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IMPROVED REPRESENTATION FOR THE FIRST PRECURSOR IN THE LORENTZ MEDIUM

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The best methods available nowadays for modelling the propagation of very short (duration less than 100 fs), ultrawideband electromagnetic signals in physical media are the asymptotic ones. Numerical methods loose their traditional leadership due to their inability to handle rapid oscillations of the propagating signal. Therefore it is important to create accurate asymptotic models of propagation which can be used as a reference. In this paper a new description of primary precursor in a dispersive Lorentz medium is given, based on uniform asymptotic theory of evaluation of integrals and a new approximate solution to the distant saddle point equation. The new representation of the signal in the medium is illustrated graphically and compared with the Oughstun-Sherman representation.

1. Introduction

When a static electric field E is applied to a medium, it affects electric charges associated with microscopic particles constituting the medium. First, the external field modifies the charge distribution in every molecule and leads to the creation of microscopic dipole moments in the medium. Second, if particles have constant dipole moments, the applied field tends to reorder their initial distribution. On spatial averaging of the dipole moments over microscopically large, and macroscopically small volumes, the polarization P is obtained, which is an averaged dipole moment per unit volume. Assuming that the external field E is not very large, the medium is isotropic and ferroelectrics are excluded from the consideration, the relation between P and E is linear, i.e. $P = \chi E$, where χ is the medium susceptibility. The dielectric displacement D, defined as $D = E + 4\pi P$, is also linearly dependent on E, i.e. $D = \epsilon E$, with $\epsilon = 1 + 4\pi\chi$ being the medium dielectric permittivity. In most media ϵ is greater than 1. One important consequence of the presence of the medium is that, in accordance with the equation $\nabla \cdot E = 4\pi\rho/\epsilon$, the same charge distribution ρ excites the electric field smaller in the medium as compared to the vacuum by the factor $1/\epsilon$.

If the external electric field is not constant, but is rapidly varying in time, no such simple relation between P and E, and consequently between D and E, exists. The reason is that the motion of molecules and ions lags behind the variations of the electromagnetic field. As a result, $D(t)$ and $E(t)$ are related by $D(t) = \epsilon \int\limits_{-\infty}^{t} E(t') f_D(t - t')\, dt'$, with suitable pulse-response function $f_D(t)$ [1]. This means that both fields are not local in time and the former field depends on the whole history of the latter field. This feature is characteristic of *temporal dispersion.* If the external electric field varies harmonically in time with the circular frequency ω, then the integral relation simplifies to $\hat{D}(t) = \hat{\epsilon}(\omega)\hat{E}(t)$, where $\hat{\epsilon}(\omega) = \epsilon \mathcal{L}_{i\omega}(f_D)$ and $\mathcal{L}_{i\omega}(f_D)$ is the Laplace transform of f_D. If ω is sufficiently low, it is customarily assumed that $\hat{\epsilon}(\omega) = \epsilon$.

Evolution of an electromagnetic signal propagating in a Lorentz medium that demonstrates both dispersive and absorptive properties have been analyzed in detail in now classical works of SOMMERFELD [2] and BRILLOUIN [3]. Their results were later collected in the monograph [4]. By examining the integral representation of a pulsed signal in the medium, Sommerfeld revealed that the front of the signal cannot propagate with a velocity exceeding the light velocity c in vacuum. Next, Brillouin, equipped with now standard asymptotic techniques, showed that after travelling a sufficient distance in the medium, the signal splits into parts significantly differing in their properties. Those parts are the first and second precursors, and the main signal. The behaviour of each precursor is closely related to the location of corresponding saddle points in the integral describing the signal dynamics in the medium. It was shown that to each precursor there corresponds a pair of saddle points in the complex frequency plane, and that they are symmetric with respect to the frequency imaginary axis. Location of these points specifies the local frequency and attenuation of the precursor at a given time instant and a space coordinate. The relation between these coordinates and location of the corresponding saddle points is governed by the saddle point equation (see [5]). No exact, closed-form solution to this equation is known; instead, approximate solutions were found. Therefore, analytic formulas known in the literature describe the local behavior of a precursor only in approximate manner.

Brillouin's description of those signal components is relatively simple and thus suitable for physical interpretation. Unfortunately, it breaks down at certain values of the space-time coordinate, where the asymptotic apparatus used is inadequate to describe properly the actual behaviour of the field. This defect was removed by OUGHSTUN and SHERMAN [5 – 11]. With the help of contemporary asymptotic methods they obtained the signal description that is valid for any value of the space-time coordinate. The form of their description is much more complicated than that Brillouin's one, and thus is not convenient for physical interpretation of the result.

The signal components mentioned above are significantly different. In particular, the first precursor rapidly oscillates, while the second precursor is a slowly varying field. Accordingly, their analysis requires different analytical techniques, and their resulting mathematical description is also different. Therefore they can be studied as independent field entities.

In non-stationary field propagation a fundamental issue is the velocity at which the field propagates in the medium. In lossless media the group velocity $v^g = [dk(\omega)/d\omega]^{-1}$ is a suitable quantity describing the velocity of a wave-packet amplitude. This concept is no longer valid in a region of anomalous dispersion, where v^g can be negative, zero or infinite [10]. More generally, in a medium exhibiting absorption, i.e. when the wave number $k(\omega)$ takes complex values for real ω, the concept of the group velocity breaks down [12]. Here, a fundamental difficulty arises because lossless, dispersive media are non-casual, and thus unphysical [13]. In order to extend the possibility of physical interpretation of signal propagation in lossy, dispersive media, OUGHSTUN and SHERMAN [10] approximated the actual signal components by waves with suitably chosen real or imaginary frequencies and the same rate of attenuation. The velocity of propagation of those waves was then determined from $v^E(\omega) = S(\omega)/u(\omega)$, where $S(\omega)$ is time-averaged magnitude of the Poynting vector, and $u(\omega)$ is the time-averaged energy density.

