

HEAT TRANSFER PROBLEMS IN ORTHOPAEDICS

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The present paper presents a comprehensive review of the heat transfer problems in orthopaedics. The emphasis is put on presenting clinically relevant issues along with the purpose and motivation for studying the presented problems. The available experimental methods and results are presented and the modelling approaches are described – mathematical formulations and numerical results.

The first of the problems studied is the bone cement heating during cemented implantations, special attention being paid to modelling the kinetics of the acrylic bone cement polymerisation. Next, the heat production during drilling and sawing of the bone is discussed. Eventually, the results concerning frictional heating of the articulating joints are presented.

1. INTRODUCTION

In the present paper thermal problems specific to orthopaedics are reviewed. They can be distinguished as the heat transfer phenomena occurring during and after orthopaedic operations in the direct vicinity of the operation site. The investigations of these processes are important for the following reasons:

- development of “thermally safe” surgical procedures such as implant fixation, bone drilling etc.,
- manufacturing implants that have better thermal properties,
- assessment of the properties of the existing prosthetic systems, their working temperatures and heat-accelerated wear rates.

The heat loads pertinent to orthopaedics can be of short duration and high intensity (heating encountered during operation or treatment) or can consist of prolonged and cyclic elevation of temperature during the functioning of an artificial joint. In the first case, the negative consequence of the heating manifests after a quite short period of time — the bone cells die and are replaced by the fibrous bone tissue. This may lead to instability of the fixation and loosening.

On rare occasions of the thermal treatment of the bone, dying of the bone cells is the desired effect, [70].

In the case of the long term, cyclic thermal loads the thermal damage may be manifested postoperatively after a quite long time.

In the present paper we concentrate on three main areas, namely the heat dissipation after the bone cement polymerisation, the dissipation of the heat produced during bone drilling and sawing and dissipation of the frictional heat produced during artificial joint articulation. Main features of these problems are outlined in Table 1.

Table 1. Basic types of heat transfer problems related to joint replacement

	bone cement polymerisation	bone drilling and sawing	frictional heating of the implant
range of application	all cemented implants, hip endoprostheses (acetabular and femoral parts)	joint replacements of any kind; fixation of screws in bone	joint replacements of any kind
time and duration of exposure to elevated temperature	during and immediately after implantation, duration: several minutes	prior to implantation, duration: a few minutes	during joint operation, temperature cycles repeated many times during the service life of artificial joint
location	bone tissue-bone cement interface, bone cement domain	cut surface/drilled hole	articulating surfaces interface (acetabular cup and prosthesis stem in hip implants)
other physical phenomena affected	bone cement polymerisation (quantity of toxic monomer leftover), thermal bone necrosis, aseptic loosening of the prosthesis	thermal bone necrosis, surgical trauma	acetabular cup wear and material degradation
materials	metals, bone cement mixtures (PMMA based), bone and surrounding tissue	bone, metals	UHMWPE, ceramics, metal alloys, bone and surrounding tissue
max. temperatures	up to approx. 60°C at the bone surface and over 100°C within cement domain (dependent on the type of cement)	over 150°C (sawing without coolant)	around 43°C in the middle of the prosthesis head and less than 40°C on the surface of bone

Another class of problems related to the influence of high temperature on hard tissue, not included in Table 1, is related to various hyperthermic treatments of bony sarcoma or adjacent soft tissues. Laser techniques used in periodontology, in the treatment of oral diseases, are a good example, [63]. The freezing and lyophilisation of the bone fragments before the storage and implantation in the case of heterogenous bone implants is also not mentioned in Table 1 since essentially no living tissue is involved in these techniques.

In investigations of the thermal problems outlined, various experimental techniques, *in vitro* as well as *in vivo* are usually used. Also the biomaterials (i.e. the materials, produced by man and used for settlement in the living organism in the role of implant, fixation or substitution) undergo experimental testing to determine their various thermal properties.

