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# FREE VIBRATION OF THE SYSTEM OF TWO STRINGS COUPLED BY A VISCOELASTIC INTERLAYER

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This paper introduces an analytical method of solving the free vibration problem for a continuous system of two strings coupled by a viscoelastic interlayer. The phenomenon of free vibration has been described using a homogenous system of conjugate partial differential equations. After separation of variables in the system of differential equations, the boundary problem has been solved and two complex sequences have been obtained: the sequence of frequency, and the sequence of modes of free vibration. Then, the property of orthogonality of complex modes of free vibration has been demonstrated. Based on complex eigenfunctions, the polyharmonic free vibration has been expanded into the complex Fourier series, coefficients of which have been determined for arbitrarily assumed initial conditions.

## 1. Introduction

String systems coupled together by viscoelastic constraints play an important role in various engineering and building structures. Such structures are used in railway and tram tractions with live load [12, 13]. They can also be found in some ski lifts and cable car systems. Strings can work together with beams, slabs and membranes in various structures. A light roof structure of a sport arena is an example of mating of strings and membranes. Vibration analysis of complex structural systems with vibration damping poses a difficult problem. In the above complex cases, especially where viscosity and discrete elements occur, it is recommended that the method of solving the dynamic problem of a system in the domain of a real variable complex function is adopted [6, 14]. For the first time the property of orthogonality of free vibration complex modes for discrete systems with damping has been demonstrated in paper [6], and for discrete - continuous systems with damping – in paper [14]. Using a complex function, a description of free vibration of a beam supported on viscoelastic, continuous Winkler foundation [8] has been developed in papers [1, 2], and [3]. In paper [9], the problem of complex continuous system dynamics has been solved using the classical method [10] with the complete theory of non-damped vibration. In paper [1], the uniform method of solving a vibration problem for two beams with a viscoelastic interlayer, with manifold and boundary conditions different for both beams, has been developed.

The purpose of this paper is to conduct a mathematical analysis of a solution of the free vibration problem for continuous one- and two-dimensional structural systems with damping, with manifold boundary conditions and different initial conditions, in a set of complex functions. The mathematical analysis presented in the paper has been developed for a system of two strings with a viscoelastic interlayer. The analysis has then been verified.

## 2. Formulation of the problem

The physical model of the structural system consists of two homogenous, parallel strings of equal length, coupled together by a viscoelastic interlayer (Fig. 1). It has been assumed that strings are made of viscoelastic material, and their equation of state is assumed in accordance with the Voigt–Kelvin model [7, 11]. However, the viscoelastic interlayer possesses the characteristics of a homogenous, continuous, one-directional Winkler foundation, and is described by the Voigt–Kelvin model [5, 7, 11]. The mathematical model constitutes a system of the following conjugate partial differential equations, describing small transverse vibration of the physical model system:

$$
\begin{aligned}
&S_1\left(1+c_1\frac{\partial}{\partial t}\right)\frac{\partial^2 w_1}{\partial x^2}-\mu_1\frac{\partial^2 w_1}{\partial t^2}-k(w_1-w_2)-c\frac{\partial}{\partial t}(w_1-w_2)=0,\\
&S_2\left(1+c_2\frac{\partial}{\partial t}\right)\frac{\partial^2 w_2}{\partial x^2}-\mu_2\frac{\partial^2 w_2}{\partial t^2}+k(w_1-w_2)+c\frac{\partial}{\partial t}(w_1-w_2)=0,
\end{aligned}
\tag{2.1}
$$

