ENGINEERING TRANSACTIONS • *Engng. Trans.* • **46**, 1, 27–44, 1998
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.657.1998

PLASTIC DESIGN OF ELEMENTS OF BOLT AND PIN JOINTS SUBJECTED TO COMPRESSIVE LOADS

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Two types of statically admissible stress fields for plastic design of are proposed. They are constructed by the method of characteristics (slip-lines), known from the theory of plastic flow. The structure of the fields is described and examples of possible applications are presented.

1. Introduction

Fasteners of circular cross-sections such as bolts, pins or rivets are most widely used for connecting various structural elements in mechanical and civil engineering. Plastic design of such connections was presented in a number of papers [1–5, 7–10]. Theoretical solutions were found to be in a good agreement with experimentally determined optimum dimension ratios. All these solutions were confined to the case, when the external loads transmitted by pins or bolts generate mostly tension within the critical cross-sections of designed elements. In practical applications, however, such elements as connecting rods or truss members loaded by compressive forces are met equally often. For slender elements, stability determines the minimum dimensions of critical cross-sections. But for short elements and in the vicinity of bearing surfaces, the possibility of damage by excessive plastic deformation without the stability loss should be taken into consideration.

The present paper deals with this case. Plastic design of elements subjected to compressive loads is proposed and new statically admisible stress fields are constructed. According to the extremum principles of the mechanics of plastic flow, a statically admissible stress field (i.e. stress field that satisfies equilibrium equations and stress boundary conditions and does not violate the yield condition), determines the upper bound of the load carrying capacity of the structural element under consideration. If the shape of the element is not prescribed in advance, statically admissible stress field gives safe estimate of it. This preliminary design must then be corrected from the point of view of structural integrity, possibility of buckling, fatigue strength etc., so that the final contour could satisfy all the structural and strength requirements. The principles and methods of plastic design of structural elements of complex shape may be found in review papers and monographs by SZCZEPIŃSKI and his collaborators (see e.g. [3 – 5]).

The structural elements presented below are plane, and the stresses perpendicular to the plane of the drawing vanish. The material is assumed to obey the Tresca yield criterion.

2. THE METHOD OF CHARACTERISTICS (SLIP-LINES)

Two general methods of construction of statically admissible stress fields are applied for plastic design – the piecewise-homogeneous stress field technique [4] and the slip-line technique. The latter method gives better (more economical) estimates of shape and is more convenient to apply in the vicinity of curvi-linear bearing surfaces. It was proved useful in plastic design of elements of pin joints loaded by tensile forces [9, 10].

The slip-line technique is the method of solving plane strain boundary-value problems in the mechanics of plastic flow. It can also be applied to plane stress problems, provided the principal stresses are of opposite signs, so for the Tresca yield conditions, there is

$$\sigma_1 - \sigma_2 = \sigma_{\mathrm{pl}} = 2k\,, \tag{2.1}$$

where σ_{pl} is the yield stress under uniaxial tension, k – the yield stress under pure shear. The equilibrium equations together with the yield condition form a system of partial differential equations of hyperbolic type, therefore it has two families of real characteristics. They are determined by the equations

$$\frac{dy}{dx} = \mathrm{tg}\left(\varphi + \frac{\pi}{4}\right), \qquad \chi + \varphi = \mathrm{const} \quad (\alpha\text{-lines}), \tag{2.2}$$

$$\frac{dy}{dx} = \mathrm{tg}\left(\varphi - \frac{\pi}{4}\right), \qquad \chi - \varphi = \mathrm{const} \quad (\beta\text{-lines}), \tag{2.3}$$

where the function χ is proportional to the sum of the principal stresses

$$\chi = \frac{1}{2\sigma_{\mathrm{pl}}}(\sigma_1 + \sigma_2), \tag{2.4}$$

and φ is the angle between the x-axis and the direction of σ_1 (larger principal stress).

Both families of characteristics form together an orthogonal net inclined at angles $\pi/4$ to the direction of principal stresses, so they coincide with the lines of maximum shear stress (slip-lines). The solution of a boundary-value problem consists in numerical integration of equations of characteristics. The coordinates of all the nodes of the net and the corresponding values of χ and φ are calculated. These values uniquely determine the state of stress at each node. More detailed description of the slip-line method can be found in [6] or in textbooks or monographs concerning the mechanics of plastic flow.

3. Standard stress field of type X

Assume that the structural element under consideration is in contact with the bolt of circular cross-section, which exerts pressure of the maximum permissible value $\sigma_{\rm pl}$, uniformly distributed along the arc AA' (Fig. 1); the contact surface is assumed to be smooth.

This distribution defines the Cauchy boundary value problem – its solution uniquely determines the stresses within the whole area below AA', bounded by the outer characteristics AE and $A'E'$. The stress field is axially-symmetrical in respect to the centre O of the circle, the principal directions being the directions of the polar coordinate system. The principal stresses are expressed by the formulae

$$\sigma_1 = \sigma_{\rm pl} \ln \frac{r}{\varrho}, \tag{3.1}$$

$$\sigma_2 = \sigma_{\rm pl} \left(\ln \frac{r}{\varrho} - 1 \right), \tag{3.2}$$

where ϱ is the distance from the given point to the point O, and r is the radius of the bolt. The characteristics are logarithmic spirals inclined at angles $\pi/4$ to the radial direction. Their equations in polar coordinates ϱ, γ, are

$$\varrho = ce^{-\gamma} \qquad (\alpha\text{-line}), \tag{3.3}$$

$$\varrho = ce^{\gamma} \qquad (\beta\text{-line}), \tag{3.4}$$

where c is a constant, assuming different values for different characteristics.