In this work we reconsider the evolution of the first precursor as it propagates in the dispersive Lorentz medium. In [15] a new approximate solution to the saddle point equation was obtained, which relates saddle point location in the complex frequency plane and the space-time coordinate. For Brillouin choice of the medium parameters, the approximation appeared to be very satisfactory in a wide range of this coordinate. Here, we employ the result obtained in [15] to obtain better approximation of the first precursor generated by initial tangent hyperbolic modulated signal, than it would follow from using the results known so far in the literature. At the onset of the precursor, where the above approximation is less accurate, generalized Brillouin approximate solution to the saddle point equation is used instead. Both approximations are combined into one formula, describing a smooth function of the space-time parameter. Bleistein-Handelsman asymptotic technique rather than standard asymptotic non-uniform approach is applied to handle properly the onset of the precursor.

2. Integral representation of the signal

Assume that the electromagnetic signal propagation occurs in a linear, homogeneous and isotropic medium whose dispersive properties are described by the Lorentz model of resonance polarization. The complex index of refraction in the medium is given by the following, frequency-dependent function

$$n(\omega) = \left(1 - \frac{b^2}{\omega^2 - \omega_0^2 + 2i\delta\omega}\right)^{1/2}, \tag{2.1}$$

where $b^2 = 4\pi N e^2/m$, N, e and m standing, respectively, for the number of electrons per unit volume, electron charge and its mass, δ is a damping constant and ω_0 is a characteristic frequency.

Any electromagnetic field in the medium satisfies the Maxwell equations

$$\nabla \times E(\mathbf{r},t) - \frac{1}{c}\frac{\partial H(\mathbf{r},t)}{\partial t} = 0, \qquad \nabla \times H(\mathbf{r},t) - \frac{1}{c}\frac{\partial E(\mathbf{r},t)}{\partial t} = 0,$$

$$D(\mathbf{r},t) = \int_{-\infty}^{t} \tilde{\epsilon}(t-\tau)E(\mathbf{r},\tau)d\tau, \qquad B(\mathbf{r},t) = \mu H(\mathbf{r},t),$$

where $\tilde{\epsilon}(t)$ is a real function and μ is a real constant (hereafter assumed to be equal 1). By Fourier transforming the equations with respect to t and assuming that the fields depend on one spatial coordinate z only, we obtain the following equations for transforms of the respected fields:

$$\hat{\mathbf{z}} \times \mathcal{H}(z,\omega) = -\frac{i\omega\epsilon(\omega)}{c}\mathcal{E}(z,\omega), \qquad \hat{\mathbf{z}} \times \mathcal{E}(z,\omega) = \frac{i\omega\mu}{c}\mathcal{H}(z,\omega),$$

where $\hat{\mathbf{z}}$ is the unit vector directed along z-axis and $\epsilon(\omega) = n^2(\omega)/(c^2\mu)$ is the Fourier transform of $\breve{\epsilon}(t)$. It then follows that $\hat{\mathbf{z}}$, $\mathcal{E}$ and $\mathcal{H}$ are mutually perpendicular. Moreover, if $\mathcal{E}$ is known then $\mathcal{H}$ is also known, and *vice versa*. It is also true for the electromagnetic field components, which are the inverse Fourier transforms of $\mathcal{E}$ and $\mathcal{H}$. Therefore, the knowledge of the electric (magnetic) field is sufficient to determine the full electromagnetic field. To make the calculations as simple as possible, it is advisable that the x (or y) axis be directed to coincide with the electric or magnetic field.

Suppose that the selected Cartesian component (say x-component) of one of these fields in the plane $z = 0$ is given by

$$A(0,t) = \begin{cases} 0 & t < 0 \\ u(t)\sin(\omega_c t) & t \geq 0, \end{cases} \tag{2.2}$$

where ω_c is a fixed carrier frequency. Assume also that $A(z,0) = 0$ for $z > 0$. In the theory of partial differential equations the problem analyzed here is referred to as a mixed problem.

We specify the function $u(t)$ to be described by a hyperbolic tangent function, i.e.

$$u_\beta(t) = \begin{cases} 0 & t < 0 \\ \operatorname{th}\beta t & t \geq 0. \end{cases} \tag{2.3}$$

With this specification the rate of impulse growth can be controlled with proper selection of the parameter $\beta \geq 0$.

