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# RELIABILITY-BASED STRUCTURAL OPTIMIZATION ACCOUNTING FOR MANUFACTURING AND MATERIAL QUALITY

K. DOLIŃSKI and R. STOCKI

**Institute of Fundamental Technological Research**
**Polish Academy of Sciences**
Świętokrzyska 21, 00-049 Warsaw, Poland

The reliability-based optimization problem in which the structural material and manufacturing quality is assumed to be uncertain is considered. Moreover, the quality cost is appropriately taken into consideration in the formula determining the total cost of the structure. Some parameters describing the quality, namely the means and variances of the material properties and the variances of the precision of assembling of the structure are considered as the design parameters in the optimization procedure, in which the required structural reliability level determines some constrains to be satisfied in searching for the minimum structural cost.

**Key words:** structural reliability, structural stability, reliability-based optimization

## 1. Introduction

Advances in reliability methods and development of numerical algorithms together with the computational power and computer availability allow us to approach the problems of structural safety in a more rational way than before. It is well known that deterministic optimization may lead to optimal structures that are very sensitive to changes of the values of design parameters. Since many of the design parameters are of random nature, it is extremely important to take into account this feature in the design process and, eventually, to apply the reliability analysis to assure an appropriate safety level of the optimal structure. Such an approach should be especially preferred for structures subject to stability loss when even small geometrical imperfections can dramatically change the structural reliability. The imperfection influence can be reduced by more precise manufacturing and assembling and by using the materials of better quality. All of these, however, increase the cost of the structure. In order to minimize the cost and to preserve a required reliability level of the structure while both the imperfection and material properties are random, some parameters characterizing the imperfection intensity and material quality are considered as design variables in the optimization procedure, called the reliability-based structural optimization (RBSO).

## 2. Structural reliability problem

It is fairly typical in civil and mechanical engineering that some quantities which describe a structural system and applied loads should be modeled as random variables, $X_1, X_2, \ldots, X_n$. They are called the basic variables and constitute a random vector $\boldsymbol{X}$ whose samples $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ belong to the Euclidian space. In the space $\boldsymbol{\mathcal{X}}$ the probability measure is defined by the joint probability density function $f_{\boldsymbol{X}}(\boldsymbol{x})$ of the random vector $\boldsymbol{X}$. Depending on the sample values of the basic variables, the structural system will satisfy the required work conditions and be safe and intact or not. The criterion of structural failure is usually expressed by the equality $g(\boldsymbol{x}) = 0$ that defines a hypersurface, called the limit state surface, in the space $\boldsymbol{X}$. It divides the space $\boldsymbol{X}$ into two regions: the failure domain $\Omega_f = \{\boldsymbol{x} : g(\boldsymbol{x}) \leq 0\}$ and the safe domain $\Omega_s = \{\boldsymbol{x} : g(\boldsymbol{x}) > 0\}$. Hence, the failure probability of the structural system is determined by the following integral:

$$P_f = \mathbb{P}[\boldsymbol{X} \in \Omega_f] = \mathbb{P}[g(\boldsymbol{X}) \leq 0] = \int_{\Omega_f} f_{\boldsymbol{X}}(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}, \tag{2.1}$$

where $\mathbb{P}[A]$ means the probability of the random event $A$. In application, where the number $n$ of basic variables $X_i$ can be great, the integral domain $\Omega_f$ complex and the calculation of the limit state function $g(x)$ cumbersome, e.g. involving a finite element numerical procedure, the direct integration appears to be impractical. Therefore, some approximate methods have been developed that allow us to effectively involve the reliability assessment in structural analysis.

In the approach that is most commonly used in application, the problem of the reliability calculation is appropriately transformed, $\boldsymbol{U} = \boldsymbol{T}(\boldsymbol{X})$ (see e.g. [1, 2, 3]), into the space $\boldsymbol{\mathcal{U}}$ where the probability measure is defined by probability density function $f_{\boldsymbol{U}}(\boldsymbol{u}) = \prod_{i=1}^{n} \varphi(u_i)$ being the product of the $n$ one-dimensional standard normal probability density functions of random variables $U_i = T_i(\boldsymbol{X})$. Since the limit state condition is also transformed into $\boldsymbol{\mathcal{U}}$, $g(\boldsymbol{x}) = 0 \to h(\boldsymbol{u}) = g[\boldsymbol{T}^{-1}(\boldsymbol{u})] = 0$, the failure probability can be calculated as follows:

$$P_f = \mathbb{P}[h(\boldsymbol{U}) \leq 0] = \int_{\{\boldsymbol{u}: h(\boldsymbol{u}) \leq 0\}} f_{\boldsymbol{U}}(\boldsymbol{u})\, \mathrm{d}\boldsymbol{u}. \tag{2.2}$$