The stress field in the region $ACED$ is the solution of the inverse Cauchy problem, based on the known stress distribution along the slip-line ACE and on the condition, that the boundary AD (*a priori* unknown) is stress-free. The curve AD is a trajectory of principal stress σ_2, which on the stress-free contour is equal to the yield stress, so it must satisfy the relations

$$\frac{dy}{dx} = \mathrm{tg} \left(\varphi + \frac{\pi}{2} \right), \qquad \chi = 0.5. \tag{3.5}$$

The numerical procedure of solving the inverse Cauchy problem and determining the free boundary is described in [5].

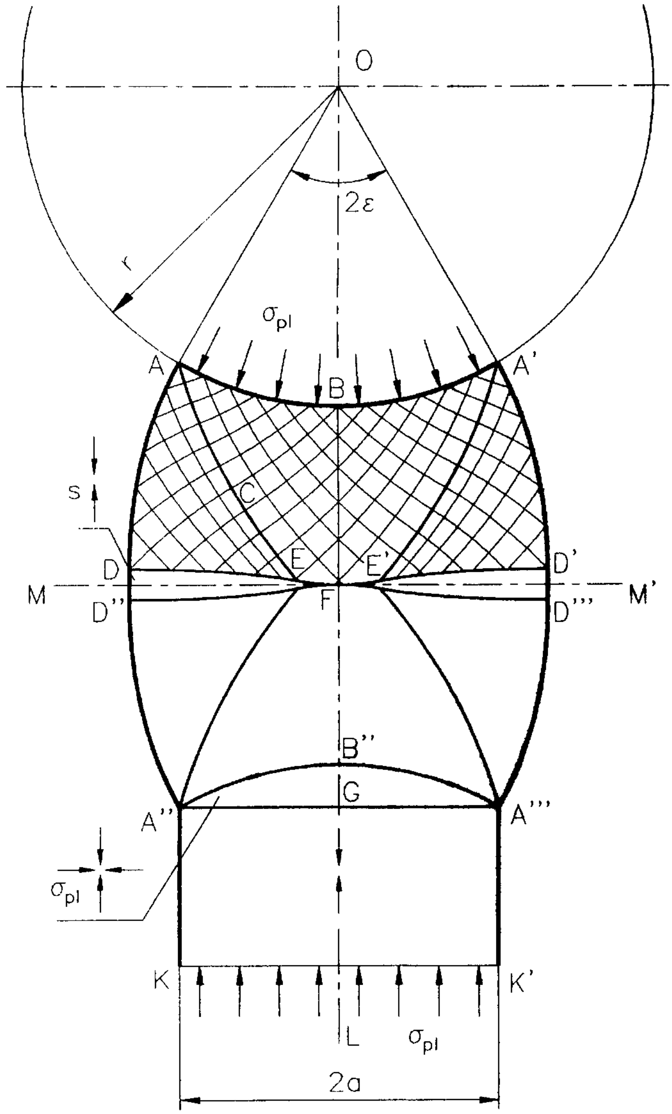

FIG. 1.

The point D is situated at the intersection of the free contour and the slip-line BCD, starting at the point B in the middle of AA'. This slip-line belongs to the α-family. It follows from $(2.2)_2$, that

$$\chi_B + \varphi_B = \chi_D + \varphi_D \,. \tag{3.6}$$

The direction of the larger principal stress ($\sigma_1 = 0$) at B is horizontal, so we have $\varphi_B = 0$. The second principal stress assumes the value $\sigma_2 = -\sigma_{\text{pl}}$, therefore, according to (2.4), $\chi_B = -0.5$. It follows from (3.5), that also $\chi_D = -0.5$. So, in order to satisfy the relation (3.6), there must be $\varphi_D = 0$. Thus, the tangent to the stress-free contour at D (direction of σ_2) is vertical. This point was chosen as the begining of discontinuity line DEF. The state of stress in the region below is assumed to be uniaxial compression s in vertical direction. On the basis of this assumption and the known stress distribution within the slip-line net, we can determine the course of discontinuity line. Two equilibrium conditions enable us to calculate at each consecutive point, the local direction of the curve and the unknown value of s. The numerical procedure necessary to perform these calculations was used in several other problems of plastic design [8–10]. The discontinuity line intersects the symmetry axis at the point F. The values of s change from $-\sigma_{\text{pl}}$ at the point D to $\sigma_{\text{pl}}(\ln(\varrho_F/r) - 1)$ at the point F (according to the formula (3.2)).

The whole stress field is symmetrical with respect to the axis OL, so the discontinuity line $D'E'F$ is symmetrical to DEF. The line MFM' perpendicular to OL is assumed as another axis of symmetry. Thus, the stress field $FD''A''B''A'''D'''$ is identical with $FDABA'D'$. The stress distribution along $A''B''A'''$ is the same as along ABA', i.e. the uniformly distributed pressure equal to the yield stress $-\sigma_{\text{pl}}$.

