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THE TORSION WITH BENDING BEHAVIOUR OF A TUBULAR TRUSS

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In this study, elastic and elastic-plastic analysis of tubular square space trusses consisting of hollow bars has been carried out. The main purpose of the study is to obtain information useful in practice without making long calculations, and also to gain detailed information on the behaviour of the mentioned systems under torsion with bending in elastic and elastic-plastic region as well. In the elastic analysis, effects of variable span, existence of inner diagonals, different support conditions, vertical bar spacing and cross-sections on deflection, rotation and distortion are investigated. And, applying elastic-plastic analysis, it is examined how the effects mentioned above change the safety, ductility and the rotation capability of the truss.

1. Introduction

Space trusses which are widely used in technology have found wide application in the fields of construction of buildings, industrial plants, highway and railway bridges, antenna towers, off-shore platforms and space constructions. It is possible that larger spans will require less material with smaller cross-sections in space trusses than plane trusses and solid beams. The aim of this study is to investigate the elastic and elastic-plastic behaviour of square tubular space trusses subject to torsion with bending.

A typical example of a tubular space truss is given in Fig. 1.

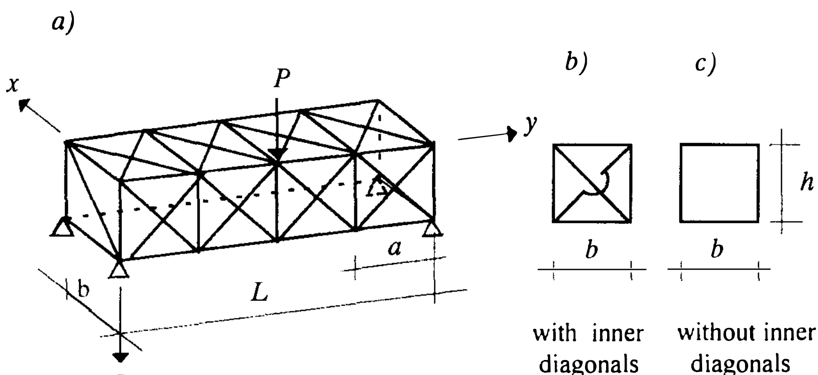


FIG. 1. Tubular space truss.

The dimensions of the space truss are assumed as follows:

span length	L	$= 4\,\mathrm{m}, \quad 6\,\mathrm{m}, \quad 8\,\mathrm{m}, \quad 12\,\mathrm{m};$
cross-section	b/h	$= 1\,\mathrm{m}/1\,\mathrm{m}, \quad 2\,\mathrm{m}/2\,\mathrm{m},$
vertical bar spacing	a	$= 1\,\mathrm{m}, \quad 2\,\mathrm{m}.$

The tubular space trusses consist of rods with hollow cross-sections. The type of steel used in the rods is St 37. Cases of trusses with or without inner diagonals are considered.

As shown in Fig. 2, tubular space trusses are considered under three different support conditions corresponding to the roller, hinged and fixed supports.

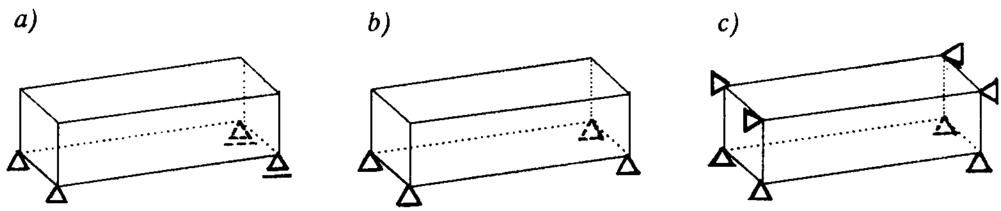


FIG. 2. Support conditions; a) roller, b) hinged, c) fixed.

Roller system. It is supported at four points at the lower ends. Both the right-hand supports are roller supports in two directions. One of the left end supports is hinged and the other one is free to roll in the x-direction, and hinged in the other directions.

Hinged system. It is supported at four points at the lower ends. The supports are hinged in all directions.

Fixed system. It is supported at eight points at the upper and lower ends. The supports are hinged in all directions.

Loading conditions. Three different kinds of loading are applied to the tubular space trusses (Fig. 3). Loads are applied to the joints and consist of one or two equal vertical forces or a force couple.

Service loads which act on tubular space trusses are taken as $P_s = 120\,\mathrm{kN}$ for all trusses.