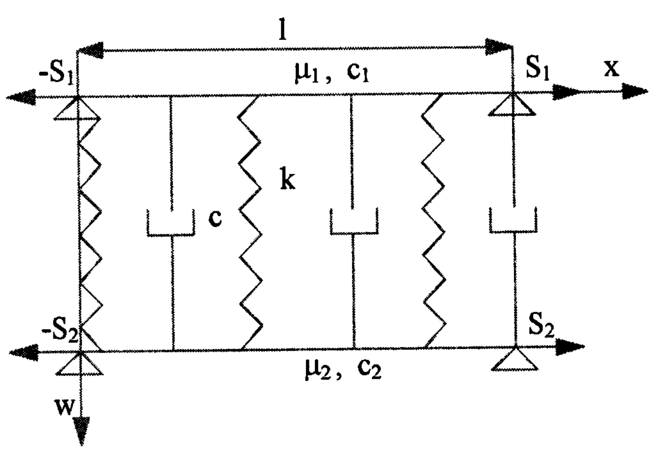


Fig. 1. Dynamical model of the system of two viscoelastic strings coupled by a viscoelastic interlayer.

where: $w_1 = w_1(x,t)$, $w_2 = w_2(x,t)$ – deflection of strings I and II, $S_1$, $S_2$ – tensile force of strings I and II, $\mu_1$, $\mu_2$ – mass of strings I and II per unit of length, $c_1$, $c_2$ – relative coefficient of viscosity in strings I and II, $c$ – coefficient of viscosity of the interlayer, $k$ – coefficient of elasticity of the interlayer, $l$ – length of strings I and II.

## 3. Solution of the boundary value problem

Substituting for $w_1$ and $w_2$ [6, 7, 14, 3] in the system of differential equations (2.1):

$$w_1 = W_1(x)\exp(i\nu t), \qquad w_2 = W_2(x)\exp(i\nu t), \tag{3.1}$$

the homogenous system of conjugate ordinary differential equations describing the complex modes of vibration of the strings is obtained:

$$\begin{aligned} &\frac{d^2W_1}{dx^2} + [S_1(1+ic_1\nu)]^{-1}\left[(\mu_1\nu^2 - k - ic\nu)W_1 + (k+ic\nu)W_2\right] = 0,\\ &\frac{d^2W_2}{dx^2} + [S_2(1+ic_2\nu)]^{-1}\left[(\mu_2\nu^2 - k - ic\nu)W_2 + (k+ic\nu)W_1\right] = 0, \end{aligned} \tag{3.2}$$

where: $W_1(x)$, $W_2(x)$ – the complex mode of vibration of strings I and II, $\nu$ – the complex frequency of vibration of the strings, $t$ – time.

Searching for a particular solution of the system of differential equations (3.2) in the form of [4]:

$$W_1 = Ae^{rx}, \qquad W_2 = Be^{rx}, \tag{3.3}$$

the homogenous system of linear algebraical equations is obtained:

$$\begin{aligned} &A\left[S_1(1+ic_1\nu)r^2 + \mu_1\nu^2 - k - ic\nu\right] + B\,[k+ic\nu] = 0,\\ &A\,[k+ic\nu] + B\left[S_2(1+ic_2\nu)r^2 + \mu_2\nu^2 - k - ic\nu\right] = 0. \end{aligned} \tag{3.4}$$

Expanding the determinant of the characteristic matrix of the system of equations (3.4) and equating it to zero, the characteristic equation in the form of the following biquadratic algebraical equation is obtained:

$$\begin{aligned} r^4 + \Big[(\mu_1\nu^2 - k - ic\nu)(S_1(1+ic_1\nu))^{-1} \\ + (\mu_2\nu^2 - k - ic\nu)(S_2(1+ic_2\nu))^{-1}\Big]r^2 \\ + \nu^2\left[\mu_1\mu_2\nu^2 - (\mu_1+\mu_2)(k+ic\nu)\right][S_1S_2(1+ic_1\nu)(1+ic_2\nu)]^{-1} = 0, \end{aligned} \tag{3.5}$$

with the following roots:

$$r_j = \pm i\lambda_\upsilon\,, \qquad j = 1,2,3,4, \qquad \upsilon = 1,2, \tag{3.6}$$