From the physical point of view, the solution of the problems characterized in Table 1 consists in finding the usually transient temperature distribution in highly heterogenous tissue domains subject to heat fluxes of various origin. The practical information to be obtained is usually the value of the highest temperature within the living tissue and the duration of the heating. Additional analysis can provide the degree and extent of the thermal damage, deterioration of the mechanical properties (e.g. creep of the polyethylene acetabular cups, microcracking of the bone cement mantle), possible chemical damage (e.g. by the bone cement radicals) etc.

Below we describe the issues contained in Table 1 presenting the specific needs and purposes of the related research, the experimental works and the developed theoretical models along with the results of numerical simulations. Each time we conclude with the list of applications and improvements possible thanks to the presented investigations.

2. HEAT GENERATION DURING BONE CEMENT POLYMERISATION - HIP ENDOPROTHESIS

The physical situation is sketched in Fig. 1 (intramedullary component of cemented hip prosthesis). The stem of the endoprosthesis is fixed in the medullar cavity of the femur with the aid of polymethylomethacrylate (PMMA) bone cement which is inserted by the surgeon in a "doughty" state and then polymerises *in situ*. The same pertains to the acetabular cup. Cemented implants are also used in the case of the knee joint reconstruction, [55, 56]. The polymethylomethacrylate bone cement is also often used for filling the voids in bone and for fixation of joints other than hip joints (acromioclavicular joint, elbow joint etc.), see [45] for an example of use of PMMA in stabilization of fossa component of temporomandibular joint prosthesis.

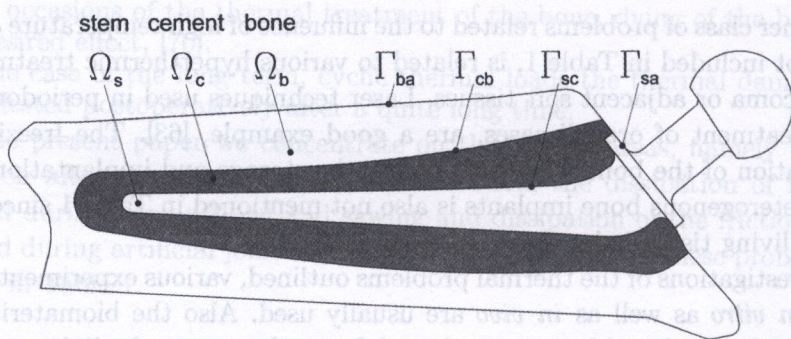


FIG. 1. Intramedullary cemented part of femoral prosthesis, the respective domains are labelled with Ω and domain boundaries with Γ supplemented with obvious subscripts.

The popularity of cemented implants originates from the fact that the undesired effect of "stress-shielding" is reduced in such a setup, when compared to the cementless ones. Stiffness of the cement mantle is intermediate between that of the metal prosthesis stem and that of the bone. Statistical data on long-term survival also support this point [52].

2.1. Purpose of the investigations

Due to the implantation of the cemented prosthesis several dangers may arise. Here, we concentrate on those associated with thermal problems. The exothermal polymerisation of the bone cement *in-situ* can be the cause of:

- *Thermal bone necrosis* – the elevated temperature at the site of the implantation is applied directly to the bone. In consequence bone, resorption may occur leading to aseptic loosening [44].
- *Chemical damage to the bone* – some researchers support the view that the free radicals, present at the early stages of the polymerisation along with the leftover monomer substantially contribute to the bone necrosis [40, 45, 47].
- *Pre-cracking of the cement mantle* – large elevations and substantial gradients of the temperature during the polymerisation are supposed to cause a high level of stress and consequently the damage of this layer. This hypothesis is supported by the experimental and numerical findings of LENNON and PRENDERGAST [38].

Thus the analysis of the temperature distribution in the implant system during and after polymerisation would allow the assessment of the thermal and chemical dangers to the living tissue as well as the assessment of the mechanical dangers to the load-bearing cement mantle. It is also worth noting that phys-

ical properties of the cement are most probably dependent on the history of polymerisation and hence on the temperature history.