In order to transmit these stresses to the compressed strip $A''A'''KK'$, additional stress field is required. The region $A''B''A'''G$ is assumed to be under biaxial uniform compression equal to the yield stress. Other possible cases of support require other configurations of the region of biaxial compression.

The total load transmitted by the presented stress field is equal

$$F = \sigma_{\text{pl}} g 2a = \sigma_{\text{pl}} g 2r \sin\varepsilon \tag{3.7}$$

where r is the radius of the bolt, g – thickness of the designed element, $2a$ – width of the compressed strip $A''A'''KK'$. The ratio $a/r = \sin\varepsilon$ is the characteristic parameter of the field. The solution presented in Fig. 1 was constructed for the particular value of this parameter, namely $a/r = 0.5$.

Table 1.

a/r	c/r	h/r	ε
0	1	1	0
0.1	1.034	1.102	0.100
0.2	1.069	1.208	0.201
0.3	1.106	1.319	0.305
0.4	1.145	1.436	0.412
0.5	1.188	1.561	0.524
0.6	1.235	1.696	0.643
0.7	1.290	1.846	0.775
0.8	1.356	2.020	0.927

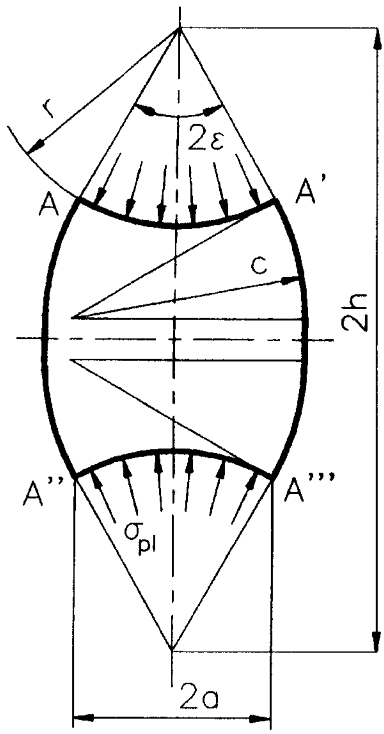

FIG. 2.

For practical purposes, it is convenient to approximate the designed contour as shown in Fig. 2. The data necessary to draw this approximation for $0 \le a/r \le 0.8$ are summarized in Table 1 (the limit case $a/r = 0$ may be useful for interpolation). In analogy to the notation introduced in [5] and applied in [10], the present stress field will be denoted as the standard stress field of type X.

4. Plastic design of a bolt connection

The stress field presented above may be used for plastic design of various elements of bolt and rivet connections. A simple example of such element is shown in Fig. 3.

The external load is applied by means of three bolts, each of them exerting the force of the magnitude

$$F = \sigma_{\mathrm{pl}} g 2a. \tag{4.1}$$

The bolt *1* is positioned on the axis of symmetry; the bolts *2* and *3* are situated below. The spacing of bolts is determined by the requirement that the material

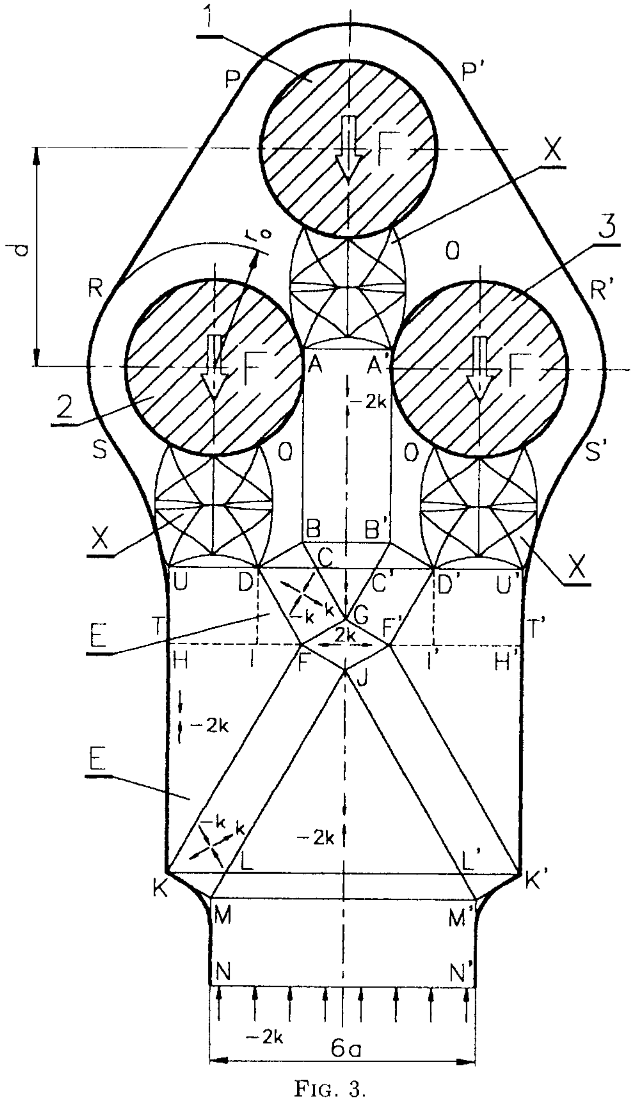

FIG. 3.

between them must carry the force F exerted by the bolt *1*. The width of the compressed strip $AA'B'B$ must be $2a$, so the distance between the centers of bolts *2* and *3* equals $2(r + a)$. The minimum distance d between the center of the bolt *1* and the horizontal line joining the centers of the bolts *2* and *3* results from the condition that the stress field of type X must be situated between these three bolts. The minimum takes place when the bolts *2* and *3* are tangent to the external contour of the field of type X. For the case shown in Fig. 3, i.e. $a/r = 0.5$, there is $d/r = 2.48$. This is the upper (safe) estimate of this distance; a smaller value of d may cause that the designed element will not carry the prescribed load without considerable plastic deformation that will change either the dimensions or the flow of forces within the element.