where

$$\lambda_\upsilon = \sqrt{0.5\left\{[(\mu_1\nu^2 - k - ic\nu)(S_1(1+ic_1\nu))^{-1} + (\mu_2\nu^2 - k - ic\nu)(S_2(1+ic_2\nu))^{-1}] \pm \sqrt{\Delta}\right\}}, \tag{3.7}$$

and

$$\Delta = \Big[(\mu_1\nu^2 - k - ic\nu)(S_1(1+ic_1\nu))^{-1} - (\mu_2\nu^2 - k - ic\nu)(S_2(1+ic_2\nu))^{-1}\Big]^2 + 4[k+ic\nu]^2\,[S_1S_2(1+ic_1\nu)(1+ic_2\nu)]^{-1} > 0 \tag{3.8}$$

is a discriminant of the biquadratic equation (3.5).

After applying the Euler formulas, the solution of the system of differential equations (3.2) consists of the fundamental system of solutions:

$$\begin{aligned} W_1(x) &= \sum_{\upsilon=1}^{2} A_\upsilon^* \sin\lambda_\upsilon x + A_\upsilon^{**}\cos\lambda_\upsilon x, \\ W_2(x) &= \sum_{\upsilon=1}^{2} B_\upsilon^* \sin\lambda_\upsilon x + B_\upsilon^{**}\cos\lambda_\upsilon x, \end{aligned} \tag{3.9}$$

where: $A_\upsilon^*$, $A_\upsilon^{**}$, $B_\upsilon^*$, $B_\upsilon^{**}$ – are constants.

In agreement with (3.4), there exist the following relations between constants of (3.9)

$$a_\upsilon = \frac{B_\upsilon^*}{A_\upsilon^*} = \frac{B_\upsilon^{**}}{A_\upsilon^{**}}\,, \tag{3.10}$$

where:

$$a_\upsilon = \frac{S_1(1+ic_1\nu)\lambda_\upsilon^2 - \mu_1\nu^2 + k + ic\nu}{k+ic\nu} = \frac{k+ic\nu}{S_2(1+ic_2\nu)\lambda_\upsilon^2 - \mu_2\nu^2 + k + ic\nu}\,. \tag{3.11}$$

After incorporating (3.10) in (3.9), the general solution of the system of differential equations (3.2) takes the following form:

$$\begin{aligned} W_1(x) &= \sum_{\upsilon=1}^{2} A_\upsilon^* \sin\lambda_\upsilon x + A_\upsilon^{**}\cos\lambda_\upsilon x, \\ W_2(x) &= \sum_{\upsilon=1}^{2} a_\upsilon(A_\upsilon^* \sin\lambda_\upsilon x + A_\upsilon^{**}\cos\lambda_\upsilon x). \end{aligned} \tag{3.12}$$

In order to solve the boundary-value problem, the following boundary conditions are applied:

$$W_1(0) = 0, \qquad W_1(l) = 0, \qquad W_2(0) = 0, \qquad W_2(l) = 0. \tag{3.13}$$

Substituting (3.12) in (3.13), the homogenous system of linear algebraical equations is obtained, which in the matrix notation has the following form:

$$\mathbf{Y}\,\mathbf{X} = 0, \tag{3.14}$$

where $\mathbf{X} = [A_1^*, A_1^{**}, A_2^*, A_2^{**}]^T$ is a vector of unknowns of the system of equations and

$$\mathbf{Y} = \left[Y_{i\times j}\right]_{4\times 4} \tag{3.15}$$

is the characteristic matrix of the system of equations (3.14) which is presented in Table 1.