The axial symmetry of the probability density function $f_{\boldsymbol{U}}(\boldsymbol{u})$ assures for any linear function $l(\boldsymbol{u}) = \beta - \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{u} = 0$, the following equality to be true

$$\mathbb{P}[l(\boldsymbol{U}) \leq 0] = \int_{\{\boldsymbol{u}: l(\boldsymbol{u}) \leq 0\}} f_{\boldsymbol{U}}(\boldsymbol{u})\, \mathrm{d}\boldsymbol{u} = \Phi(-\beta), \tag{2.3}$$

where the coefficients, $-\alpha_i, i = 1, 2, \ldots, n$, are the components of the normalized gradient of the hyperplane $l(\boldsymbol{u}) = 0$, i.e. $\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{\alpha} = 1$, $\beta = \mathrm{sign}[l(\boldsymbol{0})]\delta$ is the signed distance $\delta$ between the hyperplane and the origin in $\boldsymbol{\mathcal{U}}$ and $\Phi(\cdot)$ is the standard normal distribution. Thus, the linear approximation of the transformed limit state surface $h(\boldsymbol{u}) = 0$ in the point closest to the origin provides a simple estimate of the probability of structural system

$$P_f = \mathbb{P}[h(\boldsymbol{U}) \leq 0] \approx \mathbb{P}[l(\boldsymbol{U}) \leq 0] = \Phi(-\beta), \tag{2.4}$$

where $\beta$ is called the reliability index. The approach based on the linear approximation of the transformed limit state surface is called the first order reliability method (FORM). The reliability index is determined as a solution of the following optimization problem:

$$\begin{gathered} \beta = \operatorname{sign}[h(\mathbf{0})]\delta^* \\ \delta^* \equiv \|\boldsymbol{u}^*\| = \min\|\boldsymbol{u}\| \quad \text{subject to:} \quad h(\boldsymbol{u}) \leq 0. \end{gathered} \tag{2.5}$$

This problem can be solved by any nonlinear optimization algorithm (cf. [4] for comparison of various methods) but two algorithms, namely the Rackwitz-Fiessler algorithm [5] and NLPQL algorithm [6] are considered to be the most efficient ones. In the paper the Rackwitz-Fiessler algorithm is used.

## 3. RBSO PROBLEM FORMULATION

Before formulating the RBSO problem it is necessary to realize the variety of parameters that enter the problem (see e.g. [7]). Size, shape and loading parameters defining a structural model can be considered either deterministic or random. Design parameters can be assigned to deterministic parameters, denoted by $\boldsymbol{x}^d$, or to the mean values or standard deviations of the random variables $\boldsymbol{x}^\mu$ or $\boldsymbol{x}^\sigma$, respectively.

The RBO problem can be formulated in many ways [8, 9, 10]. In this paper the so-called componental formulation of the problem will be used which is the minimization of the cost function subjected to constraints imposed on the values of componental reliability indices

$$\text{minimize} \quad C_{\mathrm{T}}(\boldsymbol{x}^\alpha) \qquad \alpha = \{d, \mu, \sigma\}, \tag{3.1}$$

$$\text{subject to:} \quad \beta_i(\boldsymbol{x}^\alpha) \geq \beta_i^{\min} \qquad i = 1, \ldots, m_r, \tag{3.2}$$

$$c_j(\boldsymbol{x}^\alpha) \geq 0 \qquad j = 1, \ldots, m_d, \tag{3.3}$$

$$^l x_k^\alpha \leq x_k^\alpha \leq {}^u x_k^\alpha \qquad k = 1, \ldots, n, \tag{3.4}$$

where $C_{\mathrm{T}}$ represents the total cost, $\beta_i$, $i = 1, \ldots, m_r$ are the reliability indices corresponding to the elemental failure modes, $\beta_i^{\min}$ are the minimal admissible $\beta$-values, chosen according to the required safety level, $c_i$, $i = 1, \ldots, m_d$ are deterministic constraints, $m_r$ is the number of reliability constraints, $n$ is the number of design variables and ${}^l x_i^\alpha$, ${}^u x_i^\alpha$ are the lower and upper bounds, respectively, imposed on design variables.

To solve the RBSO problem (3.1)–(3.4) by the most efficient iterative gradient methods, values of the cost function and reliability constraints as well as their gradients with respect to the design parameters are needed. Because the computation of reliability index itself is the optimization problem (cf. (2.5)), the RBSO problem is sometimes called nested optimization problem. Since the most computationally demanding tasks of failure function evaluation (involving e.g. finite element analysis) and the parametric sensitivity analysis are performed in the inner optimization loop (design point search problem), it is crucial to implement in the RBSO system the most efficient analytical sensitivity methods (see. [11]). The other methods of improving the efficiency of the RBSO process can be found in [10].

The total cost function (3.1) that combines the initial costs of the structure and the costs of failures is given as

$$C_{\mathrm{T}}(\boldsymbol{x}^{\alpha}) = C_{\mathrm{I}}(\boldsymbol{x}^{\alpha}) + \sum_{i=1}^{m_r} C_{\mathrm{F}}^{i}\,\Phi(-\beta_i(\boldsymbol{x}^{\alpha}))\,, \tag{3.5}$$

where $C_{\mathrm{I}}$ is the initial cost, $C_{\mathrm{F}}^{i}$, $i = 1,\dots,m_r$, are the cost coefficients associated with elemental failures, $\Phi(\cdot)$ is the standard normal distribution function and $\Phi(-\beta_i(\boldsymbol{x}^{\alpha}))$ are the failure probabilities. The values of coefficients $C_{\mathrm{F}}^{i}$ can only be estimated with some uncertainty since, apart from the direct economical consequences, they have to take into account hardly measurable, social and ethical aspects of the structural failure. On the other hand, due to the very small values of probabilities of failure (usually $10^{-7} \div 10^{-4}$) it is important to consider only these failure modes, for which the corresponding $C_{\mathrm{F}}^{i}$ are by orders of magnitude greater than $C_{\mathrm{I}}$. Typically the initial cost is associated with structural weight. However, in the framework of RBSO, it is also possible to account for manufacturing as well as material quality. For the case of truss type structures, the extended initial cost function has the form (cf. [10])