In the elastic analysis, the values of deflection, rotation and distortion, at the centre of the span of the space truss, subject to torsion with bending have been found. The effects of span, inner diagonals, support conditions, vertical bar spacing and size of the cross-section of space truss, on the values of deflection, rotation and distortion are investigated.

The load carrying capacity of fixed systems has been calculated by means of elastic-plastic analysis by loading the system step by step. Also the diagrams "load – deflection" ($P-\delta$), "load – rotation" ($P-\theta$) and "load – distortion" ($P-\gamma$) in the middle of the systems are drawn. Then the safety factor, ductility and

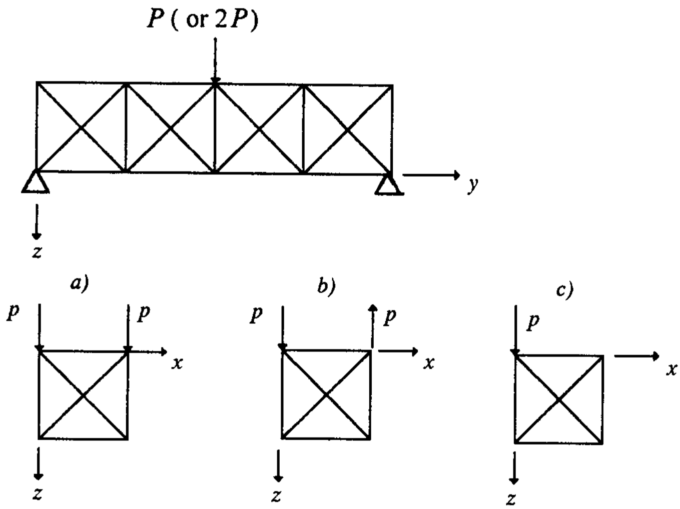


FIG. 3. Loading conditions; a) bending, b) torsion, c) torsion with bending.

rotation capability of space truss are obtained and it is examined how they are affected by the inner diagonals, support conditions, vertical bar spacing and size of the cross-section.

Basic assumptions of the analysis are given below:

- Joints are ideal hinges.
- In horizontal and vertical planes of space truss and in the plane of the cross-section of the truss it is assumed that diagonals do not intersect each other.
- The $\sigma - \varepsilon$ diagram of steel is assumed to be ideal elastic-plastic (Fig. 4).

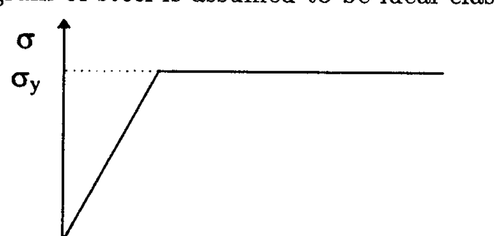


FIG. 4. Stress–strain relation.

- It is assumed that cross-sectional areas of bars can be arbitrary.
- It is assumed that the system is not subject to any lateral buckling.

2. Elastic analysis

Tubular square space trusses described in the first section have been designed for three loading cases (bending, torsion with bending, torsion), under the service load of $P_s = 120\,\mathrm{kN}$ using the allowable stress method. Firstly, the system is solved as elastic by the computer program, using the matrix deflection method, by assuming an arbitrary cross-sectional size (area A, radius of inertia i) for each bar group (bar groups: vertical bars, transversal bars, upper chords, lower chords, horizontal, vertical and inner diagonals). Cross-sections of the bars are designed according to the forces assumed as $\sigma_{\max} = \sigma_{\text{all}}$, and the system is solved with these new dimensions. This succesive approximation process is repeated until the condition $0.9\sigma_{\text{all}} < \sigma_{\max} < 1.05\sigma_{\text{all}}$ is obtained for all bar groups with cross-sectional areas of bar groups satisfying these conditions. Because $\sigma_{\max}$ is very close to σ_{all} for all bar groups with respect to these dimensions, the truss system has been designed optimally. Here σ_{all} is taken as $140\,\text{N/mm}^2$ and buckling calculations have been performed by ω numbers method. Space trusses, which are investigated, are designed as explained above and the cross-sectional areas of the bar groups in each system are given in Table 1.

Designing process of the square space truss is performed by the allowable stress method at service loading. At the same time, horizontal and vertical displacements of the joints have also been determined under combined loading of torsion with bending (Fig. 5). We have assumed that $E = 21000\,\text{kN/cm}^2$.