It was shown in [14] that continuation of the signal (2.2) to arbitrary $z > 0$ is given by

$$A(z,t) = \frac{1}{2\pi}\operatorname{Re}\left\{ i \int\limits_{ia-\infty}^{ia+\infty} \left[\frac{1}{\beta}\mathcal{B}\left(-\frac{i(\omega-\omega_c)}{2\beta}\right) - \frac{i}{\omega-\omega_c} \right] e^{\frac{z}{c}\phi(\omega,\theta)}\, d\omega \right\}. \tag{2.4}$$

Here, the complex phase function $\phi(\omega,\theta)$ is defined by

$$\phi(\omega,\theta) = i\,\frac{c}{z}\,[\tilde{k}(\omega)z - \omega t] = i\omega[n(\omega) - \theta], \tag{2.5}$$

$\mathcal{B}(x)$ is the beta function defined by the psi function as

$$\mathcal{B}(x) = \frac{1}{2}\left[\psi\left(\frac{x+1}{2}\right) - \psi\left(\frac{x}{2}\right)\right], \tag{2.6}$$

and

$$\theta = \frac{ct}{z} \tag{2.7}$$

is a dimensionless parameter that characterizes a space-time point (z, t) in the field. The contour C is the line $\omega = \omega' + ia$, a being a constant greater than the abscissa of absolute convergence for the function in square brackets in (2.4), and ω' ranges from negative to positive infinity.

The integral (2.4) represents the exact solution to the Maxwell equations in the dispersive medium with initial-boundary condition given by (2.21). It provides no direct information on the physical structure of the signal nor its form as it propagates in the dispersive medium. One of possible approaches to study the dynamics of the signal is to evaluate the integral numerically. In the case at hand such an approach is not effective, owing to rapid oscillations of the integrand. Instead, a different approach is used, which is based on methods of asymptotic evaluation of integrals.

In his monograph [4] BRILLOUIN applied the method of steepest descents to evaluate asymptotically the integral (2.4) for large values of the phase function in the integrand. He revealed three components into which the propagating signal deforms: the first precursor, the second precursor and the main signal. The precursors are contributions of two pairs of saddle points in the complex frequency plane, symmetrically located with respect to the frequency imaginary axis. As the front of the signal departs from the plane $z = 0$, the saddle points change their location in the frequency plane. The equation describing the location of both pairs of saddle points follows from the requirement that the phase function (2.5) is stationary at those points. Hence this equation can be formulated as [4]

$$n(\omega) + \omega n'(\omega) - \theta = 0, \tag{2.8}$$

or, when $n(\omega)$ is eliminated from the above equation, as [5]

$$\left[\omega^2 - \omega_1^2 + 2i\delta\omega + \frac{b^2\omega(\omega + i\delta)}{\omega^2 - \omega_0^2 + 2i\delta\omega}\right]^2 = \theta^2(\omega^2 - \omega_1^2 + 2i\delta\omega)(\omega^2 - \omega_0^2 + 2i\delta\omega), \tag{2.9}$$

where $\omega_1^2 = \omega_0^2 + b^2$. By solving approximately the Eq. (2.8) and applying the steepest descents method, Brillouin revealed the forms of both precursors in the medium.

In this paper we confine our interest to the first precursor only. Brillouin's solution corresponding to this precursor suffers from two shortcomings. First, his approximate solution of the saddle point Eq. (2.8) is valid only for $\theta \approx 1$, i.e. for frequencies tending to infinity. Second, it is at infinity that the saddle points change their order to infinite one, in which case the saddle point method becomes invalid.

In their recent works Oughstun and Sherman improved Brillouin's results. They found a different approximation to the saddle point Eq. (2.8) which provides good accuracy in the whole interval of θ variation. They also employed modern asymptotic techniques, that enable asymptotic evaluation of integrals with saddle points of changing order. Comparison of Brillouin's and Oughstun-Sherman's results can be found in [5, 11] (see also [15]).

In [15] we offered an approximate solution to the saddle point equation which is different from both Brillouin's and Oughstun-Sherman's results. For Brillouin's choice of medium parameters our approximation appeared to be superior over those results in a wide range of θ variation, except for very low values of θ, i.e. for $\theta \approx 1.3$ and below. These low values are important if the front of the first precursor is examined. For this reason, for small values of θ we apply a different approximate solution to the saddle point equation, which can be viewed as a generalized Brillouin's approximation. We combine both approximations into one formula which is described by a smooth function of θ varying in the range $[1, \infty)$. This hybrid approximation is constructed in the following section.

3. Approximate solution to the saddle point equation

In [15] a new approximate formula was found that describes the location of saddle points of the integral (2.4) in the complex frequency plane as a function of the space-time parameter θ. It applies to the case of Lorentz medium wherein the index of refraction is given by (2.1). For the distant saddle points the approximation reads

(3.1)
$$\begin{aligned} \omega^{+}(\theta) &= -i\delta + \sqrt{\omega_0^2 - \delta^2 - \frac{b^2}{n^2 - 1}}, \\ \omega^{-}(\theta) &= -(\omega^{+})^{*} = -i\delta - \sqrt{\omega_0^2 - \delta^2 - \frac{b^2}{(n^{*})^2 - 1}}. \end{aligned}$$

where n is given by

(3.2)
$$n = g - \sqrt{\frac{1}{4a}\left(\frac{\theta}{g} - 2c\right) - g^2},$$

with

(3.3)
$$g = \frac{1}{2\sqrt{3a}}\sqrt{\frac{2^{-1/3}}{(u + \sqrt{u^2 - v^3})^{1/3}}\left[(u + \sqrt{u^2 - v^3})^{2/3} + v\right] - 2c},$$

