ENGINEERING TRANSACTIONS • *Engng. Trans.* • **50**, 1-2, 3–17, 2002
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.504.2002

BEAM-ELEMENT MODEL OF BRIDGE TRUSS GIRDERS REGARDING JOINT DIMENSIONS

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The paper concerns modelling of bridge truss girders with regard to joint (gusset plates) dimensions. Modification of the classical beam-element numerical model is presented. It is based on introduction of additional near-node beam elements of lengths corresponding to joint dimensions. Their stiffnesses are determined in the iterative procedure. Results of finite element method analyses of the "accurate" (shell element) and dynamically modified "simplified" (beam element) models of structural joint with a part of connected members are compared. Laboratory tests that verify the iterative procedure are described. The calculated and recorded displacements and internal forces are compared.

1. Introduction

Bridge truss girders consist of straight bars usually rigidly connected at joints of relatively large sizes. Principles of design of the girders (Fig. 1) are given and discussed in [5, 9, 11], for example.

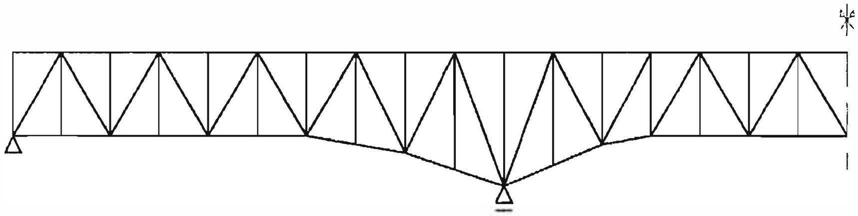


Fig. 1. Scheme of bridge truss girder.

Inspection of design documentation of several bridge truss girders, that had been carried out, revealed that the length of bridge truss girder element (taken as the clearance between gusset plates) may be even by 20 – 30% smaller than their theoretical length (between geometrical nodes). Thus, the assumption on negligible joint dimensions, which is usually made in numerical modelling that uses beam elements, is dubious. The computations may lead to wrong assessment of structural global stiffness [6]. A proper estimation of the stiffness of such structures is crucial for design, rehabilitation and loading tests of truss bridges.

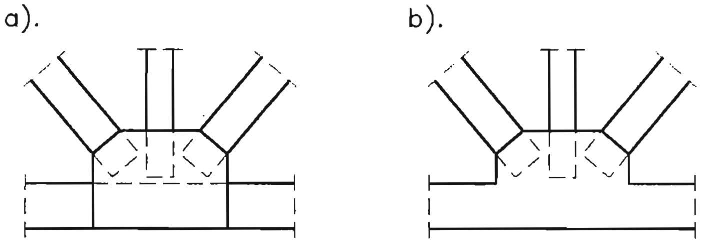


FIG. 2. Gusset plates of bridge truss girders: a) placed over truss flange webs ("overplate"), b) inserted as a part of webs ("insert").

The improvement of global stiffness assessment among others may be achieved through the following:

a) application of more accurate finite element method (FEM) models (such as using shell-element models), and

b) modification of classical beam-element models.

Since the costs of beam-element models are lower and the models are the most popular in Polish design practice, the latter approach will be followed in the paper.

It is possible to modify the beam-element FEM model of bridge truss girder to regard the influence of joints dimensions on global stiffness. The aim may be achieved by introduction of additional short near-node beam elements. The procedure of determining their $(E \cdot A)$ and $(E \cdot I)$ stiffnesses is described and applied in the paper (for details see also [10]).

2. Finite dimensions of joints in numerical modelling

The problem of the influence of the dimensions of structural joint on the magnitude of internal forces and their distribution was discussed in dozens of scientific papers. For example, the papers [3] and [4] refer to the problem to FEM beam-element model of plane frame of stocky members, and the paper [8] concerns grilles. In both cases two joint models were analysed: "accurate" (consisting of plane-stress or "brick" elements) and "simplified" (consisting of beam-elements). The influence of joint dimensions was reproduced in the "simplified" model by near-node elements of infinite flexural (and torsional [8]) stiffnesses. The element lengths were determined assuming equivalence of elastic energies in both the above models. Solutions of "simplified" models, with various combinations of near-node element lengths, were compared to the accurate model "solution". In [4] the criterion of the smallest sum of square errors of node displacement assessment in "simplified" model with respect to the "accurate" solution was applied.

In [6], the near-node element stiffness, representing the joint dimensions, was determined basing on the equilibrium of displacements of cantilevers tips, according to Fig. 3. Cross-sectional area A_z and the moment of inertia I_z are the suggested design parameters.