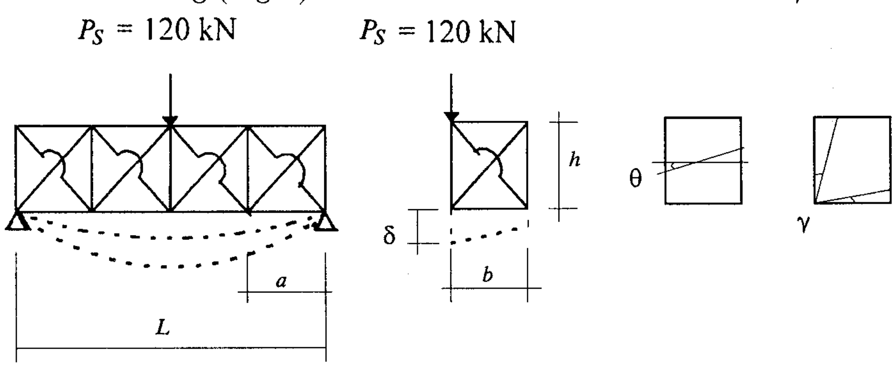


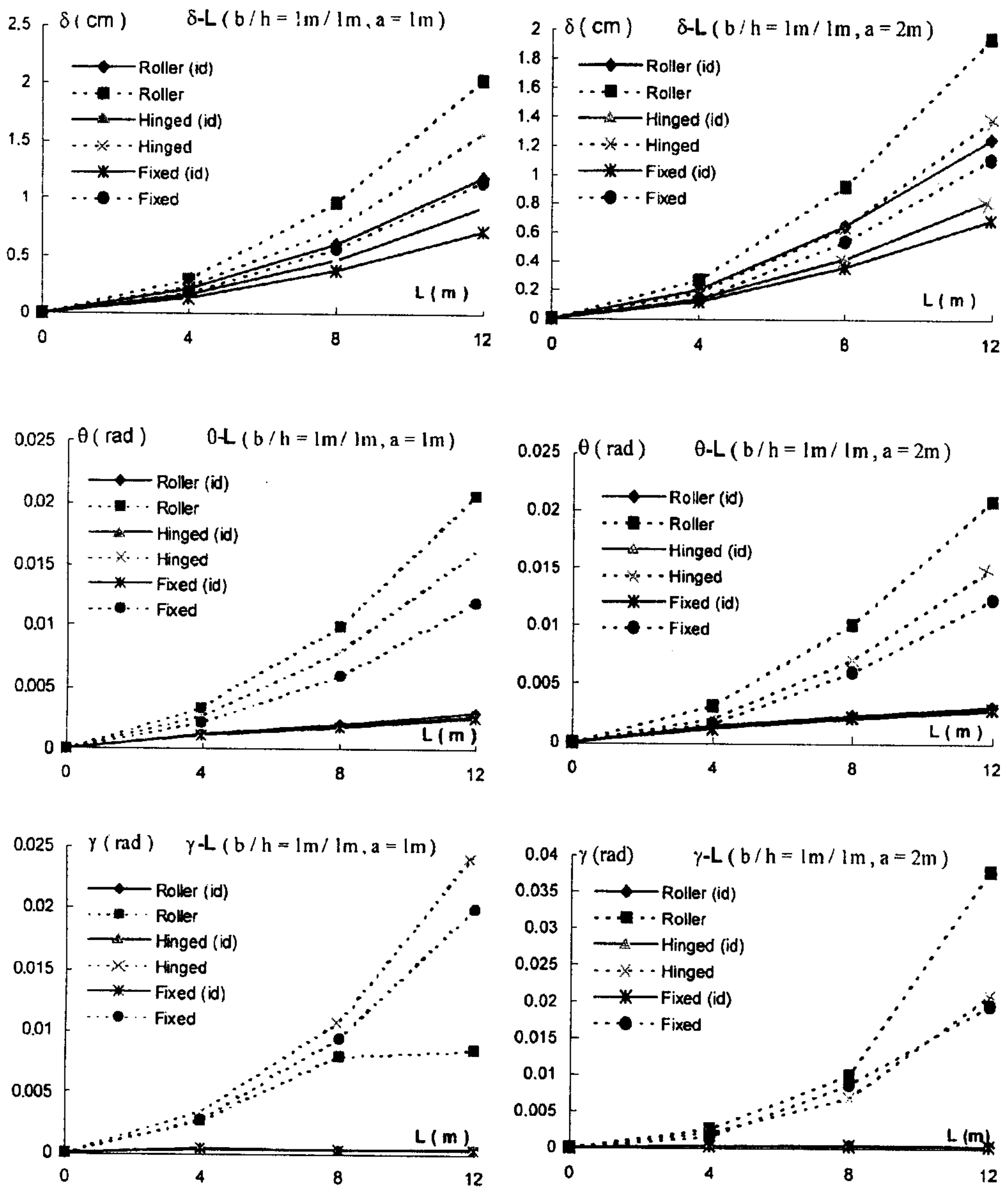
FIG. 5. Loading and deformation of the truss.