**Table 1. Matrix of coefficients $Y_{i\times j}$.**

| $i$ \ $j$ | 1. | 2. | 3. | 4. |
|---|---|---|---|---|
| 1. | 0 | 0 | 1 | 1 |
| 2. | 0 | 0 | 1 | 1 |
| 3. | $\sin\lambda_1 l$ | $\sin\lambda_2 l$ | $\cos\lambda_1 l$ | $\cos\lambda_2 l$ |
| 4. | $a_1\sin\lambda_1 l$ | $a_2\sin\lambda_2 l$ | $a_1\cos\lambda_1 l$ | $a_2\cos\lambda_2 l$ |

The condition for solving the system of equations (3.14) is vanishing of the characteristic determinant, i.e.

$$\det \mathbf{Y} = 0. \tag{3.16}$$

The identity $A_2^* = A_2^{**}$ obtained from the first two equations of the system (3.14) leads to the reduction of the characteristic equations (3.16) to the following form:

$$\begin{vmatrix} \sin\lambda_1 l & \sin\lambda_2 l \\ a_1\sin\lambda_1 l & a_2\sin\lambda_2 l \end{vmatrix} = 0, \tag{3.17}$$

where

$$\lambda_1 = \alpha_1 + i\beta_1\,, \qquad \lambda_2 = \alpha_2 + i\beta_2 \tag{3.18}$$

in the general case are complex numbers.

Expanding the determinant (3.17), the following equation is obtained:

$$[\sin(\alpha_1 + i\beta_1)l \sin(\alpha_2 + i\beta_2)l] = 0, \tag{3.19}$$

which has the following roots:

$$\alpha_{1n} = \alpha_{2n} = \alpha_n = \frac{s\pi}{l}, \qquad \beta_{1n} = \beta_{2n} = \beta_n = 0, \qquad s = 1, 2, 3, \ldots \tag{3.20}$$

where

$$n = 2s - \delta_{n,(2s-1)}, \tag{3.21}$$

and $\delta_{n,(2s-1)}$ is the Kronecker number.

Substituting (3.20) in (3.18), the following identity is obtained:

$$\lambda_{1n} = \lambda_{2n} = \lambda_n = \alpha_n = \frac{s\pi}{l}. \tag{3.22}$$

Substituting for $r = i\lambda_n$ in the equation (3.5) and carrying out all the transformations, the following equation of frequency is obtained:

$$\begin{aligned} \nu^4 - \Big[(S_1(1+ic_1\nu)\lambda_n^2 + k + ic\nu)\mu_1^{-1} + (S_2(1+ic_2\nu)\lambda_n^2 + k + ic\nu)\mu_2^{-1}\Big]\nu^2 \\ + \lambda_n^2\Big[S_1S_2(1+ic_1\nu)(1+ic_2\nu)\lambda_n^2 \\ + (k+ic\nu)(S_1(1+ic_1\nu) + S_2(1+ic_2\nu))\Big](\mu_1\mu_2)^{-1} = 0, \end{aligned} \tag{3.23}$$

from which a sequence of complex natural frequencies is determined

$$\nu_n = i\eta_n \pm \omega_n. \tag{3.24}$$

Substituting equation (3.24) in equation (3.11), the following formulas for coefficients of amplitudes are obtained:

$$\begin{aligned} a_n &= \frac{S_1(1+ic_1\nu_n)\lambda_n^2 - \mu_1\nu_n^2 + k + ic\nu_n}{k + ic\nu_n} \\ &= \frac{k + ic\nu_n}{S_2(1+ic_2\nu_n)\lambda_n^2 - \mu_2\nu_n^2 + k + ic\nu_n}. \end{aligned} \tag{3.25}$$

Incorporating the sequences $\lambda_n$ and $a_n$ in (3.12), the two following sequences of modes of free vibration for two strings are obtained:

$$\begin{aligned} W_{1n}(x) &= \sin\lambda_n x, \\ W_{2n}(x) &= a_n \sin\lambda_n x. \end{aligned} \tag{3.26}$$

## 4. Solution of the initial value problem

The complex equation of motion

$$T = \Phi \exp(i\nu t), \tag{4.1}$$

in the case of $\nu = \nu_n$ can be written in the following form:

$$T_n = \Phi_n \exp(i\nu_n t), \tag{4.2}$$

where $\Phi_n$ – the Fourier coefficient.