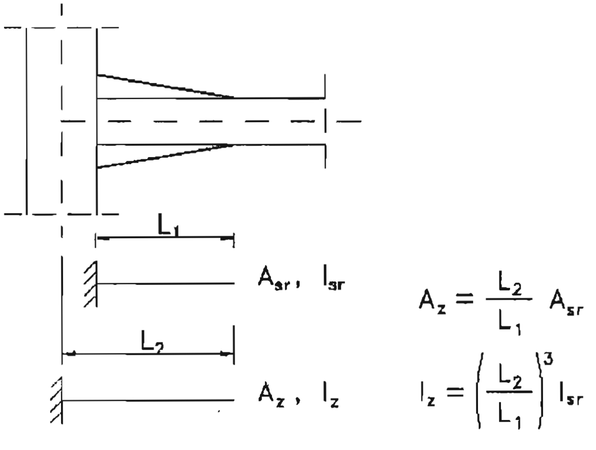


FIG. 3. Computational model of bridge truss girder joint with regard to its dimensions [6].

3. Modelling of joints

3.1. *Aim and scope of the analysis*

Introduction of near-node elements that represent joint dimensions in beam-element model has been already applied and described. However, so far, the stiffnesses of the additional near-node elements were determined either arbitrarily [2] or in a simplified manner [6].

In the paper, the iterative procedure of determining near-node elements stiffnesses is presented. It is based on comparative analysis of two FE models of joints: the one which is called "accurate", and the "simplified" one. Stiffnesses of the additional near-node beam elements in the "simplified" model are dynamically modified according to the differences in the distribution of internal forces, displacements and elastic energies obtained from the analysis of both models.

Steel truss girder joints with gusset plates are analysed (Fig. 4). The support conditions and the load schemes ("H", "V", "M") taken into consideration are given in Fig. 4b. In both models ("accurate" and "simplified"), linearly-elastic material behaviour and small strain assumptions are used.

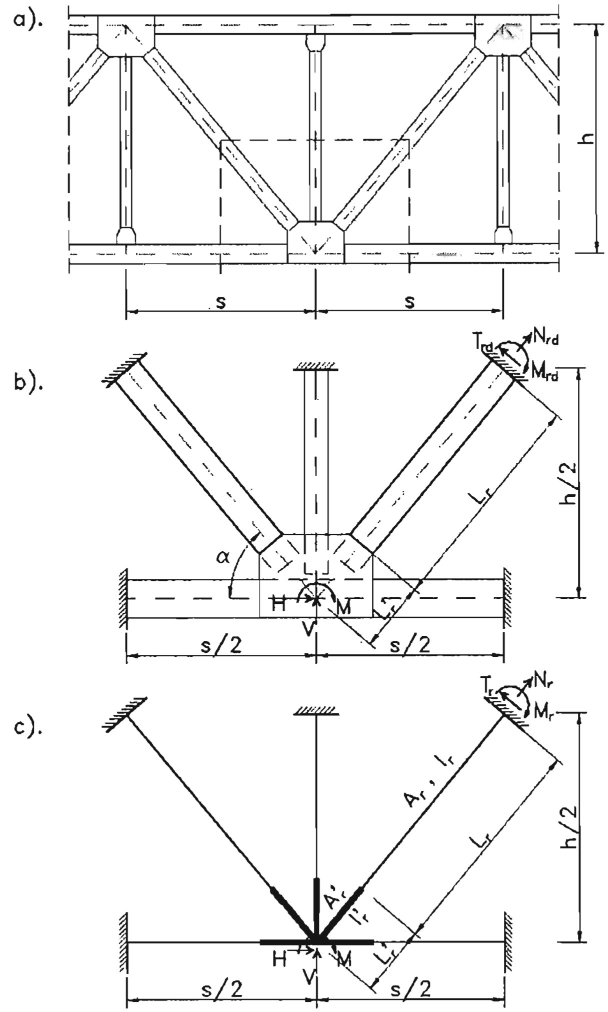


FIG. 4. Comparative analysis of computational FEM models a) part of steel bridge truss girder (analysed joint in the frame), b) contour of "accurate" model (of shell elements), c) "simplified" model (of beam elements).

The 8-node shell element model (Fig. 5) serves as the "accurate" one. It represents flanges and webs of truss members and gusset plates (Fig. 4b) in the