Vertical displacements of lower points of middle cross-section of space trusses under torsion with bending are given in Table 2. According to these results, deflection-span ($\delta - L$) diagrams are drawn for each system (Fig. 6).

Rotations and distortions have also been calculated and they are given in the same Tab. 2.

According to these results, the rotation-span ($\theta - L$) and distortion-span ($\gamma - L$) relations are shown by the diagrams in Fig. 6.

Deflections, rotations and distortions at the middle of the span of a tubular space truss loaded by service load ($P_s = 120\,\text{kN}$) are given in Table 2. According to these values, span-deflection, span-rotation and span-distortion diagrams are shown in Fig. 7. The equations of the curves obtained by the method of least squares have been found as follows.



Note: Systems with inner diagonals are denoted by (id).

FIG. 6. Graphs of deflections, rotations and distortions.

Table 1. Cross-sectional areas of bars (cm^2) ($b/h = 1\,\mathrm{m}/1\,\mathrm{m}$).

Vertical spacing		$a = 1\,\mathrm{m}$						$a = 2\,\mathrm{m}$					
Support Conditions		Roller		Hinged		Fixed		Roller		Hinged		Fixed	
Inner Diagonals	L	exist	absent	exist	absent	exist	absent	exist	absent	exist	absent	exist	absent
Vertical	4 m	5.13	4.95	4.92	4.68	5.01	5.01	5.46	5.55	4.65	3.93	5.34	5.31
	6 m	4.95	4.77	4.86	4.65	4.95	4.92	–	–	–	–	–	–
	8 m	4.89	4.68	4.86	4.62	4.95	4.86	5.61	5.37	5.43	4.62	5.55	5.46
	12 m	4.86	4.62	4.80	4.59	4.92	4.83	5.55	5.37	5.43	4.95	5.52	5.66
Transversal bars	4 m	2.58	2.37	1.80	2.07	1.98	2.16	2.91	2.91	0.63	0.63	0.63	0.63
	6 m	2.90	2.43	2.13	2.28	2.13	2.28	–	–	–	–	–	–
	8 m	2.97	2.76	2.37	2.40	2.40	2.43	5.34	4.44	3.12	3.03	2.52	2.91
	12 m	3.15	2.58	2.79	2.58	2.82	2.70	6.03	4.77	4.11	4.05	3.69	4.02
Upper chords	4 m	7.44	7.68	6.99	7.47	3.18	3.72	6.54	7.56	5.31	5.61	0.63	0.63
	6 m	11.28	11.46	11.10	11.52	5.16	5.82	–	–	–	–	–	–
	8 m	15.24	15.24	15.18	15.48	7.23	7.98	14.01	16.20	12.51	15.48	6.18	8.08
	12 m	22.71	23.19	22.63	23.31	11.22	12.09	21.63	24.69	20.31	24.30	9.72	12.39
Lower chords	4 m	5.85	8.55	3.30	3.84	3.51	4.14	4.35	8.85	0.63	0.63	0.63	0.63
	6 m	10.26	12.78	4.98	6.06	5.40	6.33	–	–	–	–	–	–
	8 m	14.58	17.24	6.90	8.34	7.47	8.52	8.01	17.64	5.85	7.65	6.44	8.40
	12 m	23.20	25.62	10.77	12.51	11.43	12.63	15.44	25.92	8.94	11.43	9.87	12.57
Vertical Diagonals	4 m	5.85	6.30	6.66	6.66	6.18	6.21	9.48	9.87	11.46	11.73	9.75	9.81
	6 m	6.12	6.48	6.96	6.93	6.24	6.36	–	–	–	–	–	–
	8 m	6.18	6.66	7.05	6.99	6.24	6.45	9.12	9.87	11.52	12.12	9.60	9.66
	12 m	6.27	6.54	7.17	7.05	6.24	6.39	9.27	9.87	12.12	12.39	9.60	9.57
Horizontal Diagonals	4 m	4.02	2.70	3.81	2.61	3.93	2.67	6.33	5.10	5.37	3.81	5.46	0.63
	6 m	4.14	2.88	4.17	2.85	4.08	3.00	–	–	–	–	–	–
	8 m	4.08	3.30	4.08	3.03	4.08	3.24	8.10	7.38	7.65	6.90	6.63	5.85
	12 m	4.14	3.15	4.11	3.51	4.08	3.60	8.85	7.95	8.67	7.80	7.80	7.35
Inner Diagonals	4 m	4.71	–	4.56	–	4.62	–	5.13	–	4.29	–	4.35	–
	6 m	4.56	–	4.56	–	4.59	–	–	–	–	–	–	–
	8 m	4.53	–	4.50	–	4.56	–	5.19	–	5.04	–	5.07	–
	12 m	4.44	–	4.41	–	4.53	–	5.07	–	5.01	–	5.04	–