The external loads are transmitted by means of stress fields of type X to the compressed strips $UDIH$, $AA'B'B$ and $U'D'I'H'$, respectively. The stresses $-\sigma_{\text{pl}}$, uniformly distributed along BB', are then transmitted to the sections IF and $I'F'$ by means of the stress field $BDIFF'I'D'B'$. This stress field, shown in some detail in Fig. 3, is piecewise-homogeneous. It is presented in monograph [5] as the basic stress field of type E and will not be described here.

The strip $UDIH$ transmits the stresses $-\sigma_{\text{pl}}$ into the segment HI, so along the whole segment HF there are uniformly distributed stresses equal to the yield load. The same state of stress is distributed along the symmetrical segment $H'F'$. These stresses are now transmitted to the segment MM' by means of the stress field $MKHFF'H'K'M'$, also of type E (this field is turned upside-down in relation to $BDIFF'I'D'B'$). Thus the whole load exerted by the three bolts is transmitted to the compressed strip $MM'N'N$.

The upper contour of the element is designed in view of its integrity rather than strength. The arcs RS, PP' and $R'S'$ are assumed to have the radius r_0. The outer radius r_0 may result from the condition of minimum width of the material around the holes, or simply be taken as equal to the radius of the head of the bolt or rivet. The arcs are connected by straight lines PR and $P'R'$. At the point S the contour follows another arc ST, which is tangent to the arc RS, the segment KE and the stress field X. The arc $S'T'$ is symmetrical to ST.

5. Standard stress field of type V

The standard stress field of type X was constructed under the assumption, that the tractions exerted by the bolt are equal to the yield stress. The field of type X may thus be used to design connections in which such stresses are admissible – as it is in most permanent joints. In many cases, however, the admissible bearing tractions are much lower. This applies especially to movable connection, when the pin can rotate in relation to the designed element. The smaller bearing tractions result in smaller friction and reduce wear. The stress field for the case, when the pressure on the bearing surface is lower than the yield stress, will be described in the present section.

Figure 4 shows analogous solution for the plane bearing surface. It was originally proposed by Winzer and Carrier [6] and is denoted in [5] as the basic stress field of type D. The whole stress field is composed of four homogeneous subfields, each of them being in the limit state. The pressure $q > -\sigma_{\text{pl}}$ distributed along the line AA' is transmitted into the region $AA'B$. The principal stress in

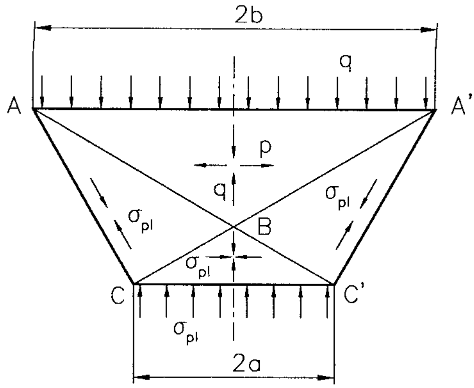

FIG. 4.

vertical direction in this region is $p = q + \sigma_{\text{pl}}$. The discontinuity line AB forms an angle ω with the loaded surface AA'. The value of ω may be calculated from the relation

$$q = -\sigma_{\text{pl}}(1 - \cos 2\omega). \tag{5.1}$$

The other principal stress is related to ω by the formula

$$p = \sigma_{\text{pl}} \cos 2\omega. \tag{5.2}$$

The free contour AC and the line AB also form the angle ω. The region ABC undergoes uniaxial compression of the limit value $-\sigma_{\text{pl}}$. The same state of stress exists in the region $A'BC'$ – symmetrical to ABC. The discontinuity line BC is perpendicular to AC. The region BCC' is in the state of biaxial isotropic compresion equal to the yield stress $-\sigma_{\text{pl}}$. The same stress is applied along the other loaded surface CC'. The condition of general equilibrium requires that

$$q/\sigma_{\text{pl}} = a/b. \tag{5.3}$$

In Fig. 5 we have a similar stress field; the bearing surface AA' in this case however, is not plane but cylindrical with the centre of curvature at the point O. The regions $AA'C$, ABC and $BB'C$ are thus not homogeneously stressed and the discontinuity lines AC and BC are curvi-linear rather than straight segments.

In the region $AA'B$ an axially-symmetrical stress field in respect to the centre O is assumed. The principal directions are the directions of the polar coordinate system, and the principal stresses are expressed by the formulae

$$\sigma_\varrho = q + \sigma_{\text{pl}} \ln \frac{\varrho}{r}, \tag{5.4}$$

$$\sigma_\upsilon = -\sigma_\varrho + \sigma_{\text{pl}}, \tag{5.5}$$

where ϱ is the radius vector; for $\varrho = r$, there is $\sigma_r = q$.