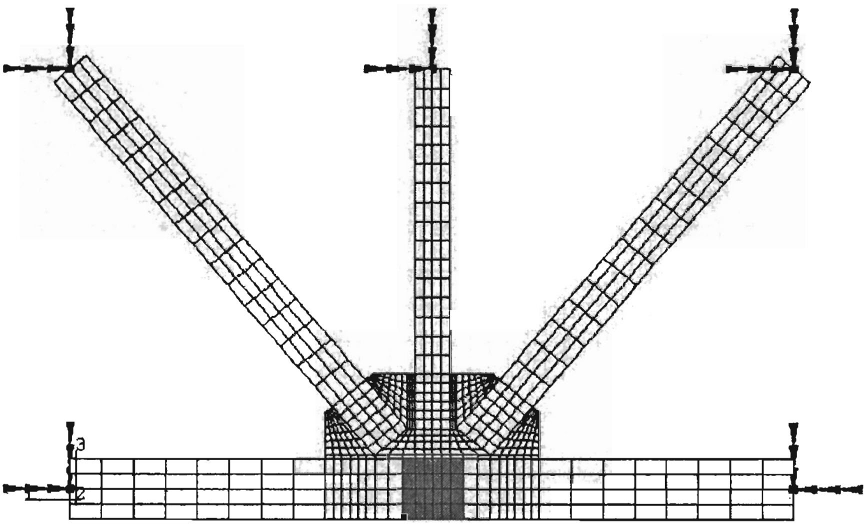


FIG. 5. Meshing of the "accurate" FEM model (support conditions are indicated, H, V, M loads were divided and applied to the nodes within the shaded area).

most adequate manner. Concentrated forces H and V, as well as concentrated bending moment M applied at the node, were replaced by groups of forces (or force pairs) and applied at nodes of the FE model in the vicinity of geometrical node, within the area shaded in Fig. 5. The models were analysed in the environment of FE code Abaqus [1], in the Poznań Supercomputing and Networking Centre.

To prove the accuracy and convergence of the results, three FE meshes were tested (models I, II and III). Figure 6 shows the considered types of meshing for a half of the gusset plate. Table 1 gives brief description of the models computational parameters. On this basis, the model II (Fig. 5) was chosen for further analysis.

Table 1.

Feature	Model I	Model II	Model III
Number of elements / number of nodes	472 / 2579	1266 / 5644	4204 / 15968
Total amount of degrees of freedom	12102	28524	86112
Computation time of a single loading scheme [s]	33	70	226
Maximum error of assessment of elastic energy, internal forces at supports, and displacements of theoretical node with respect to the model III [%]	2.5	1.5	–

The "simplified" model consists of simplified beam elements only (Fig. 4c). The length of near-node elements is related to the respective gusset plate di-

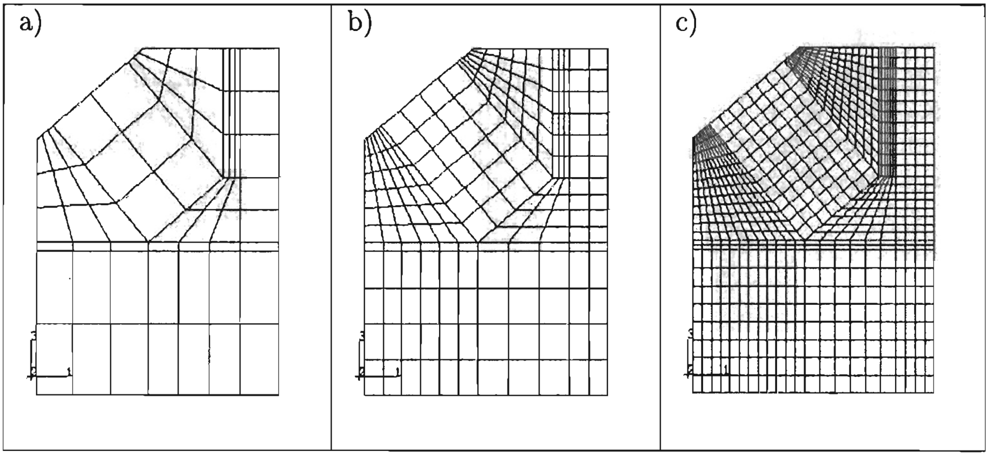


FIG. 6. Meshing of the gusset plate in the "accurate" models: a) model I, b) model II, c) model III.

mensions (Fig. 4c). The model was analysed using the force method [7]. Normal and transverse forces and bending moments were taken into account. Transversal stiffnesses $(G \cdot A)$ were assumed to be constant over theoretical lengths of the truss members.

3.2. Mathematical formulation

To find the near-node beam element stiffnesses, we compare the results obtained for both models, "accurate" and "simplified". As a representative volume we take into consideration the joint and its vicinity (half of the connected beams).

For the given loading schemes (application of horizontal and vertical forces "H", "V" and bending moment "M" at joints – Fig. 4), the following are known from the "accurate" model analysis:

$$M_{rd},\ N_{rd},\ T_{rd},\ \phi_d,\ u_d,\ v_d,\ U_d,\ U_{pd}, \tag{3.1}$$

where:

- r truss member number; $r = 1, 2, \ldots, p$,
- p number of truss members connected at the joint,
- M_{rd}, N_{rd}, T_{rd} bending moment, normal and transversal force at the support of the truss member "r" in the "accurate" model, respectively, in loading schemes "H", "V" or "M" (Fig. 4b),
- ϕ_d, u_d, v_d rotation angle, horizontal and vertical displacements of theoretical structure node in the "accurate" model, respectively, in loading schemes "H", "V" or "M",

U_d, U_{pd} elastic energy of the whole model and of its part within gusset plate boundaries in the "accurate" model, respectively, in loading schemes "H", "V" or "M".