and

(3.4)
$$u = 2c^3 - 72ace + 27a\theta^2,$$

(3.5)
$$v = 2^{2/3}(c^2 - 12ae).$$

The constant coefficients are given by

$$(3.6) \qquad a = \left[\omega_0^2 - \delta^2 + \frac{i\delta b^2}{\sqrt{\omega_1^2 - \delta^2}} - i\delta\sqrt{\omega_1^2 - \delta^2} - \frac{i\delta b^2[3b^2 + 4(\omega_0^2 - \delta^2)]}{8(\omega_1^2 - \delta^2)^{3/2}}\right]\frac{1}{b^2},$$

$$(3.7) \qquad c = -\frac{i\delta}{2\sqrt{\omega_1^2 - \delta^2}} - \frac{2}{b^2}\left(\omega_0^2 - \delta^2 - i\delta\sqrt{\omega_1^2 - \delta^2}\right),$$

and

$$(3.8) \qquad e = \frac{\omega_1^2 - \delta^2 - i\delta\sqrt{\omega_1^2 - \delta^2}}{b^2}.$$

The expression on the right-hand side of (3.2) is an approximation of the index of refraction $n(\omega)$ as given by (2.1).

The formulas (3.1) through (3.8) yield good approximation for the location of the distant saddle points for $\theta > 1.3$. For smaller values of θ we can find an approximation which is a generalization of Brillouin's result. On expanding the left-hand side of the Eq. (2.4) into negative powers of ω we obtain

$$(3.9) \qquad 1 + \frac{b^2}{2\omega^2} - \frac{2ib^2\delta}{\omega^3} + \frac{3}{8\omega^4}b^2[b^2 + 4(\omega_0^2 - 4\delta^2)] + O\left(\frac{1}{\omega^5}\right) = \theta.$$

Here, the expansion is terminated at the term $O(\omega^{-4})$; with larger number of terms included the accuracy of the approximation deteriorates at $\theta \approx 1.1$ and higher (still Brillouin's choice of the media parameters is assumed).

Inverting of this series leads to the formula

$$(3.10) \qquad \omega_B(\theta) = -2i\delta + \frac{b^2(1+3\theta) + 12\omega_0^2(\theta - 1)}{4\sqrt{2}b\sqrt{\theta - 1}} + O(\theta - 1),$$

which describes the location of the saddle point in the right ω half-plane. Corresponding location of the saddle point in the left half-plane is given by $-\omega^*$. It is seen that for $\theta \approx 1$ this formula passes into Brillouin's approximation

$$(3.11) \qquad \omega(\theta) = -2i\delta + \frac{b}{\sqrt{2(\theta - 1)}}.$$

Now we combine the approximations given by (3.1) and (3.10) into one formula:

$$\omega_{SD}(\theta) = \omega^{+}(\theta)\left[H(\theta-1.08) - \frac{\operatorname{sign}(\theta-1.08)}{2}\frac{\rho(\theta-1.08)}{\rho(0)}\right] + \omega_B(\theta)\left[H(1.08-\theta) - \frac{\operatorname{sign}(1.08-\theta)}{2}\frac{\rho(1.08-\theta)}{\rho(0)}\right], \tag{3.12}$$

where $H(x)$ is a unit-step function,

$$\rho(x) = g(x+0.29)g(x-0.29), \tag{3.13}$$

and

$$g(x) = \begin{cases} e^{-\dfrac{1}{x^2}}, & x > 0, \\ 0, & x \le 0. \end{cases} \tag{3.14}$$

The factors in square brackets provide smooth transition between two approximations. The numerical parameters were optimized for best accuracy of the resulting approximation. Graphical presentation of this approximation is given in Fig. 1.

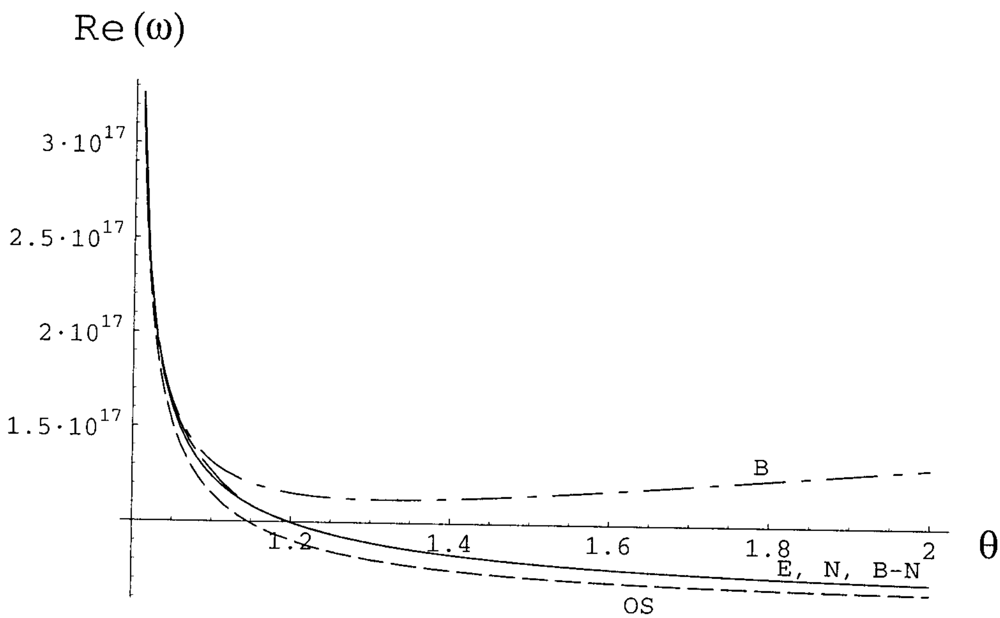

FIG. 1. a) Re(ω) versus θ for small values of θ. E — exact values, B — Brillouin's approximation, OS — Oughstun-Sherman's approximation, N — approximation based on Eq. (3.1), B-N — combined approximation based on Eq. (3.12).