(2.1) $\delta = cL^d$,

(2.2) $\theta = 10^{-5}(eL + f)$ (systems with inner diagonals),

(2.3) $\theta = 10^{-5}eL^f$ (systems without inner diagonals),

(2.4) $\gamma \cong 0$ (systems with inner diagonals),

(2.5) $\gamma = 10^{-5}mL^n$ (systems without inner diagonals).

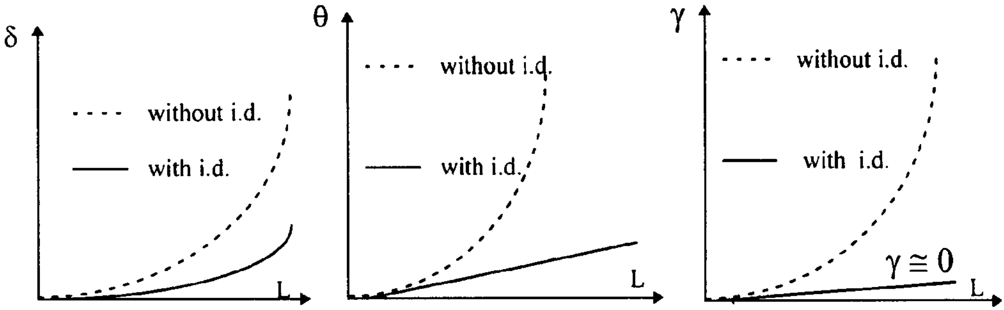
Table 2. Deflection, rotation and distortion values at the middle of the space trusses for $P_s = 120\,\text{kN}$.

L	b/h	a	inner	Roller			Hinged			Fixed		
m	m/m	m	diagonals	δ	θ	γ	δ	θ	γ	δ	θ	γ
4	1/1	1	exist	0.21	120	30	0.16	115	35	0.13	100	35
			absent	0.29	325	260	0.23	260	335	0.17	205	275
		2	exist	0.20	140	30	0.14	120	25	0.12	115	25
			absent	0.27	315	265	0.18	200	210	0.14	160	160
	2/2	2	exist	0.09	45	17	0.07	40	12	0.05	30	15
			absent	0.11	70	60	0.08	50	53	0.05	40	40
6	1/1	1	exist	0.38	160	35	0.29	150	35	0.24	140	35
			absent	0.58	610	535	0.45	480	660	0.35	380	570
8	1/1	1	exist	0.60	200	30	0.46	185	30	0.37	175	30
			absent	0.96	990	795	0.74	775	1065	0.56	590	940
		2	exist	0.65	230	35	0.42	205	30	0.36	215	35
			absent	0.92	1015	1005	0.64	715	690	0.54	605	880
	2/2	2	exist	0.32	75	17	0.21	73	18	0.16	60	20
			absent	0.42	230	210	0.32	178	236	0.23	135	140
12	1/1	1	exist	1.18	280	35	0.92	265	35	0.71	250	25
			absent	2.02	2060	845	1.56	1600	2450	1.14	1185	1995
		2	exist	1.25	305	35	0.83	275	40	0.69	290	35
			absent	1.94	2080	3795	1.38	1500	2105	1.11	1235	1965
	2/2	2	exist	0.61	100	17	0.42	100	22	0.31	75	10
			absent	0.89	475	370	0.67	360	478	0.48	265	403

δ values are given in cm, θ and γ values are given in 10^{-5} rad.

Coefficients c, d, e, f, m and n appearing in the relations depend on the size of cross-section, vertical bar spacing, on whether there are inner diagonals or not, and on the support conditions (Table 3). Values of L, δ, θ and γ are given in meters, centimeters and radians, respectively.