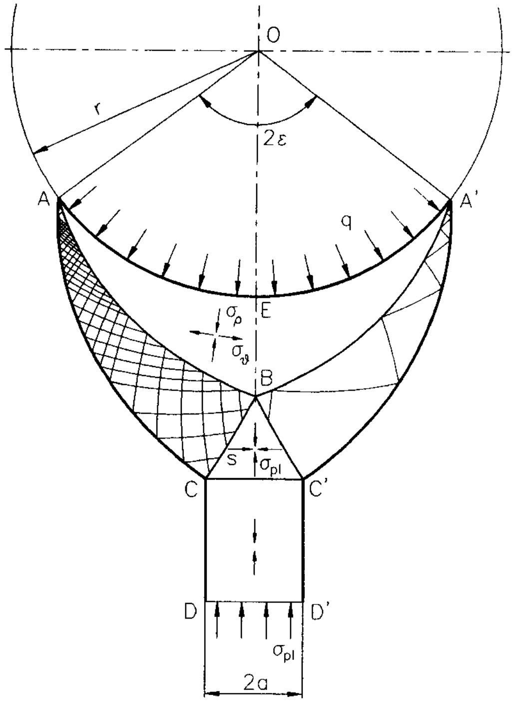

FIG. 5.

The stress field in the region ABC is constructed with the use of the slip-line method. It is assumed, that the stress field in the vicinity of the point A is identical to that at the corner A in Fig. 4. Figure 6 shows it in more detail. The state of stress in the region AFK is described by the formulae (5.1) and (5.2), and the region AKL undergoes uniaxial compression of the value $-\sigma_{pl}$.

The line KL forms an angle of $\pi/4$ with the stress-free contour AL and with the direction of principal stress $-\sigma_{pl}$. This is therefore the slip-line (of the α-family). The stress field between the curve n and the *a priori* unknown discontinuity line KMP is defined by (5.1) and (5.2). On the basis of the known (uniform) distribution of stresses along KL and the condition of equilibrium, that must be satisfied along the segment KM of the discontinuity line, the inverse Cauchy boundary value problem is solved. The solution gives us the run of the discontinuity line KM and of the slip-line LM (belonging to the β-family) together with the whole slip-line net, which determines the stress distribution within the region KLM.

The slip-line net and the stress distribution in the region LMN is found as the solution of another inverse Cauchy boundary problem, based on the known form of the slip-line LM and the condition, that the boundary LN is stress-free. The sequence of solving the inverse Cauchy problems of these two types is then

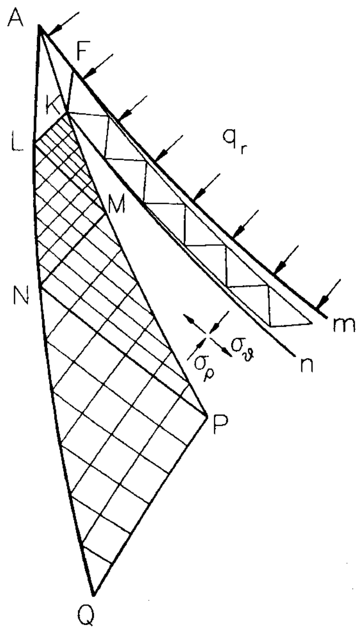

FIG. 6.

repeated until the discontinuity line AB intersects the axis of symmetry and the stress-free contour AC reaches the edge CD of the compressed strip $CC'D'D$. The width $2a$ of this strip is determined by the global equilibrium condition (similar to (5.3))

$$\frac{a}{r} = \frac{q}{\sigma_{\rm pl}} \sin \varepsilon \,. \tag{5.6}$$

The finite dimensions of the region $AFKL$ require the existence of an auxiliary stress system located between the arc m representing the bearing surface and n – the boundary of the axially-symmetrical stress field defined by the formulae (5.4) and (5.5). The structure of this auxiliary field is shown in Fig. 7. For the sake of clarity, this figure has been drawn disregarding the real proportions.

The field is composed of several regions separated by the lines of stress discontinuity. These discontinuities are of statically admissible type, i.e. when crossing a discontinuity line, the normal and shear stresses acting on the plane of discontinuity line are continuous but the normal stresses parallel to discontinuity can have different values on both sides of it.

The polygon $FKIRTSJH$ is composed of triangular elements of two different kinds. The elements are isosceles triangles, congruent within each of both families, the directions of compressive principal stresses being perpedicular to the bases of the triangles. Each element is in the limit state. The neighbouring