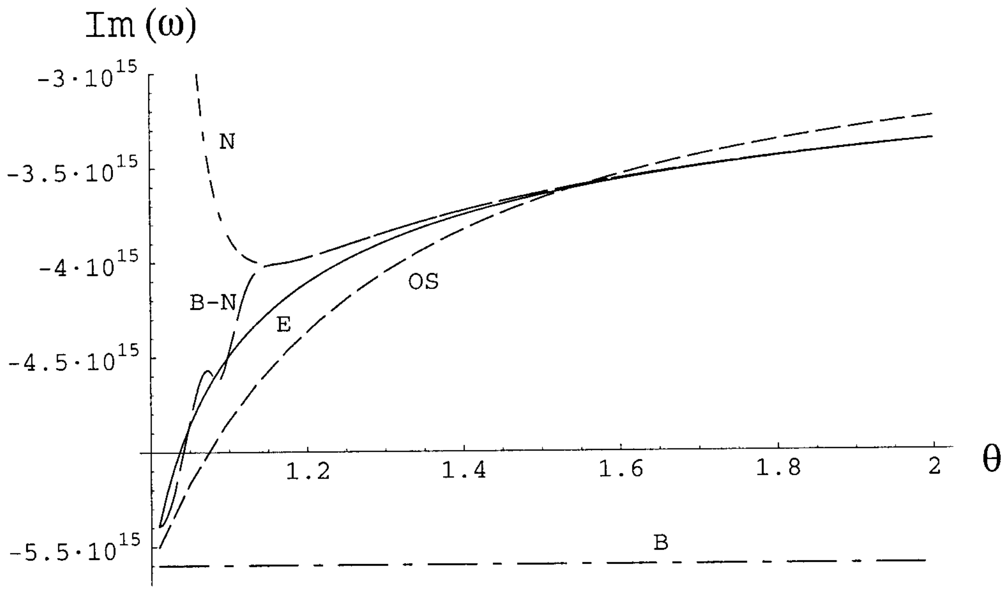

FIG. 1. b) Im(ω) versus θ for small values of θ. The legend as in Fig. 1. b).

4. Asymptotic representation for the first precursor

For $|\omega|$ sufficiently large the function $n(\omega)$ can be expanded in a convergent Laurent series and, consequently, the function $\psi(\omega,\theta) = \phi(\omega,\theta)/(iz/c)$ can be represented by the series

$$\psi(\omega,\theta) = -[\omega(\theta-1) + \sum_{n=0}^{\infty} a_n \omega^{-n}, \tag{4.1}$$

with properly determined a_n.

From this representation it is easily seen that as $\theta \to 1^+$, not only the first derivative of ψ vanishes for $\omega \to \infty$, but also all higher derivatives do. Moreover, then both distant saddle points tend to infinity. We can conclude that as $\theta \to 1^+$ the distant saddle points meet at infinity and at the same time their order changes from 1 to infinity. In these circumstances the method of steepest descents is invalid and more advanced asymptotic technique must be used to construct the asymptotic representation for the first precursor. A suitable approach was proposed by BLEISTEIN and HANDELSMAN [16, 17], see also [11]. By applying their method to the integral (2.4) we find the following uniform asymptotic representation for the first precursor

$$A_S(\omega,\theta) = c_0 I_0 + c_1 I_1, \tag{4.2}$$

valid for large values of the exponent in the integrand in (2.4). Here,

$$I_j(\lambda,\theta) = -e^{i\lambda\rho} i(2\gamma e^{-i\frac{\pi}{2}})^{1+j} J_{1+j}(\lambda\gamma), \quad j=0,1, \tag{4.3}$$

$J_m(x)$ being the Bessel function of m-th order,

$$\lambda = \frac{z}{c}, \quad \gamma(\theta) = -\mathrm{Re}\{\psi[\omega(\theta),\theta]\} \quad \rho(\theta) = -i\,\mathrm{Im}\{\psi[\omega(\theta),\theta]\}, \tag{4.4}$$

$$c_0 = \frac{1}{4\gamma^2}\left\{\sqrt{\frac{-4\gamma^3}{\psi_{\omega\omega}[\omega(\theta),\theta]}}\left[\frac{i}{\beta}\mathcal{B}\left(-i\frac{\omega(\theta)-\omega_c}{2\beta}\right)+\frac{1}{\omega(\theta)-\omega_c}\right]\right.$$
$$\left.-\left(\sqrt{\frac{-4\gamma^3}{\psi_{\omega\omega}[\omega(\theta),\theta]}}\left[\frac{i}{\beta}\mathcal{B}\left(-i\frac{\omega(\theta)+\omega_c}{2\beta}\right)+\frac{1}{\omega(\theta)+\omega_c}\right]\right)^*\right\}, \tag{4.5}$$

$$c_1 = \frac{1}{16\gamma^3}\left\{\sqrt{\frac{-4\gamma^3}{\psi_{\omega\omega}[\omega(\theta),\theta]}}\left[\frac{i}{\beta}\mathcal{B}\left(-i\frac{\omega(\theta)-\omega_c}{2\beta}\right)+\frac{1}{\omega(\theta)-\omega_c}\right]\right.$$
$$\left.+\left(\sqrt{\frac{-4\gamma^3}{\psi_{\omega\omega}[\omega(\theta),\theta]}}\left[\frac{i}{\beta}\mathcal{B}\left(-i\frac{\omega(\theta)+\omega_c}{2\beta}\right)+\frac{1}{\omega(\theta)+\omega_c}\right]\right)^*\right\}. \tag{4.6}$$

As before $\omega(\theta)$ corresponds to the location of the distant saddle point in the right ω half-plane, corresponding to a particular value of θ. The star stands for complex conjugate.