Free vibration of strings is presented in the form of the Fourier series based on the complex eigenfunctions, i.e.:

$$\begin{aligned} w_{1n}(x,t) &= \sum_{n=1}^{\infty} W_{1n}(x)\Phi_n \exp(i\nu_n t), \\ w_{2n}(x,t) &= \sum_{n=1}^{\infty} W_{2n}(x)\Phi_n \exp(i\nu_n t). \end{aligned} \tag{4.3}$$

From the system of equations (3.2), after performing algebraical transformations, adding the equations together, and then integrating them on the sides in limits from 0 to $l$, the property of orthogonality of eigenfunctions (omitting damping in the strings) is obtained [1]

$$\int_0^l \Big[\xi_1(W_{1m}V_{1n} + W_{1n}V_{1m}) + \xi_2(W_{2m}V_{2n} + W_{2n}V_{2m}) + 2\eta(W_{1n} - W_{2n})(W_{1m} - W_{2m})\Big]dx = N\delta_{mn} \tag{4.4}$$

where: $\delta_{nm}$ – Kronecker delta,

$$N_n = 2\int_0^l \Big[\xi_1 W_{1n}V_{1n} + \xi_2 W_{2n}V_{2n} + \eta(W_{1n} - W_{2n})^2\Big]\,dx, \tag{4.5}$$

$$\begin{aligned} V_{1n}(x) &= i\nu_n W_{1n}(x), \qquad & V_{2n}(x) &= i\nu_n W_{2n}(x), \\ V_{1m}(x) &= i\nu_m W_{1m}(x), \qquad & V_{2m}(x) &= i\nu_m W_{2m}(x), \end{aligned} \tag{4.6}$$

$$\xi_1 = \frac{\mu_1}{\mu}, \qquad \xi_2 = \frac{\mu_2}{\mu}, \qquad \eta = \frac{c}{2\mu}.$$

The problem of free vibration of strings is solved by applying the following conditions:

$$\begin{aligned} w_1(x,0) &= w_{01}, \qquad & w_2(x,0) &= w_{02}, \\ \overset{o}{w}_1(x,0) &= \overset{o}{w}_{01}, \qquad & \overset{o}{w}_2(x,0) &= \overset{o}{w}_{02}. \end{aligned} \tag{4.7}$$

Applying conditions (4.7) in the series (4.3) and taking into consideration the property of orthogonality (4.4), the formula for the Fourier coefficient [1] is obtained

$$(4.8)\qquad \Phi_n = \frac{1}{N_n}\int_0^l \Big\{\xi_1(V_{1n}w_{01} + W_{1n}\overset{o}{w}_{01} + \xi_2(V_{2n}w_{02} + W_{2n}\overset{o}{w}_{02} + 2\eta\,[(W_{1n} - W_{2n})(w_{01} - w_{02})]\Big\}dx.$$

Substituting (3.26), (4.2) and (4.8) in (4.3) and performing trigonometrical and algebraical transformations, the final free vibration of strings is obtained:

$$(4.9)\qquad \begin{aligned} w_{1n} &= \sum_{n=1}^{\infty} e^{-\eta_n t}|\Phi_n||W_{1n}|\,[\cos(\omega_n t + \varphi_n + \psi_{1n}) + i\sin(\omega_n t + \varphi_n + \psi_{1n})]\,,\\ w_{2n} &= \sum_{n=1}^{\infty} e^{-\eta_n t}|\Phi_n||W_{2n}|\,[\cos(\omega_n t + \varphi_n + \psi_{2n}) + i\sin(\omega_n t + \varphi_n + \psi_{2n})]\,, \end{aligned}$$

where:

$$(4.10)\qquad \begin{aligned} |W_{1n}| &= \sqrt{X_{1n}^2 + Y_{1n}^2}\,, \qquad |W_{2n}| = \sqrt{X_{2n}^2 + Y_{2n}^2}\,,\\ |\Phi_n| &= \sqrt{C_n^2 + D_n^2}\,,\\ \psi_{1n} &= \arg W_{1n}, \qquad \psi_{2n} = \arg W_{2n}, \qquad \varphi_n = \arg\Phi_n, \end{aligned}$$

and

$$(4.11)\qquad \begin{aligned} X_{1n} &= \operatorname{re} W_{1n}\,, \qquad Y_{1n} = \operatorname{im} W_{1n}\,,\\ X_{2n} &= \operatorname{re} W_{2n}\,, \qquad Y_{2n} = \operatorname{im} W_{2n}\,,\\ C_n &= \operatorname{re}\Phi_n\,, \qquad D_n = \operatorname{im}\Phi_n\,. \end{aligned}$$