$$C_{\mathrm{I}}(\boldsymbol{x}^{\alpha}) = C_{\mathrm{mat}} \sum_{i=1}^{n_e} A_i(\boldsymbol{x}^{\mu}, \boldsymbol{x}^{d})\, l_i(\boldsymbol{x}^{\mu}, \boldsymbol{x}^{d})[1 + \kappa_i(\boldsymbol{x}_{\mathrm{mat}}^{\sigma})] + C_{\mathrm{man}}\theta(x_{\mathrm{man}}^{\sigma})\,, \tag{3.6}$$

where $n_e$ is the number of truss elements, $A_i$ and $l_i$, $i = 1,\dots,n_e$, are the element cross-sectional area and length, respectively, $C_{\mathrm{mat}}$ is the cost of unit material volume and $\kappa_i(\boldsymbol{x}_{\mathrm{mat}}^{\sigma})$ are the non-dimensional functions of standard deviations of yield stress that is taken as a measure of material quality. Since the space coordinates of the structural joints are modeled as normally distributed random variables, the standard deviation of the joint position may be used as a measure of manufacturing and assembling precision. Assuming that the standard deviations of random nodal coordinates are the same and equal to $x_{\mathrm{man}}^{\sigma}$, the non-dimensional function $\theta(x_{\mathrm{man}}^{\sigma})$ is introduced in the equation (3.6) to account for the manufacturing quality costs. $C_{\mathrm{man}}$ is the respective cost coefficient.

## 4. Numerical examples

### 4.1. Example 1

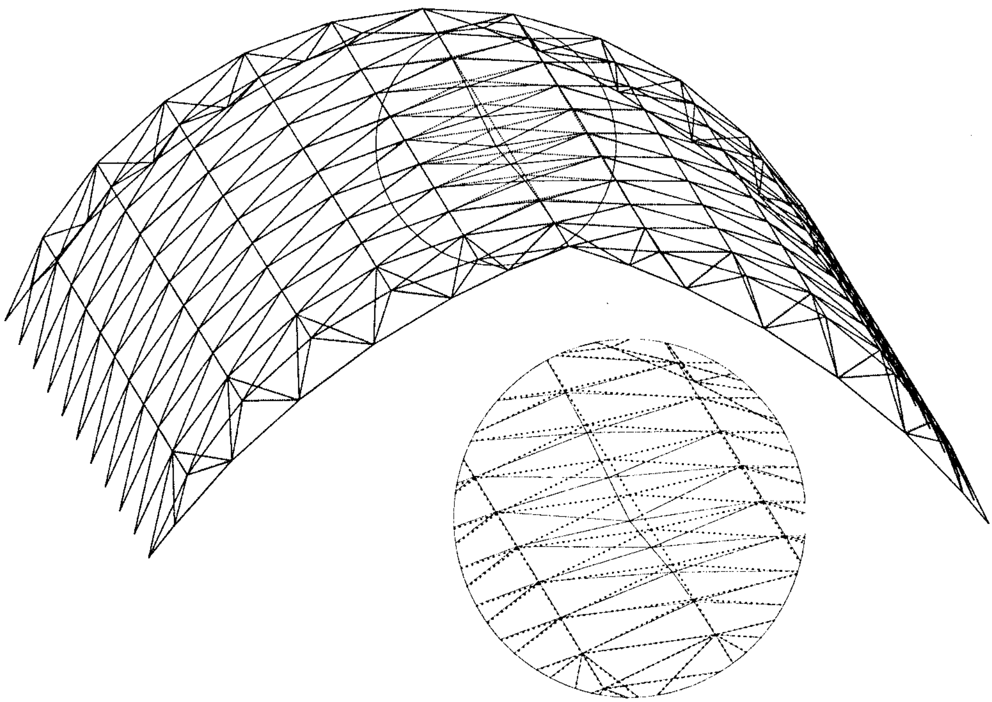

Fig. 1. Snap-through failure of the cylindrically shaped truss

In this example the RBSO of cylindrically shaped truss is carried out. As it is shown in Fig. 1, the possible failure of such a structure may be due to the snap-through effect. Since the geometrical imperfections arising from the manufacturing of structural elements as well as the assembly process may lead to the reduction of critical load, it is desirable that the manufacturing and assembly quality should be high. This certainly results in increasing the cost of the structure, and that is why the RBSO employing the extended cost function (3.6) may give the designer some knowledge about how much it is worth to invest in improving the quality.

The structure consists of 474 steel tubular elements connected at 167 nodes. It is reinforced from the two sides by two three-hinged arches. The elements are divided into 7 groups. They are shown in Fig. 2. The external uniformly distributed load acting on the structure is shown in Fig. 3. The load intensity, $p$, given per unit surface equals $0.18\,\mathrm{kN/m^2}$.