In the analysis of the "simplified" (beam-element) model with the force method, one column matrix $[\mathbf{X}]$ of unknowns may be replaced by [R], that consists of the respective values obtained form the "accurate" model (M_{rd}, N_{rd}, T_{rd}). The set of governing equations, written by means of force method, is then as follows:

$$[\mathbf{K}] \cdot [\mathbf{R}] - [\mathbf{U}] = [\mathbf{0}], \tag{3.2}$$

where:

$[\mathbf{K}]$ is the flexibility matrix,

$[\mathbf{U}]$ is the displacement vector related to unknowns of the force method, caused by external loading, calculated traditionally the in respective statically determinate structure.

The comparisons of rotation angle, horizontal and vertical displacements of nodes of the "accurate" structure with the "simplified" one, as well as elastic energy of the whole model and of its part within the gusset plate boundaries, leads to the next five equations.

In all the equations, the unknowns are: near-node element cross-sectional area A'_r (stiffness $E \cdot A'_r$) and moment of inertia I'_r (stiffness $E \cdot I'_r$). Each of the equations may be rewritten in the following way:

$$\sum_r \left(\frac{1}{I'_r} \cdot \int_{s'_r} \mu'_{ir} \cdot ds'_r \right) + \sum_r \left(\frac{1}{A'_r} \cdot \int_{s'_r} \eta'_{ir} \cdot ds'_r \right) + \sum_r \Omega_{ir} = 0, \tag{3.3}$$

where:

index "r" refers to summation over the truss members, r, p are given in (3.1), s'_r refers to the part of truss member "r" within the gusset plate boundaries, I'_r, A'_r are the cross-sectional area and the moment of inertia of the near-node element "r", $\mu'_{ir}, \eta'_{ir}, \Omega_{ir}$ are constants.

Equations of the type (3.3) are linear with respect to inverses of I'_r and A'_r. They could be rewritten in the form:

$$[\mathbf{\Psi}] \cdot [\bar{\mathbf{S}}] + [\mathbf{\Omega}] = [\mathbf{0}], \tag{3.4}$$

where:

$[\mathbf{\Psi}]$ is the coefficient matrix of $[3 \cdot (p-1)+3+2] \times (2 \cdot p)$ dimension,

$[\bar{\mathbf{S}}]$ is the matrix of inverses of the cross-sectional area and moment of inertia of near-node elements:

$$(3.5)\qquad [\bar{\mathbf{S}}] = \begin{bmatrix} (I_1')^{-1} \\ \vdots \\ (I_p')^{-1} \\ (A_1')^{-1} \\ \vdots \\ (A_p')^{-1} \end{bmatrix},$$

$[\boldsymbol{\Omega}]$ is a matrix of constants.

Given "p" truss members, there are $[3\cdot(p-1)]+3+2=3\cdot p+2$ equations (conditions to be fulfilled) and $(2\cdot p)$ unknowns (A_r' and I_r' for each truss member). It is easily observed that the description of the problem has more conditions than unknowns. It means that the problem does not have explicit solution. It should be emphasised that the problem is how to identify $2\cdot p$ unknowns satisfying $3\cdot p+2$ conditions. So the solution that meets all the conditions with the smallest error should to be found, and it is proposed and analysed in the following sections.

3.3. Iterative procedure

The stiffnesses of the near-node elements are determined by the iterative procedure. The results of the analyses of dynamically modified "simplified" model and the "accurate" one are compared. Iterational solution bases on the error estimates that includes the sum of square differences of internal forces, displacements and elastic energy assessment in the "simplified" model with respect to the "accurate" solution.

The iteration scheme is given in Box 1 ("k" is the number of iteration step).

Box 1.

1. analysis and solution of the "accurate" model $\Rightarrow [\mathbf{D}]$,
2. k:=0; assumption of initial near-node element stiffnesses $(E\cdot I_r')$, $(E\cdot A_r')$,
3. k:=k+1; analysis and solution of the "simplified" model $\Rightarrow [\mathbf{U}]_k$,
4. calculation of $[\mathbf{U}]_k$ error with respect to $[\mathbf{D}] \Rightarrow \Delta_k$,
5. $\Delta_k<\Delta_{k-1} \rightarrow$ modification of $(E\cdot I_r')$ and $(E\cdot A_r') \rightarrow$ 3. (loop)
$\quad\Delta_k>\Delta_{k-1} \rightarrow$ 6.,
6. solution = stiffness distribution as in the iteration step number $(k-1)$.