5. Numerical results

As shown in [15], for Brillouin choice of medium parameters the formulas (3.1) yield very good approximation for the location of the distant saddle points in the complex ω-plane in a wide range of θ. This approximation, however, fails to hold for values of θ close to 1. Figure 1 depicts a comparision of various approximations for small values of θ.

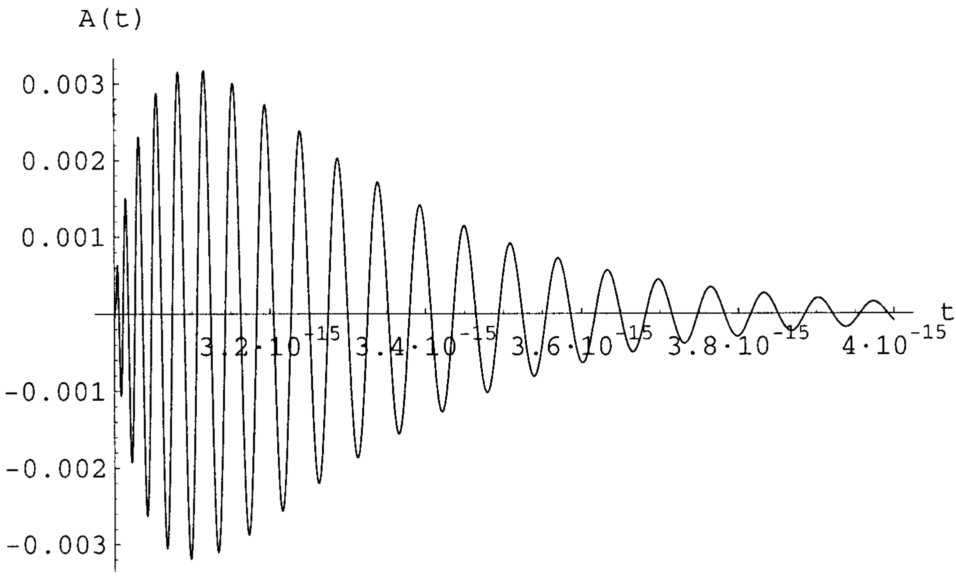

FIG. 2. a) Asymptotic description of the dynamic evolution of the first precursor corresponding to the hyperbolic tangent signal in the Lorentz medium. $\omega_c = 2.0 \times 10^{16}$ Hz, $\lambda = 0.3 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{16}$ s^{-1}.

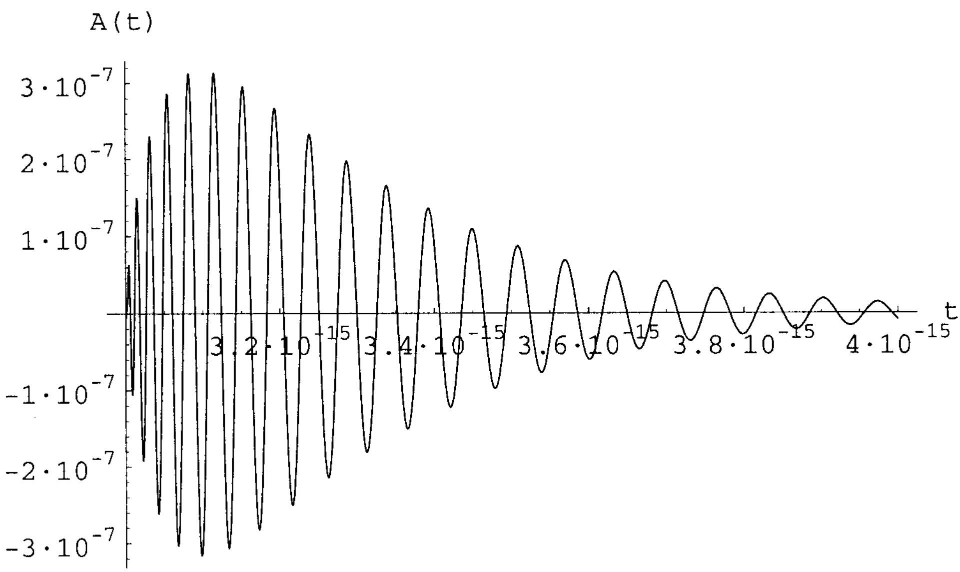

FIG. 2. b) Asymptotic description of the dynamic evolution of the first precursor corresponding to the hyperbolic tangent signal in the Lorentz medium. $\omega_c = 2.0 \times 10^{16}$ Hz, $\lambda = 0.3 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{12}$ s^{-1}.