Deflection, rotation and distortion values are determined according to the existence or absence of inner diagonals, different support condition, different vertical bar spacing and different sizes of the cross-sections. Then the effects of inner diagonals, support conditions, vertical bar spacing and size of the cross-sections on deflection, rotation and distortion are investigated.



Note: Inner diagonals are denoted by i.d.

FIG. 7. Span-deformations ($\delta - L, \theta - L, \gamma - L$) relations of the truss.

Table 3. c, d, e, f, m and n **coefficients.**

Inner Diagonals	b/h m/m	a m		Support Condition: Roller	Hinged	Fixed
exist	1/1	1	c ; d	0.0232; 1.572	0.0172; 1.593	0.0152; 1.543
			e ; f	30.8 ; 1.681	25.5 ; 1.655	22.1 ; 1.593
			m ; n	66.7 ; 1.092	26.7 ; 1.802	22.6 ; 1.800
		2	c ; d	0.0198; 1.672	0.0148; 1.616	0.0132; 1.591
			e ; f	29.1 ; 1.715	15.7 ; 1.834	12.1 ; 1.859
			m ; n	9.12 ; 2.370	11.4 ; 2.057	6.77 ; 2.207
	2/2	2	c ; d	0.0081; 1.751	0.0073; 1.626	0.0050; 1.663
			e ; f	6.25 ; 1.704	4.14 ; 1.800	3.68 ; 1.725
			m ; n	6.06 ; 1.672	3.31 ; 2.018	2.16 ; 2.071
absent	1/1	1	c ; d	0.0248; 1.766	0.0202; 1.742	0.0156; 1.726
			e ; f	20.0 ; 40.0	18.8 ; 37.9	18.6 ; 26.4
		2	c ; d	0.0224; 1.792	0.0138; 1.851	0.0103; 1.891
			e ; f	20.6 ; 60.0	19.4 ; 45.0	21.9 ; 31.7
	2/2	2	c ; d	0.0079; 1.906	0.0055; 1.942	0.0029; 2.074
			e ; f	6.9 ; 18.3	7.5 ; 11.0	5.6 ; 10.0