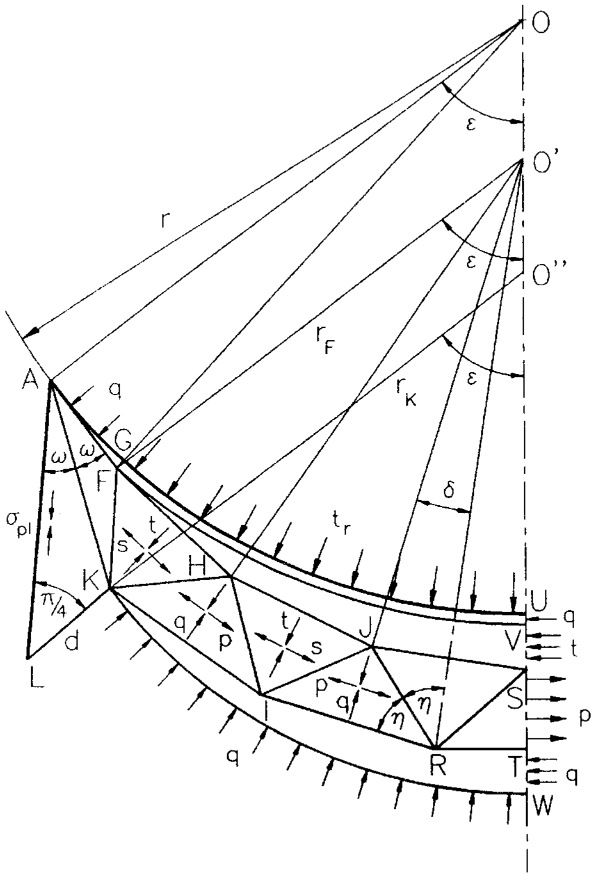

FIG. 7.

elements are separated by the lines of stress discontinuity. These lines form angles η with the corresponding principal directions (see e.g. [5]). The directions of the compressive principal stresses in the neighbourig elements form angles δ. It is easy to see that the relation holds

(5.7) $$2\eta + \delta = \pi/2.$$

The triangles HIK, IJR and RST (the second half of it is on the other side of the axis of symmetry) of the first family are subject to the same state of stress as is the region AFK, so the principal stresses follow the formulae (5.1) and (5.2). The principal stresses within the triangles FHK, HJI and JSR are given by the following expressions:

(5.8) $$s = p - \sigma_{\rm pl} \cos 2\eta,$$

and

(5.9) $$t = q - \sigma_{\rm pl} \cos 2\eta.$$

The region $TWKIR$, determined by the arc WK and the line KI, RT, and RT, is subject to biaxial isotropic compression under stresses q. The same state of stress exists in the region AGF. The region $FVSJH$, bounded by the arc FV and the segments FH, HJ and JS, undergoes biaxial isotropic compression under stresses t. The state of stress in the region $GUVF$ is axially symmetrical, with the centre at the point O. The circumferential stresses are constant and equal q, while the radial stresses change from the value t at the arc FV to

$$t_r = \frac{r_F}{r} + q\left(\frac{r_F}{r} - 1\right), \tag{5.10}$$

at the arc GU (r_F is the radius vector of the point F in relation to O).

The stress field of Fig. 5 may be considered as the limit case, when the dimensions of the region $AFKL$ (e.g. the length d of the slip-line KL) are vanishingly small in comparison to the radius r. We have then $r_F \to r$, $\delta \to 0$, $\eta \to \pi/4$, $t_r \to t \to q$, $d \to 0$, and the arcs m and n (Fig. 6) coincide. In numerical calculations the dimensions of the region $AFKL$ must be small but finite; the computation of the stress field in Fig. 5 has been performed for $d = 0.000\,05r$.

The point B of intersection of the discontinuity line AB and the symmetry axis (Fig. 5) is the begining of another discontinuity line BC. On the right-hand side of this line, in the region BCC', the principal directions are assumed to be constant. It is moreover assumed that the principal stress in vertical direction is equal $-\sigma_{\mathrm{pl}}$. Two equilibrium equations that must be satisfied along the discontinuity line enable us to calculate at each point of it the unknown local direction of the line and the local value of s, the principal stress in horizontal direction.

The discontinuity line BC intersects the stress-free contour AC at the point C. The point of intersection must be located at the distance a from the axis of symmetry (see (5.6)). This condition may be used to check the accuracy of numerical procedures. The line CC' is another discontinuity line. The material below is subject to uniaxial compression equal to the yield stress.

In analogy to the earlier notation, the stress field just described will be denoted as the standard stress field of type V. The fields of this kind are defined by two characteristic parameters a/r and $-q/\sigma_{\mathrm{pl}}$. It is difficult to find a simple and possibly universal aproximation of external contours of the fields of type V by means of straight segments and circular arcs only. It is much more convenient to use for this purpose the Archimedean spiral, defined in polar coordinates (ϱ, ψ) by the formula

$$\varrho = (m\psi + 1)r, \tag{5.11}$$

where ϱ is radius in relation to the point O, ψ – angle measured from the axis OA, and m is a parameter dependent on q/σ_{pl} (Fig. 8). The values of m for several values of q/σ_{pl} are given in Table 2.

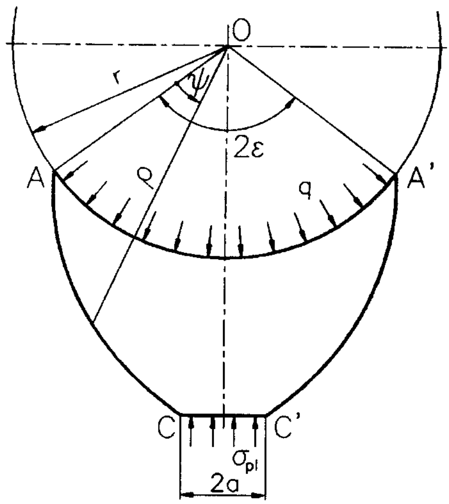

FIG. 8.