Matrices $[\mathbf{D}]$ and $[\mathbf{U}]_k$ consist of the solutions of the analysed loading schemes: "H", "V", "M", "P_r", "$H+V+M$". The schemes: "H", "V" and "M" are realised according to Fig. 4. "P_r" refers to "p" loading schemes of nodal concentrated force, coaxial with each truss member ("p" – number of truss members). "$H + V + M$" – horizontal and vertical forces and bending moment acting together.

In the "accurate" model, in addition to the values given in (3.1), the expressions referring to nodal bending moments (at the joint) in the "simplified" model are calculated (M'_{rd} – for the truss member "r").

In the "simplified" model, the iterative procedure begins with the assumption that near-node element stiffnesses are equal to the respective truss member stiffnesses at the connection with gusset plates. Besides the internal forces, displacements and elastic energies ($M_r, N_r, T_r, M'_r, u, v, \phi, U, U_p$), as for the "accurate" model, the error criterion Δ is calculated according to (3.6), in each iteration step, for the loading scheme "$H + V + M$":

$$\Delta = \sum_r \left[\left(\frac{M_{rd} - M_r}{M_{rd}} \right)^2 + \left(\frac{M'_{rd} - M'_r}{M'_{rd}} \right)^2 + \left(\frac{N_{rd} - N_r}{N_{rd}} \right)^2 + \left(\frac{T_{rd} - T_r}{T_{rd}} \right)^2 \right] + \left(\frac{u_d - u}{u_d} \right)^2 + \left(\frac{v_d - v}{v_d} \right)^2 + \left(\frac{\phi_d - \phi}{\phi_d} \right)^2 + \left(\frac{U_d - U}{U_d} \right)^2 + \left(\frac{U_{pd} - U_p}{U_{pd}} \right)^2 , \tag{3.6}$$

where:
index "r" is described in (3.3), $M_{rd}, N_{rd}, T_{rd}, M'_{rd}$ are the bending moment, the longitudinal and transversal forces at the support of truss member "r" and the expressions referring to the nodal bending moment (at the joint) in the truss member "r", respectively, in the "accurate" model, in the loading scheme "$H + V + M$", M_r, N_r, T_r, M'_r – as $M_{rd}, N_{rd}, T_{rd}, M'_{rd}$, in the "simplified" model, ϕ_d, u_d, v_d are the rotation angle, the horizontal and vertical displacements of theoretical structure node, respectively, in the "accurate" model, in the loading scheme "$H + V + M$", ϕ, u, v – as ϕ_d, u_d, v_d, in the "simplified" model, U_d, U_{pd} are the elastic energy of the whole model and within the gusset plate boundaries, respectively, in the "accurate" model, in the loading scheme "$H + V + M$", U, U_p – as U_d, U_{pd}, in the "simplified" model.

The near-node element stiffness distribution that generates the smallest Δ value (3.6) is regarded as the iteration solution.

To ensure the uniqueness of the solution, various starting stiffnesses of near-node elements were tested. In the cases under analysis, the minima of Δ (3.6) were obtained at the same point with a high accuracy. Additional analysis also proved that iterative procedure is not sensitive to variation of the loading scheme used to calculate Δ (determining the iterational solution).

The near-node element stiffnesses are modified on the basis of the error of internal forces and displacements assessment provided by the "simplified" model solution, with respect to the "accurate" model solution, according to (3.7):

$$(3.7)\qquad I'_{r\,\mathrm{mod}} = k_{rI} \cdot I'_r \qquad \text{and} \qquad A'_{r\,\mathrm{mod}} = k_{rA} \cdot A'_r \,,$$

where:

$I'_r, A'_r, I'_{r\,\mathrm{mod}}, A'_{r\,\mathrm{mod}}$ are the moment of inertia and cross-sectional area of the near-node element "r", in the current and following iteration step, respectively, k_{rI}, k_{rA} are coefficients modifying the $(E \cdot I'_r)$ and $(E \cdot A'_r)$ stiffnesses of the near-node element "r" in the "simplified" model, according (3.8) and (3.9), respectively:

$$(3.8)\qquad k_{rI} = \sqrt{\frac{\left(\dfrac{M'_{rd}}{\phi_d}\right)}{\left(\dfrac{M'_r}{\phi}\right)}} = \sqrt{\left(\frac{M'_{rd}}{M'_r}\right) \cdot \left(\frac{\phi}{\phi_d}\right)} = \sqrt{\omega_{M'r} \cdot \omega_\phi} \,,$$

$$(3.9)\qquad k_{rA} = \sqrt{\frac{\left(\dfrac{N_{rd}}{\Delta L_{rd}}\right)}{\left(\dfrac{N_r}{\Delta L_r}\right)}} = \sqrt{\left(\frac{N_{rd}}{N_r}\right) \cdot \left(\frac{\Delta L_r}{\Delta L_{rd}}\right)} = \sqrt{\omega_{Nr} \cdot \omega_{\Delta Lr}} \,,$$