3. Elastic-plastic analysis

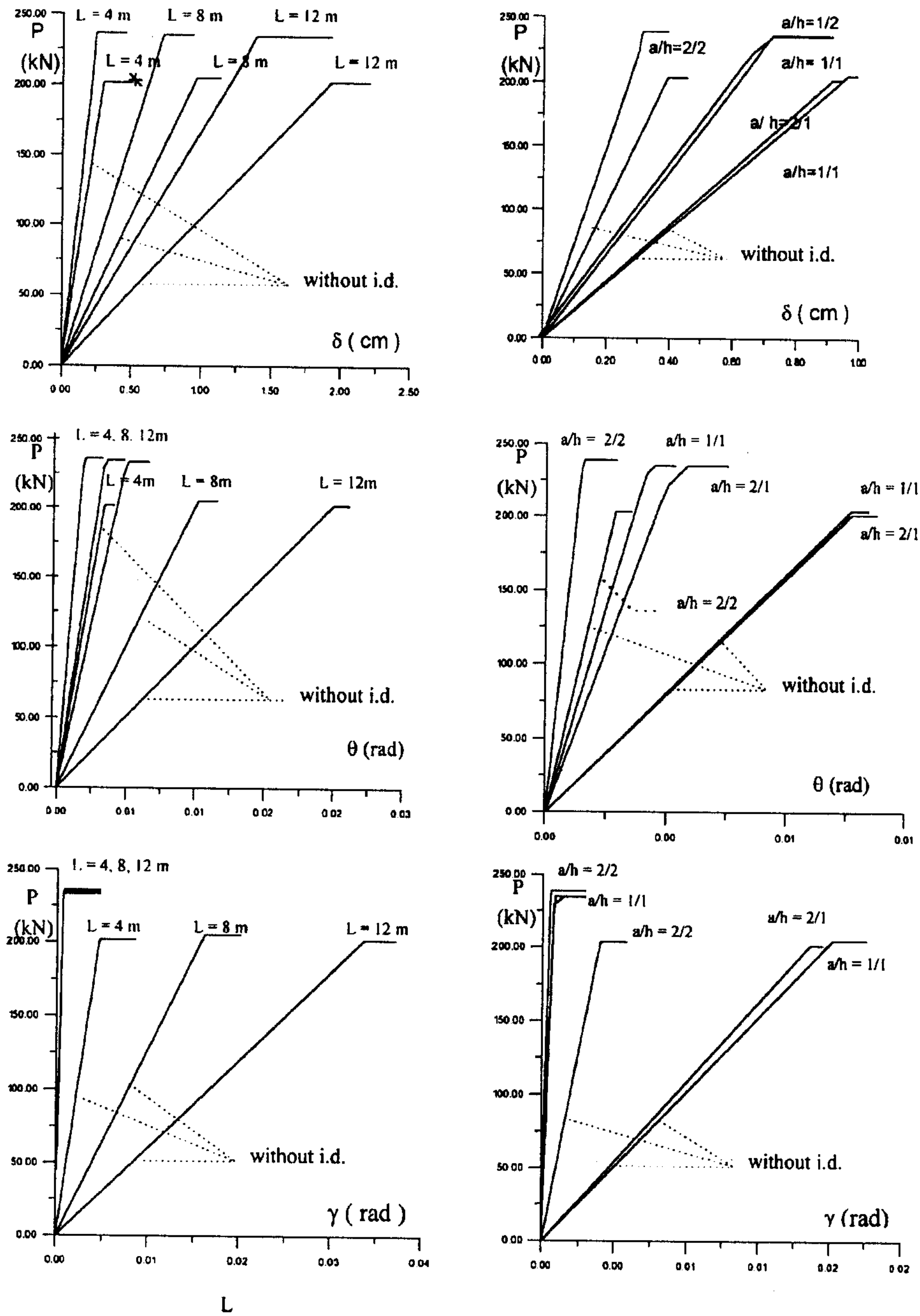
The elastic-plastic analysis has been performed for only fixed supported space trusses using the load increment technique. Here, the service loads have been increased until the maximum stress reached $\sigma_{\max} \cong \sigma_y$ in a bar (or a group of bars) and so the first yielding is obtained; the bar stresses obtained for this loading are denoted in the table by σ_1. It has been assumed that bars approaching σ_y ($\sigma \cong 235\,\mathrm{N/mm^2}$) at the end of this loading yield. Then, they are taken out from the system and the new system with other bars is loaded again from zero with an increasing load ΔP_1 and then the $\Delta\sigma$ stresses are obtained in the bars. At some bars, the value of $(\sigma_1 + \Delta\sigma_1 \cong \sigma_y)\Delta P_1$ loading is calculated while the value of $\sigma_1 + \Delta\sigma_1$ does not exceed the value of σ_y and the stresses σ_2 are obtained as a sum of σ_1 and $\Delta\sigma_1$ stresses. Bars in which the σ_2 stress approach σ_y are taken out from the system and ΔP loading is applied again. Similarly, $\Delta\sigma$ stresses have been calculated and total stresses have been obtained. ΔP loading process is repeated while the failure load of the space truss has been computed as bars in which stress approaches σ_y are taken out at each step. So, elastic-plastic analysis is performed. The values of deflection which occurs at each step are added to the previous deflection values and so deflection values at yielding are obtained.

The load capacity of space trusses at yielding (loads corresponding to yielding) and displacement (δ) in the cross-sections in the middle of the span of space truss are found; the deflections are given in the Table 4. Rotation and distortion for

Table 4. Deflection, rotation and distortion at the middle of space trusses at yielding.

L (m)	b/h (m/m)	a (m)		Systems with inner diagonals				Systems without inner diagonals	
4	1/1	1	P_i	221.80	229.50	236.02		199.45	201.68
			δ_i	0.230	0.235	0.241		0.290	0.296
			θ_i	17500	18100	19750		33500	34165
			γ_i	5500	5700	6800		45500	46325
8	1/1	1	P_i	221.25	229.08	234.67		202.64	204.63
			δ_i	0.680	0.701	0.721		0.950	0.958
			θ_i	32000	33115	35615		100500	101450
			γ_i	5500	5660	7160		159000	160500
		2	P_i	209.90	222.17	231.83	235.07	201.34	201.65
			δ_i	0.630	0.673	0.723	0.734	0.910	0.912
			θ_i	38000	40800	45300	46800	101500	101730
			γ_i	5500	5850	6350	6600	148000	148300
	2/2	2	P_i	224.97	233.99	235.09	238.82	203.17	203.22
			δ_i	0.300	0.307	0.308	0.313	0.390	0.3901
			θ_i	11250	11700	11900	12625	23000	23075
			γ_i	3500	3650	3700	3950	31500	31508
12	1/1	1	P_i	221.44	227.59	233.29		201.63	
			δ_i	1.320	1.350	1.380		1.920	
			θ_i	47500	48500	51000		199500	
			γ_i	5500	5000	5500		335500	

P_i is given in kN, δ_i in cm, θ_i and γ_i are given in 10^{-7} rad.