## 5. Numerical results

Computer calculations have been carried out for the following data: $S_1 = 50$ [N], $S_2 = 100$ [N], $\mu_1 = 10^{-2}$ [kg m$^{-1}$], $\mu_2 = 2\cdot 10^{-2}$ [kg m$^{-1}$], $k = 2\cdot 10^2$ [N m$^{-2}$], $l = 1$ [m], $c = 0.75$, $c_1 = c_2 = 0$.

In order to find the Fourier coefficient $\Phi_n$ (4.8), the following initial conditions have been assumed:

$$(5.1)\qquad w_{01} = A_s\sin\left(\frac{\pi x}{l}\right), \quad \overset{o}{w}_{01} = 0, \quad w_{02} = 0^-, \quad \overset{o}{w}_{02} = 0, \quad A_s = 0.02l.$$

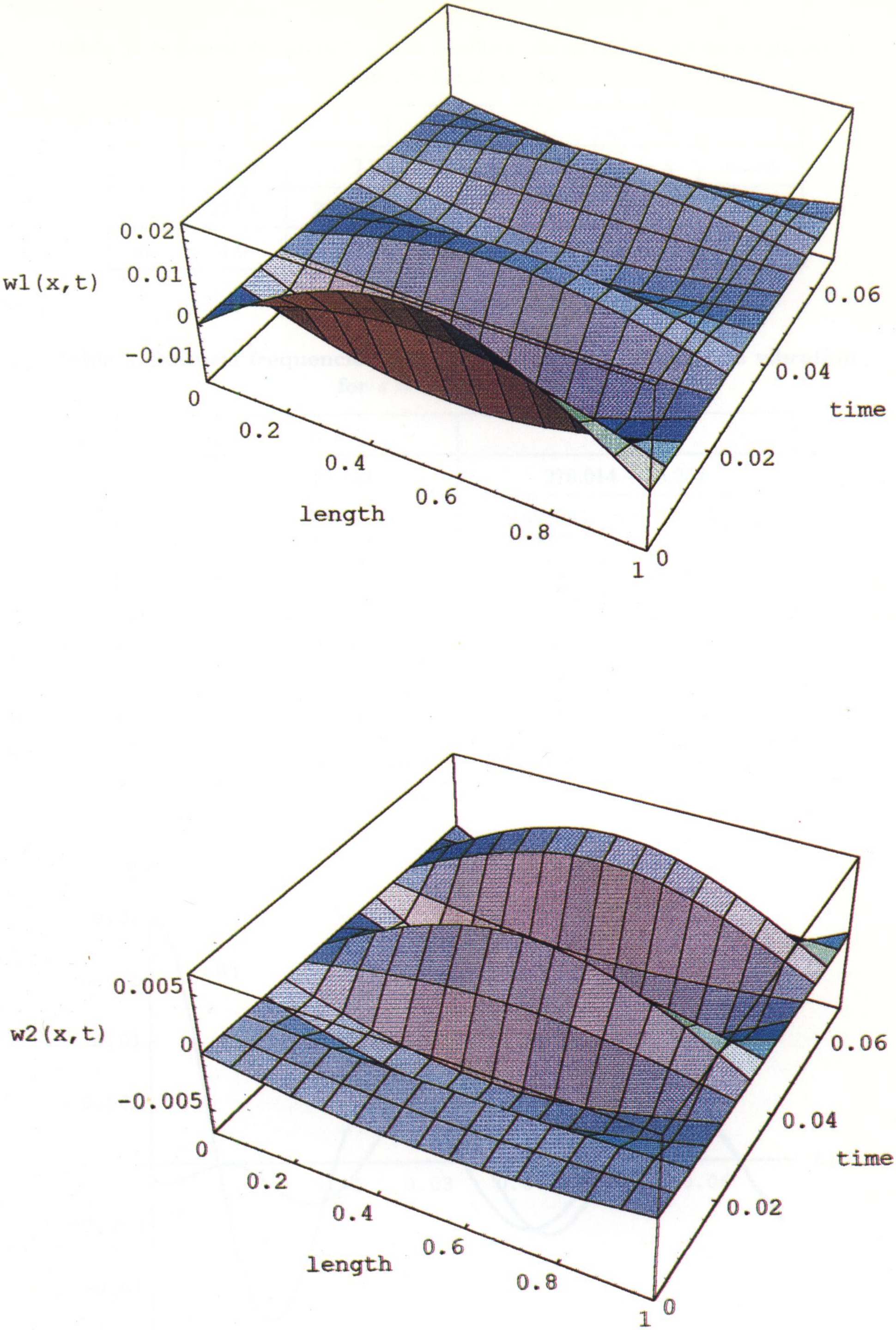


FIG. 2. Free vibration of the system of two strings coupled by a viscoelastic interlayer.

Table 2. Natural frequencies and coefficients of modes of free vibration for $s = 1, 2, 3,\ c = 0$.

| | $s = 1$ | | $s = 2$ | | $s = 3$ | |