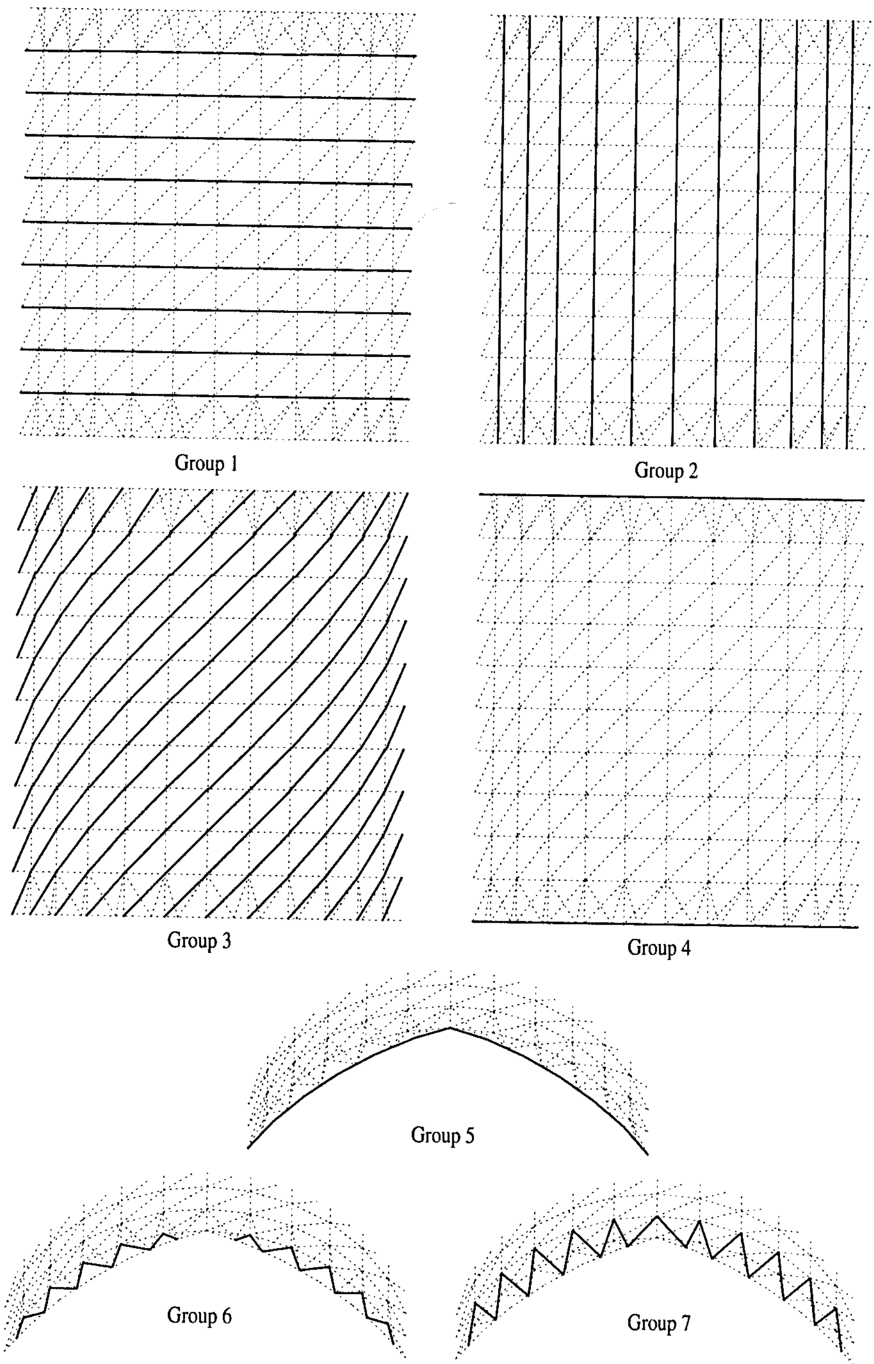


FIG. 2. Groups of elements

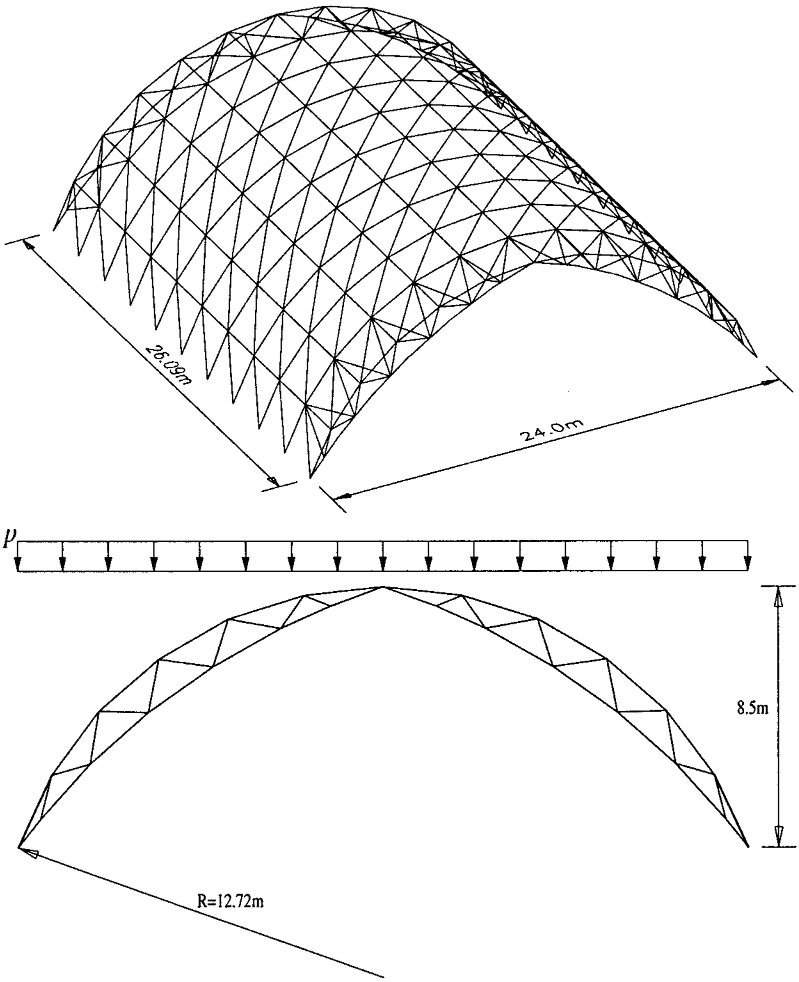


FIG. 3. Dimensions and load

Large displacement theory is employed. The limit state function corresponding to the snap-through failure has the following form:

$$g(\lambda_{\mathrm{cr}}(\boldsymbol{X},\boldsymbol{x}^{\alpha})) = \lambda_{\mathrm{cr}}(\boldsymbol{X},\boldsymbol{x}^{\alpha}) - 1\,, \tag{4.1}$$

where $\lambda_{\mathrm{cr}}$ is the critical load factor, $\boldsymbol{X}$ is the vector of random variables and $\boldsymbol{x}^{\alpha}$ is the vector of design variables. The total number of 444 parameters describing the analyzed structure and load were considered to be random. They are listed in Tab. 1, where their respective distribution types mean values and standard deviations are presented. Besides the size, material and load type random variables, the coordinates of 145 nodes were chosen as random.

For the optimization problem, 8 design variables were selected. They are as follows: variables $x_k^{\mu}$, $k = 1,\ldots,7$ are the mean values of the random cross-

**Table 1. Cylindrically shaped truss: random variables**

| Var. | Distribution | Mean value | Std. dev. | Description |
|---|---|---|---|---|
| $X_1$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 1 |
| $X_2$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 2 |
| $X_3$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 3 |
| $X_4$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 4 |
| $X_5$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 5 |
| $X_6$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 6 |
| $X_7$ | log-normal | $20.0\,\mathrm{cm}^2$ | $1.0\,\mathrm{cm}^2$ | cross sec. - gr. of elems no. 7 |
| $X_8$ | log-normal | $21000.0\,\mathrm{kN/cm}^2$ | $500.0\,\mathrm{kN/cm}^2$ | Young modulus of material |
| $X_9$ | Gumbel | 1.0 | 0.1 | load factor |
| $X_{10}$ | normal | $0.0\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $x$ coor. - first unconstr. node |