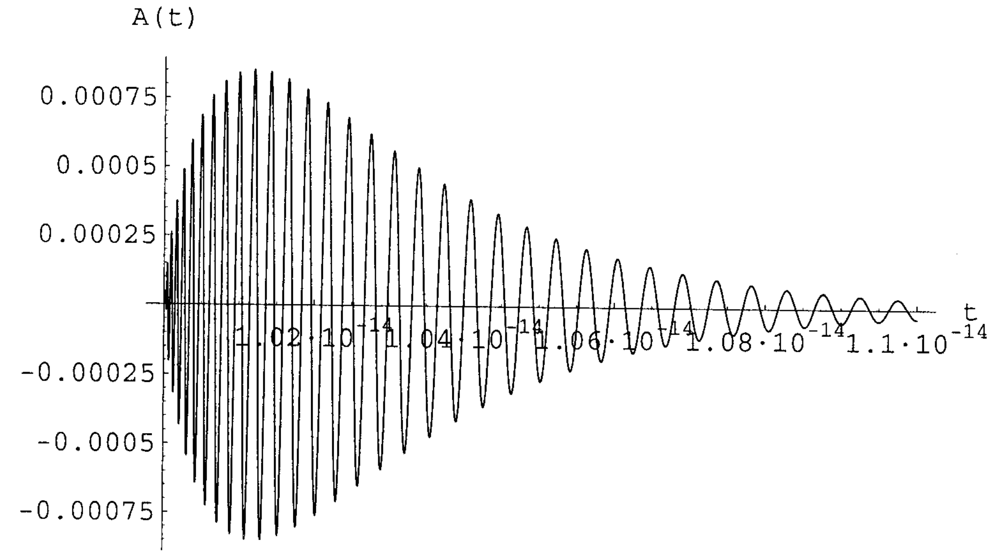

FIG. 3. a) Asymptotic description of the dynamic evolution of the first precursor corresponding to the hyperbolic tangent signal in the Lorentz medium. $\omega_c = 2.0 \times 10^{16}$ Hz, $\lambda = 1.0 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{12}$ s^{-1}.

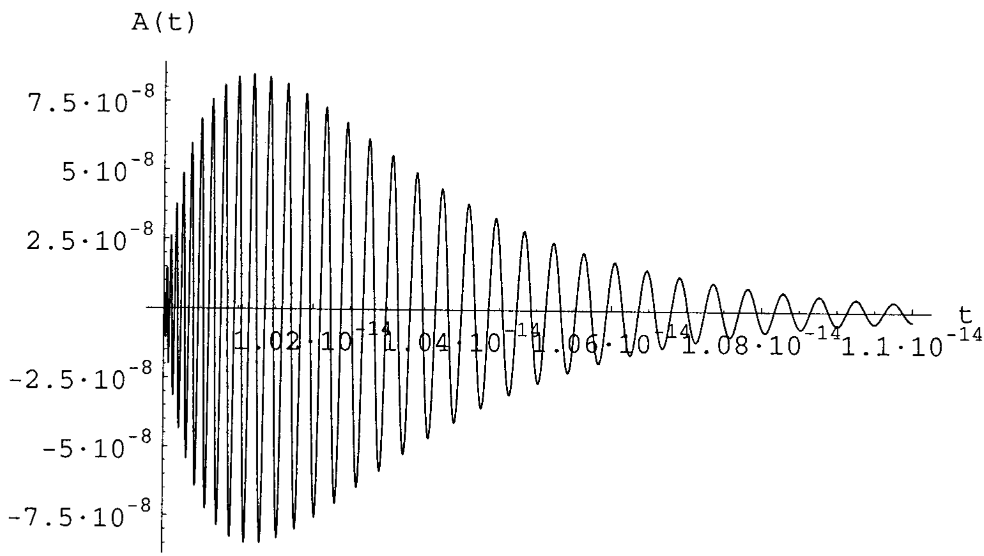

FIG. 3. b) Asymptotic description of the dynamic evolution of the first precursor. $\omega_c = 2.0 \times 10^{12}$ Hz, $\lambda = 1.0 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{12}$ s^{-1}.

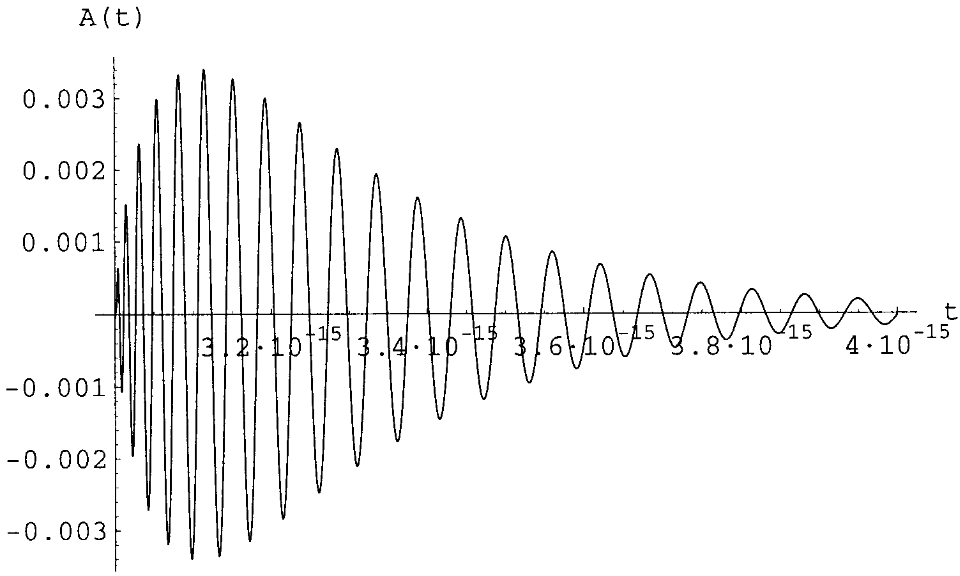

FIG. 4. a) Asymptotic representation of the dynamic evolution of the first precursor. Based on Oughstun-Sherman's approximation. $\omega_c = 2.0 \times 10^{16}$ Hz, $\lambda = 1.0 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{12}$ s^{-1}.