M'_{rd} and N_{rd} are the nodal bending moment in the element "r" in the "M" loading scheme and the longitudinal force at the support of element "r" in the "P_r" loading scheme, in the "accurate" model, respectively, M'_r and N_r – as M'_{rd} and N_{rd}, in the "simplified" model, ϕ_d and ϕ are the rotation angles of theoretical structure node in the "accurate" and "simplified" model in the "M" loading scheme, respectively, ΔL_{rd} and ΔL_r are the length variations of the element "r" in the "accurate" and "simplified" model in the "P_r" loading scheme, respectively, $\left(\dfrac{M'_{rd}}{\phi_{rd}}\right)$ and $\left(\dfrac{M'_r}{\phi_r}\right)$ – flexural stiffness ratio of near-node region of the element "r" in the "accurate" model and flexural stiffness ratio of near-node element "r" in the "simplified" model, respectively, $\left(\dfrac{N_{rd}}{\Delta L_{rd}}\right)$ and $\left(\dfrac{N_r}{\Delta L_r}\right)$ – axial stiffness ratio of near-node region of the element "r" in the "accurate" model and axial stiffness ratio of near-node element "r" in the "simplified" model, respectively, $\omega_{M'r}$ and ω_ϕ are coefficients modifying $(E \cdot I'_r)$ stiffness of the near-node element "r", depending on the nodal bending moments and rotations, respectively, ω_{Nr} and $\omega_{\Delta Lr}$ are coefficients modifying $(E \cdot A'_r)$ stiffness of the near-node element "r", depending on the axial forces and length variations, respectively.

4. Experimental verification

4.1. Scope and aim of the laboratory tests

The tests of two bridge truss girders, "A" and "B", were carried out (see Fig. 7 for details). The aim was to determine vertical displacements of the top flange nodes and internal forces distribution in the vicinity of bottom flange nodes in midspan (Fig. 7), to verify the concepts described above and to prove the applicability of numerical simulations.

The laboratory experiment, the used specimens, testing procedure and detailed results are discussed in [10].

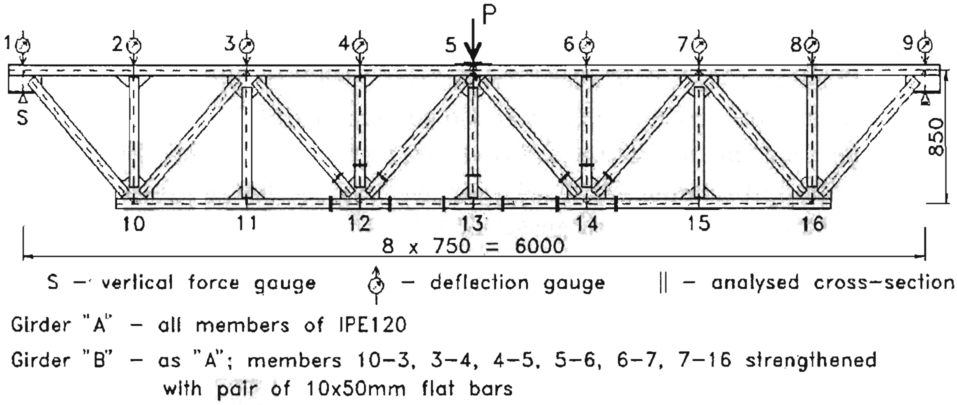


FIG. 7. Scheme of tested girders.

4.2. Iterative procedures for tested girders joints

The types of girder A nodes are divided into 4 types. Fig. 8 shows two of them: the first is called "*ksk*" (for instance the nodes 3, 12) and the second "*ks*" (10, 16). Other types are: "*s*" (for instance the nodes 2, 11) and "*k*" (1, 9). Nodes of the girder B with strengthened members were divided also into 4 types: "*ksk*" with strengthened cross-bracing and flange member (3, 7), "*ksk*" with strengthened flange members (5), "*s*" with strengthened flange members (4, 6), "*ks*" with strengthened cross-bracing (10, 16). Iterative procedure was then applied to all types of nodes.

Figure 9 shows the convergence of the iterative process for two types of nodes shown in Fig. 8. Variations of the k_I and k_A coefficients (referred to as "$k[I]$" and "$k[A]$") are given. The value of the "m" coefficient, that is the relationship between Δ at the given iteration step and the smallest Δ for the respective iteration (denoting the iteration solution), is also given. Coefficients of k_A and k_I converge unilaterally to unity and are very close to the value $(1,00 - 1,03)$ in the iteration step regarded as the solution.