| $X_{11}$ | normal | $-11.26\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $y$ coor. - first unconstr. node |
| $X_{12}$ | normal | $0.92\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $z$ coor. - first unconstr. node |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $X_{442}$ | normal | $26.09\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $x$ coor. - last unconstr. node |
| $X_{443}$ | normal | $11.26\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $y$ coor. - last unconstr. node |
| $X_{444}$ | normal | $0.92\,\mathrm{m}$ | $1.0\,\mathrm{cm}$ | $z$ coor. - last unconstr. node |

sections of groups of elements and $x_8^\sigma$ is the standard deviation of the nodal coordinates. $x_8^\sigma$ is assumed to be the same for all the random variables from $X_{10}$ to $X_{444}$. The RBSO problem can be defined as

$$\text{(4.2)} \qquad \text{minimize} \qquad C_{\mathrm{I}}(\boldsymbol{x}^\alpha) = C_{\mathrm{mat}} \sum_{i=1}^{7} x_i^\mu\, l_i + C_{\mathrm{man}} \theta(x_8^\sigma)\,,$$

$$\text{(4.3)} \qquad \text{subject to:} \qquad \beta(\boldsymbol{x}^\alpha) \geq 3.7\,,$$

$$\text{(4.4)} \qquad 5.0\,\mathrm{cm}^2 \leq x_k^\mu \leq 33.18\,\mathrm{cm}^2\,, \qquad k = 1,\ldots,7\,,$$

$$\text{(4.5)} \qquad 0.5\,\mathrm{cm} \leq x_8^\sigma \leq 4.0\,\mathrm{cm}\,,$$

where the reliability constraint (4.3) corresponds to the limit state function (4.1), $l_i$ is the total length (in cm) of elements belonging to the $i$-th group and the cost coefficients $C_{\mathrm{mat}}$ and $C_{\mathrm{man}}$ are equal to 1.0 and $2 \cdot 10^6$, respectively. The definition of function $\theta(x_8^\sigma)$ making use of the simple bounds (4.5) together with its graph are shown in Fig. 4. The initial values of the cost function, design parameters and the reliability index are given in Tab. 2. The values of cost coefficients were calibrated in order to make the manufacturing and assembling quality cost corresponding to the lower bound of $x_8^\sigma$ to be around 75% of the initial cost of material.