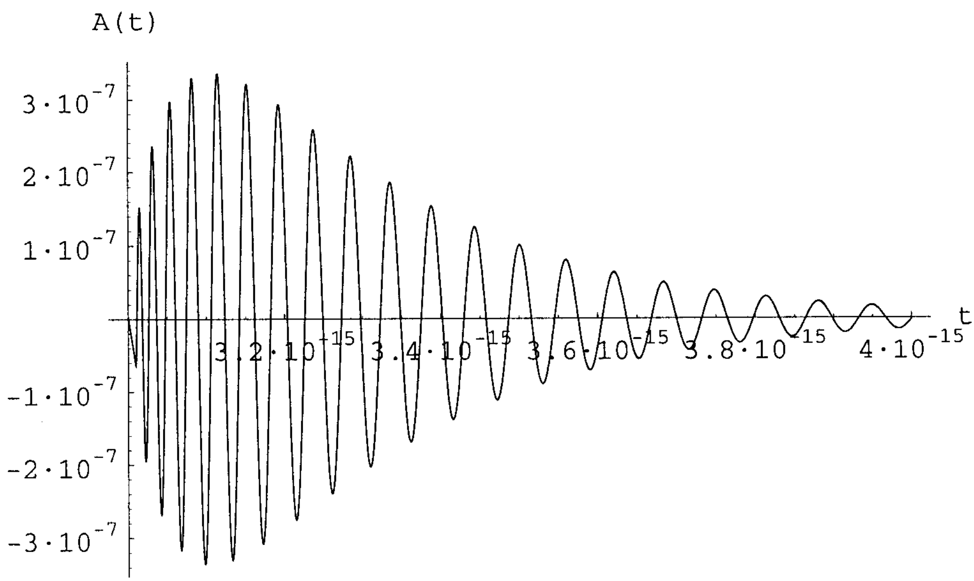

FIG. 4. b) Asymptotic representation of the dynamic evolution of the first precursor. Based on Oughstun-Sherman's approximation. $\omega_c = 2.0 \times 10^{12}$ Hz, $\lambda = 1.0 \times 10^{-14}$ s and $\beta = 1.0 \times 10^{12}$ s^{-1}.

It is seen that for θ slightly exceeding unity, the real parts of approximations based on Eq. (3.1) and (3.12) are virtually undistinguishable from the exact result. Their imaginary parts, however, exhibit observable deviations from the exact result for θ smaller than 1.3. For higher values of θ they provide a very good approximation to the exact result.

The approximation given by (3.12) was then utilized in (4.2) to obtain asymptotic representation of pulse dynamics in the Lorentz medium. Figure 2 and Fig. 3 show the first precursor as a function of θ, calculated for different values of β and λ. Its form is governed by (4.2) through (4.6). For θ close to unity the argument of the functions J_{1+j}, $j = 0, 1$, is small and they can be approximated with a power functions of their argument. It then follows that the precursor is a rapidly oscillating, growing function (the frequency of oscillations is infinite when $\theta = 1$), which is then exponentially attenuated with increasing θ. It can also be seen that the front of this precursor travels with the velocity of light c in the vacuum. The detailed description of general behaviour of the first precursor can be found in [11], Ch. 7.

In the present context one can see that changing the value of the parameter β from 10^{16} s^{-1} to 10^{12} s^{-1} does not affect the shape of the precursor, but proportionally changes its amplitude. Alternatively, the parameter λ, which is proportional to the distance traveled by the precursor, has an essential impact on both the pulse shape and its decay. With growing distance, higher frequencies in the frequency spectrum seem to be less attenuated than the lower ones.

These results, based on the approximation given by (3.12), can be compared against those obtained from the Oughstun-Sherman's approximation (see Fig. 4). From Fig. 2 and Fig. 4 some differences in precursor dynamics, especially at higher values of θ, can be observed. They seemingly result from lower accuracy of Oughstun-Sherman's approximation at higher values of θ, as seen from Fig. 1.

6. Conclusions

The problem of electromagnetic plane wave propagation in the Lorentz medium has been reconsidered. A new uniform asymptotic representation is found for the first precursor which is more accurate than the approximations known in the literature. The present result is based on the new approximate solution to the saddle point equation, as found in [15]. The precursor representation is constructed with the help of Bleistein and Handelsman's [17] asymptotic method and is illustrated graphically for the Brillouin's choice of the medium parameters. Plots are obtained for different values of the space coordinate and rate of growth in the initial pulse.

In the literature, the Debye model and Drude model of dispersive media are often used. The former is pertinent to polar liquids, and the latter applies to media with conductivity. Both models follow from the Lorentz model [18] and therefore the results of this paper apply also in this case. However, there is a number of models of various media (see [1]) to which the approximations obtained here cannot be directly applied.

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