Table 2.

$-q/\sigma_{pl}$	m
0.125	0.583
0.175	0.725
0.250	0.928
0.350	1.196
0.500	1.734

6. Plastic design of connecting rods

The stress fields of type V may be useful in plastic design of various elements of pin connections. An example of such design is shown in Fig. 9. The connecting rod transmits the compressive load F between the pair of pins. The load is applied in the form of uniformly distributed tractions q along the arc AA'' ($0 > q > -\sigma_{pl}$). It is then transmitted by a pair of stress fields of type V whose axes of symmetry are inclined at angles $\pi/6$ to the vertical axis of symmetry of the whole element. The section CC' is subject to the compressive stresses equal to the yield limit $-\sigma_{pl}$. The stress field in the region $CDEGHC'$ is the standard stress field of type C; its structure is described in detail in [5]. The compressive load in section CC' is balanced by the compressive stresses in the strip $EIJG$ and tensile stresses in the strip $GKLH$. In both strips the stresses reach the yield limit. The region KLK' is subject to bi-axial uniform compression, also equal to the yield stress. The horizontal line JJ' is the second axis of symmetry of the element. The external contour of the designed element in the neighbourhood of

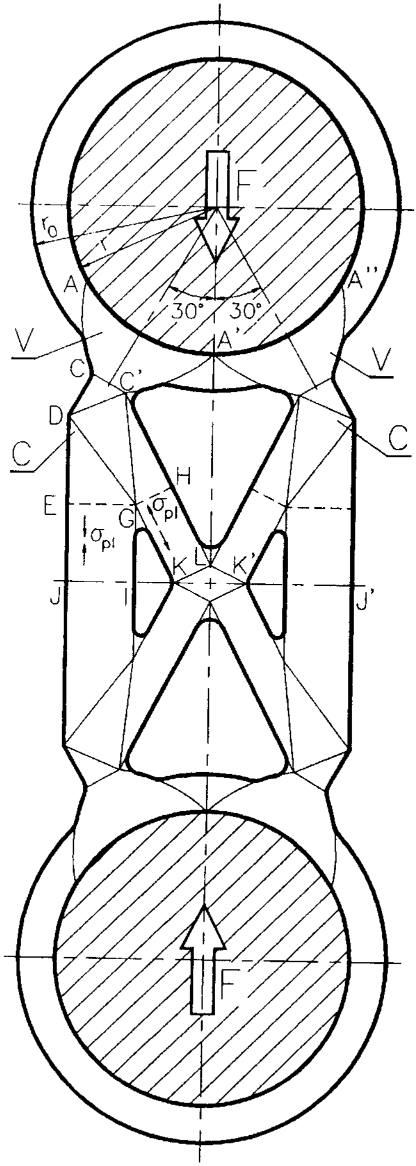

FIG. 9.

the pin is determined by the radius r_0. As it is seen in the example shown in Fig. 3, this shape does not result from the strength considerations but from the requirement of structural integrity of the whole connection.

The structural element in Fig. 9 is plane. In some cases however, three-dimensional arrangements of the plane sub-fields may be used. Figure 10 shows another example of plastic design of a connecting rod. The external load uni-

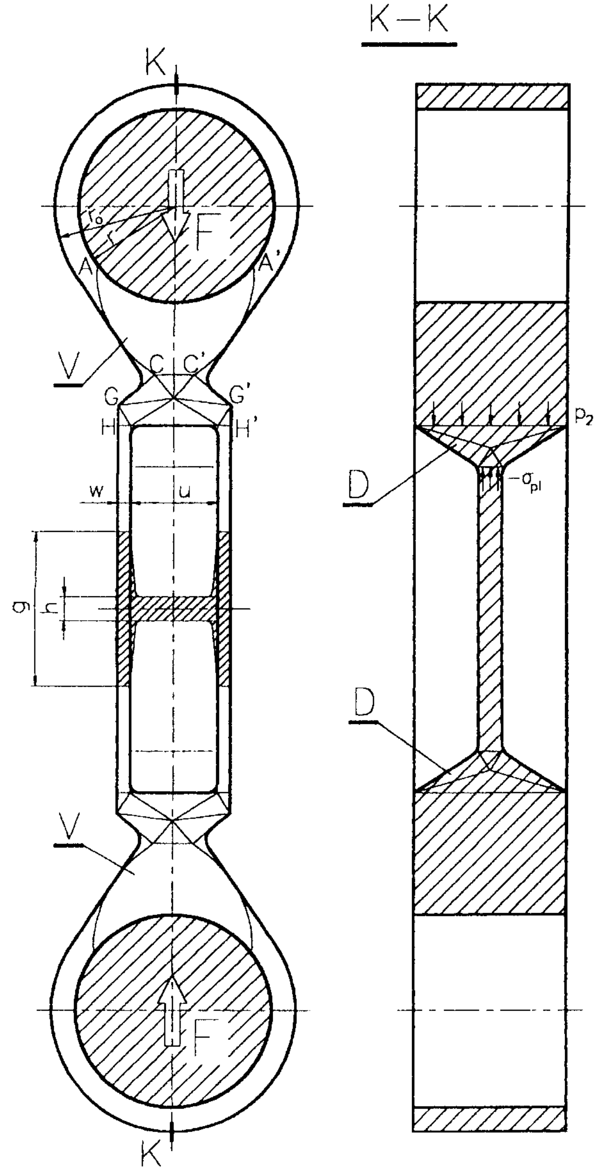

FIG. 10.