Note: Inner diagonals are denoted by i.d.

FIG. 8.

all yielding cases are calculated and given in the Table 4. Using these values, load-deflection $(P-\delta)$, load-rotation $(P-\theta)$ and load-distortion $(P-\gamma)$ curves in torsion with bending are given in the Fig. 8.

The real safety factors (P_U/P_S) are found using the ratio between the load carrying capacity at failure and the service load. Ductility (δ_U/δ_1) and rotation capability (θ_U/θ_1) of the systems are found using the ratio of the values of deflection and the rotation at failure and at the first yielding (Table 5).

Table 5. P, δ, θ **ratios.**

Ratios	Inner Diagonals	$L(m)$				
		4	8			12
			a/h 1/1	a/h 2/1	a/h 2/2	
P_u/P_s	exist	1.967	1.955	1.959	1.990	1.944
	absent	1.680	1.705	1.680	1.693	1.680
δ_u/δ_1	exist	1.226	1.060	1.165	1.043	1.045
	absent	1.019	1.008	1.002	1.000	1.000
θ_u/θ_1	exist	1.242	1.113	1.232	1.122	1.074
	absent	1.020	1.009	1.002	1.000	1.000

The changes (increase and decrease) in the real safety factors, ductility and rotation capability are found by using the ratio of real safety, ductility and rotation capability according to the existence or absence of inner diagonals with different vertical bar spacing and different size of the cross-section.

4. Results

The results of elastic and elastic-plastic analysis of tubular square space truss according to torsion with bending are found as follows.

The values of deflection, rotation and distortion at the middle of span of the truss under torsion with bending are given as functions of the span.

These relationships concerning deflections, rotations and distortions for the system without inner diagonals are exponential; in case of rotations for the system with inner diagonals, the relations are linear.

In elastic-plastic analysis, it is observed that failure occurs by yielding of bars attached to the joint where the load is applied.

The cross-sectional area of the upper and lower members of the space trusses is reduced at the presence of inner diagonals (as compared to the case without inner diagonals). It is seen that there is a 42% reduction in the deflection due to the effect of inner diagonals.

Large reduction (20% – 85%) in the rotation at the middle cross-section occurrs with the existence of inner diagonals, especially for a long span. It is possible to say that distortion is almost prevented by the inner diagonals at torsion with bending.

In the cross-section of the space trusses with inner diagonals it is observed that there is 16%, 10%, 13% increment in the real safety, ductility and rotation capability, respectively. It can be said that there is a positive effect of inner diagonals on the real safety, ductility and rotation capability. For the space trusses without inner diagonals, it is observed that failure occurs near the primary yielding loading case, there is no ductility and rotation capability.

Positive effect of the reactions on the supports of space trusses is observed at increasing the number of reactions. Deflection decreases by 27% in hinged systems as compared to roller systems; 44% and 22% in fixed systems as compared to roller and hinged systems, respectively.

There is also a positive effect of the number of reactions of the supports on the rotation. This effect increases by increasing the number of reactions and more effective for the systems with inner diagonals. Rotation decreases by 7% and 25% in hinged systems as compared to roller systems, with inner diagonals and without inner diagonals, respectively. In fixed systems, it decreases by 16% and 42% as compared to roller systems with inner diagonals and without inner diagonals, respectively; and by 9% and 21% as compared to hinged systems with inner diagonals and without inner diagonals, respectively.

It can be concluded that when the number of reactions at the supports increases, the value of distortion decreases when horizontal displacement of the support is prevented.

Generally, reduction reaching up to 23% in the deflections with the increase in the vertical bar spacing in space trusses (except the roller systems with inner diagonals) is observed. This reduction increases in cases of short spans. When the rotation is increasing in the systems with inner diagonals, it is decreasing in the systems without inner diagonals, with the increase in the vertical bar spacing.

The real safety in the trusses does not change so much when the vertical bar spacing is increased two times. However, 10% increment in the ductility for the systems with inner diagonals is found. It can be said that the increase in the vertical bar spacing has positive effect on the ductility. 11% increment in the rotation capability with the increase in the vertical bar spacing is observed. It can be said that the increase in the vertical bar spacing increases the rotation capability.

It is seen that there is no change in the real safety, 11% reduction in the ductility and 9% reduction in the rotation capability in the systems with inner diagonals, with the increase in the cross-section.

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