|---|---|---|---|---|---|---|
| | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ |
| $\omega_n$ | 222.1 | 281.7 | 444.3 | 476.9 | 666.4 | 688.6 |
| $a_n$ | 1.0 | −0.5 | 1.0 | −0.5 | 1.0 | −0.5 |

Table 3. Natural frequencies and coefficients of modes of free vibration for $s = 1, 2, 3,\ c = 0.75$.

| | | |
|---|---|---|
| $s = 1$ | $n = 1$ | $n = 2$ |
| $\nu_n$ | 222.144 | $276.014 + 56.25I$ |
| $a_n$ | $1 - 1.60488 \cdot 10^{-6} I$ | $-0.499995 - 7.06682 \cdot 10^{-6} I$ |
| $s = 2$ | $n = 3$ | $n = 4$ |
| $\nu_n$ | 444.288 | $473.527 + 56.25I$ |
| $a_n$ | $1 - 5.76002 \cdot 10^{-6} I$ | $-0.499998 - 4.8391 \cdot 10^{-6} I$ |
| $s = 3$ | $n = 5$ | $n = 6$ |
| $\nu_n$ | 666.432 | $686.271 + 56.25I$ |
| $a_n$ | $1 - 0.000010130611$ | $-0.499999 - 4.44171 + 10^{-6} I$ |