Using the interactive RBSO system POLSAP-RBO [7, 12] and the sequential quadratic programming algorithm NLPQL [6], the optimal design was obtained after 21 iterations. The optimization results are presented in Tab. 2. It is interesting to observe that the optimal value of the design variable $x_8^\sigma$ is even smaller

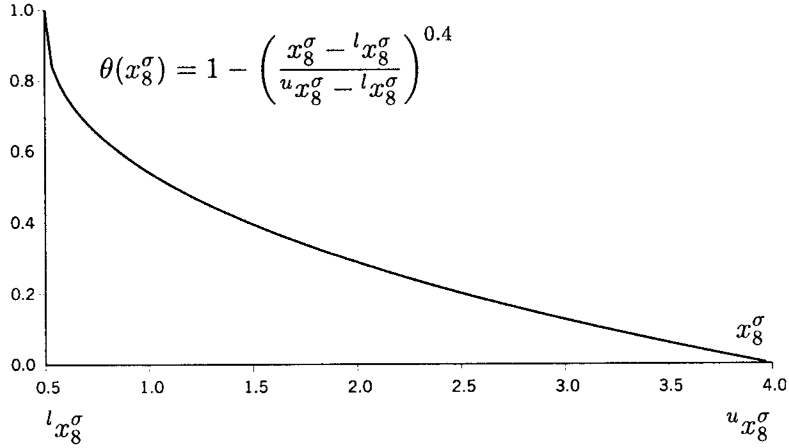


FIG. 4. Function $\theta(x_8^\sigma)$

**Table 2. Cylindrically shaped truss: RBSO results**

| | Initial design | Optimal design |
|---|---|---|
| Cost | 3740761 | 3525681 |
| $x_1^\mu$ | 20 cm$^2$ | 8.92 cm$^2$ |
| $x_2^\mu$ | 20 cm$^2$ | 30.52 cm$^2$ |
| $x_3^\mu$ | 20 cm$^2$ | 18.79 cm$^2$ |
| $x_4^\mu$ | 20 cm$^2$ | 12.43 cm$^2$ |
| $x_5^\mu$ | 20 cm$^2$ | 11.51 cm$^2$ |
| $x_6^\mu$ | 20 cm$^2$ | 5.0 cm$^2$ |
| $x_7^\mu$ | 20 cm$^2$ | 8.83 cm$^2$ |
| $x_8^\sigma$ | 10 mm | 7.85 mm |
| $\beta$ | 3.10 | 3.70 |

than its initial value and close to the lower bound. It gives the clear information that the optimal design can only be accepted provided the manufacturing and assembling quality is very high.

### *4.2. Example 2*

To show the influence of the material quality cost (as defined in (3.6)) on the initial cost of the structure, the RBSO of the truss structure shown in Fig. 5 was considered. It consists of 17 tubular elements divided into 5 groups: 1st group - elements 1...5, 2nd group - elements 5...8, 3rd - element 9, 4th - elements 10...13 and 5th - elements 14...17. The stochastic description of the structure is given in Tab. 3 and as the design variables, the following distribution parameters were selected: $x_1^\mu \ldots x_5^\mu$ - mean values of the cross-sectional areas of the groups of elements, $x_6^\sigma \ldots x_{10}^\sigma$ - standard deviations of the yield stresses of the material of element groups and $x_{11}^\sigma$ - standard deviation of shape type random variables $(X_{13} \ldots X_{27})$.