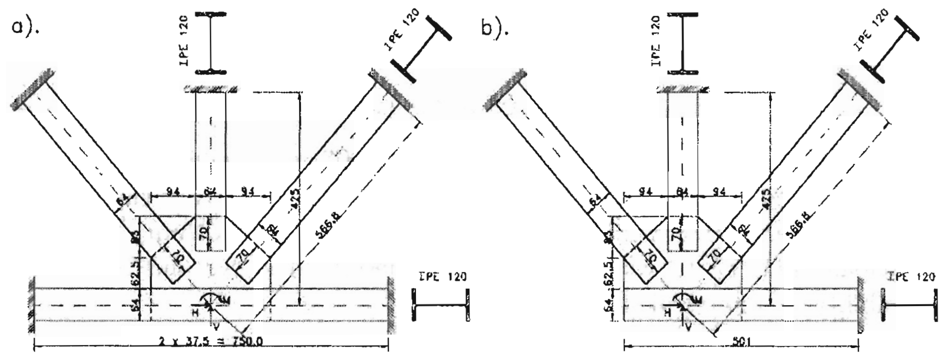
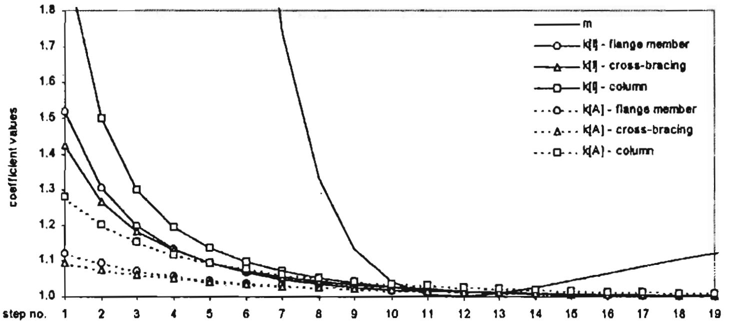
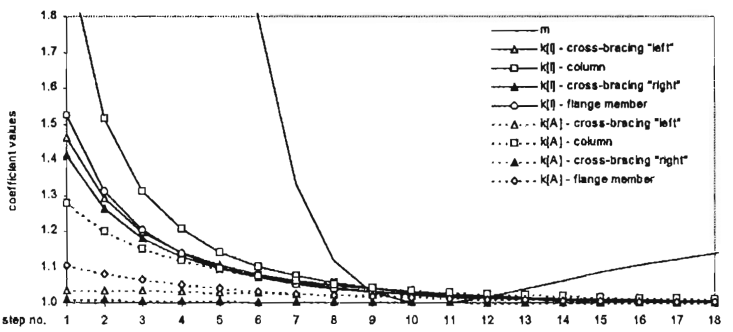


FIG. 8. Scheme of "cut-out" joints: a) "ksk", b) "ks".

Iterational solutions for the two considered types of nodes are presented in Table 2. Symbols "ksk", "ks", "left", "right" are used according to Fig. 8. I'_r, A'_r are the moment of inertia and cross-sectional area of the near-node element "r" in the "simplified" model, in the iteration step regarded as the solution. Symbols I_r, A_r are the initial values of I'_r, A'_r, respectively (as for the respective truss member at the connection with gusset plate).



a)



b)

FIG. 9. Iterations of near-node element stiffness for: a) joint "ksk" (girder A), b) joint "ks" with strengthened cross-bracing (girder B).

Table 2.

Truss member	$\frac{I'_r}{I_r}$	$\frac{A'_r}{A_r}$	Truss member	$\frac{I'_r}{I_r}$	$\frac{A'_r}{A_r}$
joint "*ksk*" (A) (column axis is the centre line)			joint "*ks*" (B) (strengthened cross-bracing "left")		
bottom flange members	3.66	1.69	cross-bracing "left"	3.56	1.31
cross-bracings	3.37	1.59	column	7.07	2.71
column	7.00	2.92	cross-bracing "right"	3.16	1.06
–	–	–	bottom flange member	3.62	1.54

4.3. Experimental verification

Two "simplified" (beam-element) numerical models of experimentally tested girders were analysed:
a) "classical" model (model C) – rigid nodes of negligible dimensions,
b) modified model (model M) – with additional near-node elements.

The obtained results were compared with laboratory evidence.

Figure 10 shows vertical displacements of girders A and B obtained in the analysis of two models: C (dashed line) and M (solid line) and recorded during the tests (marked with circles). Model C assessment is very conservative and may lead to wrong conclusion about the actual global stiffness. Theoretical results obtained using model M are in good agreement with the recorded values.