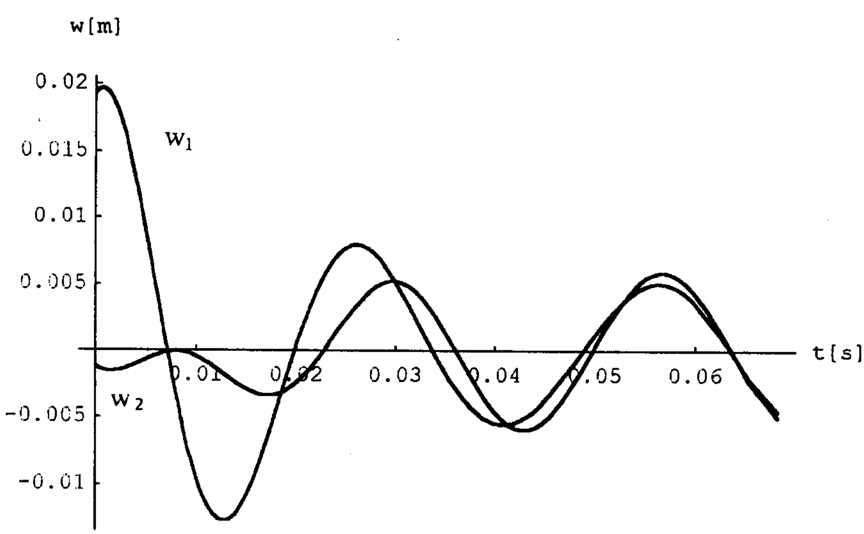


FIG. 3. Free vibration of the strings for $x = 0.5\,l$.

Table 2 contains the natural frequencies and coefficients of modes of free vibration for the case where $c = 0$. Table 3 contains the results for systems with damping, for $s = 1, 2, 3, \ldots$ . The solution of the problem of free vibration (4.9) is the real part. The space diagrams of $w(x, t)$ in Fig. 2 show the real part Re of free vibration. Figure 3 shows dynamic displacements of the strings for $x = 0.5l$.

## 6. Conclusions

1. A uniform method of solving free vibration problems for complex, continuous, one-dimensional systems with damping, for various boundary conditions and different initial conditions has been presented in this paper.

2. Verification of this method has been carried out for a system of two strings in the case where no damping occurs (Tab. 2). The results shown in Tab. 3 for natural frequencies and coefficients of amplitude, solved by means of the method presented in this paper agree with the results obtained using the classical method [9].

3. The notion of complex modes of free vibration introduced in this paper, forms the basis for solving the problems of free vibration of continuous, one- and two-dimensional structural systems with damping.

4. The method presented in this paper can be applied to solutions of free vibration of different engineering structures consisting of strings, beams and slabs coupled together by viscoelastic constraints.

## References

1. K. Cabańska–Płaczkiewicz, *Dynamics of the system of two Bernoulli–Euler's beams with a viscoelastic interlayer*, XXXVIIth Symposium of Model. in Mech., **7**, Silesian Univ. of Tech., 49–54, Gliwice 1998.

2. K. Cabańska-Płaczkiewicz, *Free vibration of the system of string-beam with a viscoelastic interlayer*, Theor. Found. in Civil Engrg., **6**, Faculty of Civil Engrg. Warsaw Univ. of Tech., 59–68, Warsaw 1998.

3. K. Cabańska-Płaczkiewicz, *Free vibration of the system of two visco-elastic beams coupled by a viscoelastic interlayer*, Qualifying Journal, Kiev 1998 (in rev.).

4. R. Gutowski, *The ordinary differential equations*, WNT, Warsaw 1971.

5. S. Kasprzyk, *Dynamics of the continuous system*, Publ. of AHG, Kraków 1989.

6. J. Niziol, J. Snamina, *Free vibration of the discrete-continuous system with damping*, J. Theor. and Appl. Mech., **28**, 1–2, 149–160, Warsaw 1990.

7. W. Nowacki, *The building dynamics*, Arkady, Warsaw 1972.

8. W. Nowacki, *The building mechanics*, Arkady, Warsaw 1976.

9. Z. Oniszczuk, *Vibration analysis of the compound continuous systems with elastic constraints*, Publ. of the Rzeszów Univ. of Tech., Rzeszów 1997.

10. Z. Osiński, *The theory of vibration*, PWN, Warsaw 1978.

11. Z. Osiński, *Damping of the mechanical vibration*, PWN, Warsaw 1979.

12. W. Szcześniak, *The selection of problems of beams and shells subjecting the inertial moving load*, Building Engineering, **125**, Publ. of the Warsaw Univ. of Tech., Warsaw 1994.

13. W. Szcześniak, *The selection of railway problems*, Building Engineering, **129**, Publ. of the Warsaw Univ. of Tech., Warsaw 1995.

14. F. Tse, I. Morse, R. Hinkle, *Mechanical vibrations theory and applications*, Allyn and Bacon, Boston 1978.

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