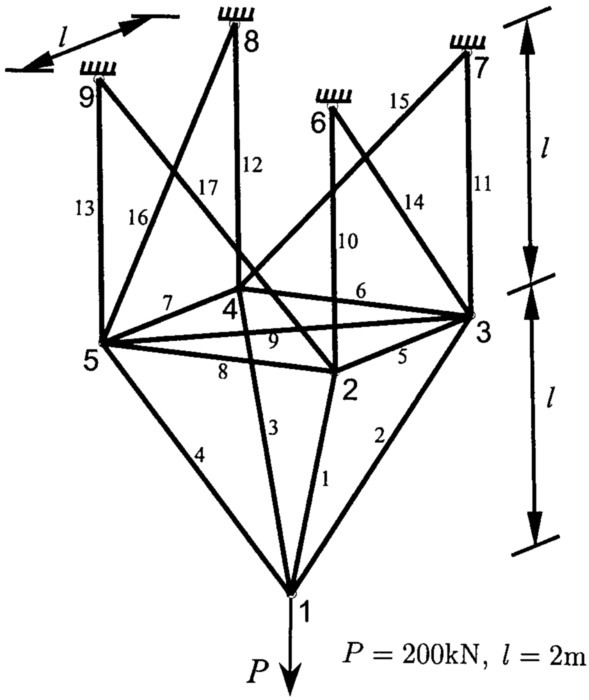


FIG. 5. Hanging truss structure

**Table 3. Hanging truss: random variables**

| Var. | Distribution | Mean value | Std. dev. | Description |
|---|---|---|---|---|
| $X_1$ | log-normal | 10.0 cm$^2$ | 0.5 cm$^2$ | cross sec. - gr. of elems no. 1 |
| $X_2$ | log-normal | 10.0 cm$^2$ | 0.5 cm$^2$ | cross sec. - gr. of elems no. 2 |
| $X_3$ | log-normal | 5.0 cm$^2$ | 0.25 cm$^2$ | cross sec. - gr. of elems no. 3 |
| $X_4$ | log-normal | 10.0 cm$^2$ | 0.5 cm$^2$ | cross sec. - gr. of elems no. 4 |
| $X_5$ | log-normal | 5.0 cm$^2$ | 0.25 cm$^2$ | cross sec. - gr. of elems no. 5 |
| $X_6$ | log-normal | 21000.0 kN/cm$^2$ | 1050.0 kN/cm$^2$ | Young modulus of material |
| $X_7$ | Gumbel | 1.0 | 0.2 | load factor |
| $X_8$ | log-normal | 30.0 kN/cm$^2$ | 3.0 kN/cm$^2$ | yield stress - gr. of elems no. 1 |
| $X_9$ | log-normal | 30.0 kN/cm$^2$ | 3.0 kN/cm$^2$ | yield stress - gr. of elems no. 2 |
| $X_{10}$ | log-normal | 30.0 kN/cm$^2$ | 3.0 kN/cm$^2$ | yield stress - gr. of elems no. 3 |
| $X_{11}$ | log-normal | 30.0 kN/cm$^2$ | 3.0 kN/cm$^2$ | yield stress - gr. of elems no. 4 |
| $X_{12}$ | log-normal | 30.0 kN/cm$^2$ | 3.0 kN/cm$^2$ | yield stress - gr. of elems no. 5 |
| $X_{13}$ | normal | 0.0 cm | 2.0 cm | $x$ coor. - node no. 1 |
| $X_{14}$ | normal | 0.0 cm | 2.0 cm | $y$ coor. - node no. 1 |
| $X_{15}$ | normal | 0.0 cm | 2.0 cm | $z$ coor. - node no. 1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $X_{25}$ | normal | 26.09 m | 2.0 cm | $x$ coor. - node no. 5 |
| $X_{26}$ | normal | 11.26 m | 2.0 cm | $y$ coor. - node no. 5 |
| $X_{27}$ | normal | 0.92 m | 2.0 cm | $z$ coor. - node no. 5 |

To introduce the reliability constraints, 5 stress/local buckling limit state functions are considered. They can be expressed as

$$g_i(\boldsymbol{q}(\boldsymbol{X},\boldsymbol{x}^\alpha),\boldsymbol{X},\boldsymbol{x}^\alpha) = 1 - \frac{|\sigma_{k_i}(\boldsymbol{q}(\boldsymbol{X},\boldsymbol{x}^\alpha),\boldsymbol{X},\boldsymbol{x}^\alpha)|}{\sigma^a_{k_i}(\boldsymbol{X},\boldsymbol{x}^\alpha)}, \qquad i=1,\ldots,5, \tag{4.6}$$

where $\sigma_{k_i}$ is the axial stresses in the $k_i$-th element, $\sigma^a_{k_i}$ is an admissible stress and $\boldsymbol{q}$ is the vector of nodal displacements. The elements corresponding to the limit states $g_1 \ldots g_5$ are 2, 5, 9, 11 and 15, respectively. For elements under tensile stress the value of $\sigma^a_{k_i}$ is assumed constant and equal to the yield stress during the $\beta$–point search, while in the compression case it is assumed to vary according to the current values of realizations of the corresponding size and shape random variables; this allows to account for local buckling. The RBSO problem is formulated as follows:

$$\text{(4.7)} \quad \text{minimize} \quad C_{\mathrm{I}}(\boldsymbol{x}^\alpha) = C_{\mathrm{mat}} \sum_{i=1}^{5} x^\mu_i \, l_i[1+\kappa_i(x^\sigma_{i+5})] + C_{\mathrm{man}}\theta(x^\sigma_{11})$$

$$\text{(4.8)} \quad \text{subject to:} \quad \beta_j(\boldsymbol{x}^\alpha) \geq 4.2 \qquad j=1,\ldots,5$$

$$\text{(4.9)} \quad 2.0\,\mathrm{cm}^2 \leq x^\mu_k \leq 30.0\,\mathrm{cm}^2, \qquad k=1,\ldots,5$$

$$\text{(4.10)} \quad 1.0\,\mathrm{kN/cm}^2 \leq x^\sigma_k \leq 5.0\,\mathrm{kN/cm}^2, \qquad k=6,\ldots,10$$

$$\text{(4.11)} \quad 0.5\,\mathrm{cm} \leq x^\sigma_{11} \leq 4.0\,\mathrm{cm}$$

where $l_i$ is the total length (in cm) of the $i$-th group elements, and the cost coefficients are $C_{\mathrm{mat}} = 1$ and $C_{\mathrm{man}} = 4000$. The proposed definitions of the functions $\kappa_i$ and $\theta$ are shown in Fig. 6. The minimal admissible $\beta$-values equal to 4.2 correspond to failure probability of $1.3 \cdot 10^{-5}$. Starting solution and the optimization results obtained after 23 iterations of NLPQL algorithm are presented in Tab. 4. Again, it shows that the manufacturing quality should be high and that the elements belonging to the 1st and 4th group should be made of high quality steel.