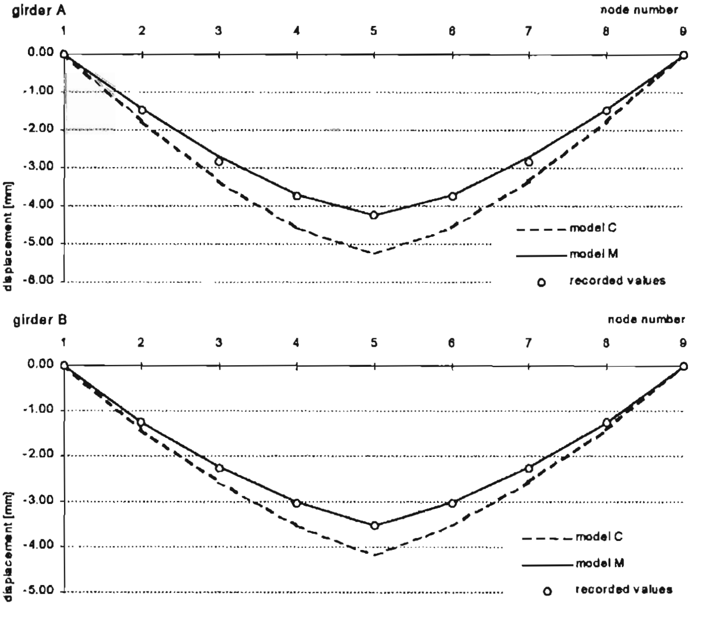


FIG. 10. Theoretical (computational) and recorded vertical displacements (top flange node numbers are given on the horizontal axis).

Longitudinal forces calculated on the basis of recorded strains and additional FEM analysis of normal stress distribution differ from the M model analysis results by 2 – 6%. Bending moments are small. Thus the values calculated on the basis of the recorded strains suffer from larger relative estimation errors than the values of longitudinal forces.

5. Conclusions

Final remarks are as follows:

1. Local variations of element stiffnesses due to gusset plate dimensions should be regarded in numerical models.

2. The influence of joint dimensions may be taken into account by introduction of stiffer near-node elements. Their lengths may be related to gusset plate dimensions and stiffnesses determined by iterative procedure. This bases on the comparison of the "accurate" (shell) and modified "simplified" (beam) model analysis results.

3. The numerical tests show that, for welded bridge truss girders, the "simplified" model with near-node elements reduces the error of assessment of structural global stiffness significantly.

4. The accuracy of internal forces assessment in the vicinity of nodes by "simplified" models requires further research.

5. The presented procedure of determining near-node element stiffness may be employed to set charts or tables helpful in the engineering design practice.

Acknowledgement

The Poznań University of Technology grants 12–929–DS/01 and 11–047–BW/01 are kindly acknowledged.

References

1. Abaqus Users Manual, version 5.8, Hibbitt, Karlsson, Sorensen, Inc., 1999.
2. J. Bień, Cz. Machelski, *Computer Models of Truss Rirder Spans of Prince J. Poniatowski Bridge in Warsaw* [in Polish], Inż. i Bud., **6**, 200-203, 1992.
3. Z. Borowiec, *Calculating Near-Node Forces in Elements of 3D Stocky Frame* [in Polish], Archives of Civil Engineering, **XVIII**, 1, 87-101, 1972.
4. Z. Borowiec, *Modification of Bar Static Scheme of 2D Stocky Frame* [in Polish], Inż. i Bud., 6, 278-282, 1974.
5. M. Cholewo, M. Sznurowski, *Railway Bridges* [in Polish], Wydawnictwa Komunikacyjne, Warszawa 1958.

6. J. KARLIKOWSKI, *Concept of Strengthening of Railway Bridge of Small Global Stiffness* [in Polish], The Ninth International Conference Metal Structures, Kraków, 26-30 June 1995.

7. W. NOWACKI, *Structural mechanics* [in Polish], PWN, 1964.

8. G. RAKOWSKI, L. SZUBERT, *Modelling of Node of Bar Structure* [in Polish], Archives of Civil Engineering, **XXIX**, 1-2, 37-44, 1983.

9. A. RYŻYŃSKI, W. WOŁOWICKI, J. SKARŻEWSKI, J. KARLIKOWSKI, Steel Bridges [in Polish], PWN, Warszawa 1984.

10. W. SIEKIERSKI, *Numerical Modelling of Bridge Truss Girders With Regard to Actual Joint Dimensions*, PhD thesis [in Polish], Warsaw Academy of Technology, 2000.

11. F. SZELĄGOWSKI, Metal Bridges [in Polish], part I, Wydawnictwa Komunikacji i Łączności, Warszawa 1966.

Received January 1, 2001; revised version July 18, 2001.