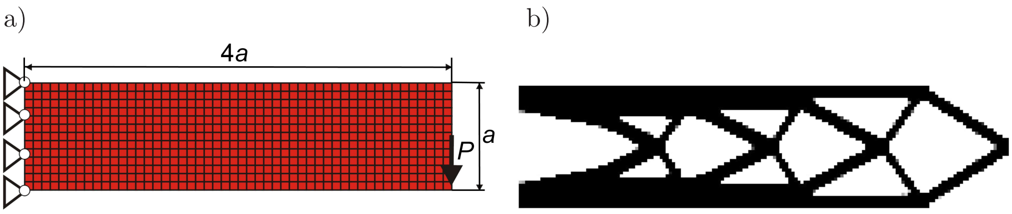

Fig. 4. a) Initial structure, and b) final topology.

A square structure supported along the left edge and loaded by two forces acting at the bottom right corner, presented in Fig. 5, is considered next. For 120×120 elements, load $P = 50$ N, $a = 60$ mm, material data: $E = 10$ GPa, $\nu = 0.3$, volume fraction 0.3, the resulting topology of compliance 116.6 $\mathrm{N \cdot mm}$ has been found, as shown in Fig. 5.

The final example is a bridge-like structure: 240×80 elements, load $p = 1$ N/mm, $a = 80$ mm, material data: $E = 10$ GPa, $\nu = 0.3$, volume fraction 0.3. The topology of minimal compliance 43.2 $\mathrm{N \cdot mm}$ has been obtained and is presented here in Fig. 6.

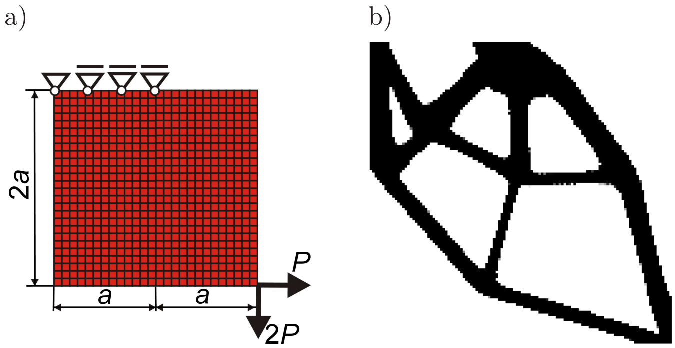

FIG. 5. Initial structure (left) and final topology (right).

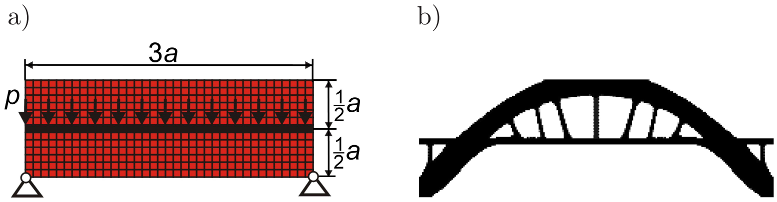

FIG. 6. a) Initial structure, and b) final topology.

For comparison, the same tasks have been solved using two other available algorithms. The very popular program described in [8] (revisited in [1]) and the heuristic technique based on the cellular automata concept presented in [4] are selected for this purpose. Applying the first one to examples 3 and 4, the minimal compliances of 118.7 N · mm and 46.1 N · mm have been obtained, respectively. As to examples 1 and 2, the cellular automata algorithm has been implemented. The compliances for final topologies generated for the two considered cases are 84.70 N · mm and 362.90 N · mm. In all the discussed cases, the compliance values are larger than the ones obtained using the novel algorithm proposed in this paper. This observation indicates that the new technique may serve as an efficient structural topology generator.

5. Conclusion

The results, obtained using the novel heuristic topology generator, are very promising. The proposed technique is easy to implement, and there are only very few parameters to adjust. What is also important, one can easily combine this type of algorithm with any finite element structural analysis code. The algorithm does not require any additional density filtering, and generated topologies are free from the checkerboard effect. Although the discussion in this paper concentrates on the topology optimization of plane structures, the algorithm needs only small adjustment regarding a neighborhood selection and can easily be applied also to spatial structures. It is worth mentioning that the obtained numerical results have been improved compared to the ones obtained with the use of other approaches. Based on the above observations, one can conclude that the topology optimization algorithm based on sorted compliances can be considered as a real alternative to other techniques used for generating minimal compliance topologies of engineering structural elements.

References

1. Andreassen E., Clausen A., Schvenels M., Lazarov B.S., Sigmund O., *Efficient topology optimization in MatLab using 88 lines of code*, Structural and Multidisciplinary Optimization, **43**(1): 1–16, 2011, doi: 10.1007/s00158-010-0594-7.

2. Bendsoe M.P., Kikuchi N., *Generating optimal topologies in optimal design using a homogenization method*, Computer Methods in Applied Mechanics and Engineering, **71**(2): 197–224, 1988, doi: 10.1016/0045-7825(88)90086-2.

3. Bendsoe M.P., Sigmund O., *Topology optimization – theory, methods and applications*, Springer, Berlin, Heidelberg, New York, 2004.

4. Bochenek B., Tajs-Zielińska K., *Novel local rules of Cellular Automata applied to topology and size optimization*, Engineering Optimization, **44**(1): 23–35, 2012, doi: 10.1080/0305215X.2011.561843.

5. Deaton J.D., Grandhi R.V., *A survey of structural and multidisciplinary continuum topology optimization: post 2000*, Structural and Multidisciplinary Optimization, **49**(1): 1–38, 2014, doi: 10.1007/s00158-013-0956-z.

6. Rozvany G.I.N., *A critical review of established methods of structural topology optimization*, Structural and Multidisciplinary Optimization, **37**(3): 217–237, 2008, doi: 10.1007/s00158-007-0217-0.

7. Sigmund O., Maute K., *Topology optimization approaches*, Structural and Multidisciplinary Optimization, **48**(6): 1031–1055, 2013, doi: 10.1007/s00158-013-0978-6.

8. Sigmund O., *A 99 line topology optimization code written in Matlab*, Structural and Multidisciplinary Optimization, **21**(2): 120–127, 2001, doi: 10.1007/s001580050176.

9. Xing B., Gao W.-J., *Innovative computational intelligence: a rough guide to 134 clever algorithms*, Springer, 2014.

Received October 11, 2016; accepted version November 9, 2016.