**Table 4. Hanging truss: RBSO results**

| | Initial design | Optimal design |
|---|---|---|
| Cost | 34982 | 21299 |
| $x^\mu_1$ | 10 cm$^2$ | 5.93 cm$^2$ |
| $x^\mu_2$ | 10 cm$^2$ | 6.10 cm$^2$ |
| $x^\mu_3$ | 5 cm$^2$ | 4.39 cm$^2$ |
| $x^\mu_4$ | 10 cm$^2$ | 4.73 cm$^2$ |
| $x^\mu_5$ | 5 cm$^2$ | 2.99 cm$^2$ |
| $x^\sigma_6$ | 3 kN/cm$^2$ | 1.59 kN/cm$^2$ |
| $x^\sigma_7$ | 3 kN/cm$^2$ | 5.0 kN/cm$^2$ |
| $x^\sigma_8$ | 3 kN/cm$^2$ | 3.62 kN/cm$^2$ |
| $x^\sigma_9$ | 3 kN/cm$^2$ | 1.55 kN/cm$^2$ |
| $x^\sigma_{10}$ | 3 kN/cm$^2$ | 4.61 kN/cm$^2$ |
| $x^\sigma_{11}$ | 20 mm | 15.3 mm |
| $\beta_1$ | 5.84 | 4.2 |
| $\beta_2$ | 7.79 | 4.2 |
| $\beta_3$ | 5.56 | 4.2 |
| $\beta_4$ | 6.69 | 4.2 |
| $\beta_5$ | 7.00 | 4.2 |

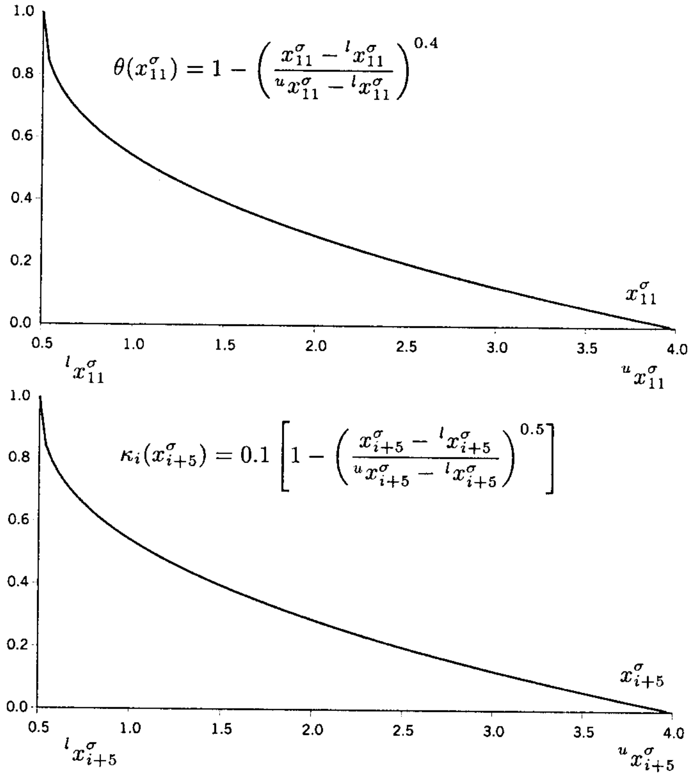


FIG. 6. Functions $\theta(x_{11}^{\sigma})$ and $\kappa_i(x_{i+5}^{\sigma})$, $i = 1, \ldots, 5$

## 5. Conclusions

The inevitable randomness of material, loading and geometrical parameters is always taken into account in design process of any structure. It is usually done implicitly by application of some safety coefficients given in codes and standard regulations. There are, however, some structures or types of structures that should be considered individually owing to the consequences of their malfunction or failure, their cost or importance for the society etc. In these cases the most rational approach to assure a sufficient or required level of the structural safety is the application of the reliability analysis that allows us to estimate the failure probability of the designed structure. There are some parameters that can be, to some extent, controlled and evidently reduce the probability of structural failure, e.g. the quality of material and of the manufacturing and assembling process. Better quality makes, however, the structure more expensive. The question is what quality may be sufficient to assure the required structural reliability level at minimum cost. The method that gives the answer to such a question is proposed in the paper in the framework of the Reliability-Based Structural Optimization (RBSO) where the cost of quality is included in the total cost of the structure. The quality is related to variances of the material parameters and the precision of assembling procedure. The less variances are assumed, the more expensive and reliable becomes the structure. The inclusion of moments of random variables describing the quality as the design parameters in the structural optimization procedure enabled us to minimize the total cost and to assure the required reliability of the structure. The relations between the cost and the moments should be defined specifically to any individual case. They will depend not only on the type of the structure but also on the local conditions. The functions $\kappa$ and $\theta$ assumed in the numerical examples reflect rather a general tendency of the cost-quality relation than any actual design situation. The results of illustrative examples clearly confirm that the savings due to material and manufacturing quality do not necessarily lead to reduction of total structural cost.

Moreover, the actual design would require much more comprehensive analysis involving some important features that have been neglected in the examples presented in the paper. It is however a specific property of a given structure what the failure modes are important, which should be taken into consideration, how they have to be combined in the analysis of the structure. It leads to the structural optimization with system reliability constrains. The approach presented in the paper provides a basis to develop the more advanced method accounting for manufacturing and material quality in structural analysis, optimization and, eventually, in the structural design.

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