formly distributed along AA' is transmitted to the section CC' by means of the single stress field of type V. The stress field in the region $CGHH'G'C'$ is shown in detail in Fig. 11. This stress field is piecewise homogeneous and may be treated as a generalization of the standard stress field of type E, described in [5]. The material in the region $CC'J$ is subject to biaxial uniform compression equal to the yield limit $-\sigma_{\mathrm{pl}}$. The regions CGJ and GHI are under uniaxial compression,

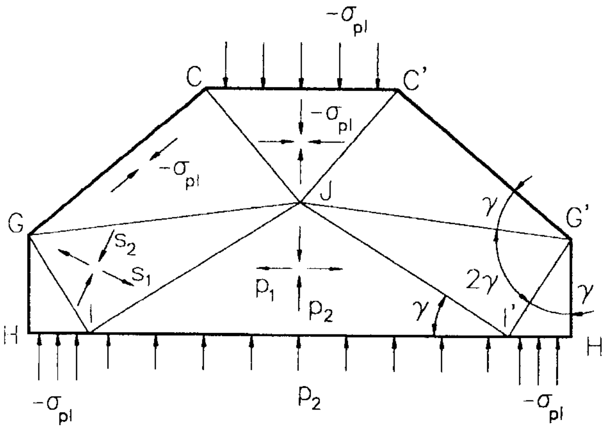

FIG. 11.

also equal to the yield stress. The principal stresses within the region GIJ are determined by the formulae

$$s_1 = \sigma_{pl} \cos 2\gamma, \tag{6.1}$$

$$s_2 = \sigma_{pl}(\cos 2\gamma - 1), \tag{6.2}$$

and the principal stresses within $II'J$ are given by the expressions

$$p_1 = \sigma_{pl} 2 \cos 2\gamma, \tag{6.3}$$

$$p_2 = \sigma_{pl}(2 \cos 2\gamma - 1). \tag{6.4}$$

The angle γ must satisfy the inequality $\pi/6 \le \gamma < \pi/4$. For the limiting value $\gamma = \pi/6$ we have $p_2 = 0$; this corresponds to the standard stress field of type E described in [5]. For $\gamma > \pi/6$ the stress p_2 becomes negative, but does not reach the yield limit

$$-\sigma_{pl} < p_2 \le 0. \tag{6.5}$$

More information on construction of piecewise-homogeneous stress fields may be found in [5].

The stresses $-\sigma_{pl}$ along CC' are in equilibrium with tractions distributed along HH'. The stresses along HI and $H'I'$ are equal $-\sigma_{pl}$, and they are transmitted downwards by means of two columns of rectangular cross-sections and dimensions $t \times w$ (Fig. 10), where g is the thickness of the element of Fig. 11. The stresses on II' attain the value p_2. They are equilibrated by the stress field of the type proposed by Winzer and Carrier (Fig. 4). The plane of this stress field is however rotated by 90° with respect to the plane of the field of Fig. 11. It is shown in the cross-section in Fig. 10 (according to notation of [5] marked by the letter D). So finally, the loads q distributed along II' are also transmitted to the column of rectangular cross-section but of the dimensions $h \times u$ (Fig. 10), where

$$h/g = -q/\sigma_{\text{pl}} . \tag{6.6}$$

Thus the connecting rod has in its middle part an I-shaped cross-section, which is advantageous from the point of view of its stability.

REFERENCES

1. L. DIETRICH, *Design of pin joints by plastic analysis methods in the light of fatigue tests* [in Polish], Engng. Trans., **25**, 3, 513–524, 1978.
2. L. DIETRICH and J. MIASTKOWSKI, *Limit load tests of pin joints* [in Polish], Engng. Trans., **18**, 4, 555–574, 1971.
3. W. SZCZEPIŃSKI, *Plastic design of machine parts* [in Polish], PWN, Warszawa 1968.
4. W. SZCZEPIŃSKI, *Limit analysis and plastic design of structural elements of complex shape*, [in:] Progress in Aerospace Sciences, D. KUCHEMANN [Ed.], Pergamon Press, **12**, pp. 1–47, 1972.
5. W. SZCZEPIŃSKI and J. SZLAGOWSKI, *Plastic design of complex shape structures*, PWN – Horwood, Warszawa 1990.
6. A. WINZER and G.F. CARRIER, *The interaction of discontinuity surfaces in plastic fields of stress*, J. Appl. Mech., **15**, 261–264, 1948.
7. W. ZOWCZAK, *Design and load carrying capacity of pin joint elements* [in Polish], Engng. Trans., **29**, 2, 267–278, 1981.
8. W. ZOWCZAK, *Design of structural elements by continuous statically admissible stress fields* [in Polish], J. Theor. Appl. Mech., **19**, 4, 563–573, 1981.
9. W. ZOWCZAK, *Design of yoke elements by the method of characteristics* [in Polish], Engng. Trans., **37**, 3, 565–574, 1989.
10. W. ZOWCZAK, *On a certain class of standard stress fields for plastic design*, Engng. Trans., **44**, 3–4, 483–497, 1996.

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Received November 28, 1997.