2.2. Experimental methods

The pertinent experimental works available can be divided into the following categories:

- measurements of the thermal properties of the biomaterials used in arthroplasty (including identification of the polymerisation kinetics), [3, 44, 9],
- measurements of the thermal properties of the bone tissue *in vitro*, [8],
- measurements of the temperature rise in the implant system *in vivo*, [54, 59, 72, 81, 18],
- measurements of the temperature rise in the simulated implant system, [19, 33, 45],
- long-term experiments on animals and follow-ups of the human cases designed to determine the effect of the arthroplasty on the bone, [59, 78].

For the task of identification of the polymerisation kinetics, the researchers nowadays usually choose *Differential Scanning Calorimetry* in the isothermal regime, [3, 44] and the references therein. The results are curve-fitted to a phenomenological model. Results of these and other experimental works are presented in Sec. 2.4.

In vivo and *in vitro* temperature measurements of hip joint have also been performed by researchers working on frictional heat generation and these experiments will be therefore described in the subsequent sections.

An extensive follow-up study of 28 human total hip arthroplasty cases was performed by WILERT *et al.* [78]. The specimens were obtained by means of biopsy during the reoperation and during the autopsy – from patients that died from reasons unconnected with the hip problems. The samples taken at various times after the implantation were prepared and the observations of the bone-cement interface were made with the use of microscope. The main conclusion of their work can be summarized as follows: in the specimens taken at no more than three weeks postoperatively, a wide zone of necrosis (3 mm) was observed in the implant bed. This was attributed mainly to the thermal effect of the polymerising cement. There were also some infarct-like areas of necrosis at some distance from the cement, caused probably by the vascular disruption during the operation. Active repair and remodelling continued for one year or longer. Necrotic bone was remodelled, reinforced or replaced.

The permanent implant bed showed a thin connective-tissue membrane surrounding the cement. The bone remodelling continued at minimum rate. For detailed, instructive observations and conclusions the reader is referred to [78].

2.3. Modelling

2.3.1. Common assumptions. When modelling the heat transfer in the cemented bone-implant system, the following simplifications are usually imposed:

1. Axial symmetry of the model along the axis of the stem, see [44, 65, 68]. In [29] a two-dimensional, axisymmetric model was considered, in [67] the plane thermal problem was considered.
2. Perfect thermal contact between the materials [68, 67]. In [44] several areas of null contact are considered on an otherwise perfect thermal interface; in [29] and in [65] nonzero thermal contact resistance was allowed.
3. Temperature-independent material moduli, see [29, 44, 65, 68, 67].
4. Material isotropy, [29, 44, 65, 68, 67]. This assumption can be easily generalized but the lack of reliable material coefficients is the major obstacle. There are a few experimental measurements of anisotropic thermal properties of the tissue and these usually do not encompass all the components of the thermal conductivity (or diffusivity) tensor, [16] and [50].
5. Constant boundary conditions, independent of temperature, see [29, 44, 65, 68, 67]. The temperature in a biological system is thus assumed to vary over a range small enough to allow for changes in the boundary convection to be disregarded. Otherwise the convection coefficient should be reiterated for every computed boundary temperature. In view of the relatively small temperature variation, this should be regarded as a reasonable assumption.
6. Excluding the surrounding muscle tissue from the model, see [29, 44, 65, 68, 67]. Modelling the soft tissue presents difficulties since the effect of vascularity must be accounted for. It can be done in a simple way by adopting one of the known continuous bio-heat models, e.g. Pennes equation, Weinbaum-Jiji equation, hybrid models, etc., [51, 76, 84]. Other possibility is to construct a *vascular model* accounting for the exact architecture of blood vessels in the limb. For a review of modelling of the soft tissues see [66].

Various simplifications are also often introduced at the stage of modelling of the thermal effect of bone cement polymerisation, i.e. PMMA polymerisation kinetics. These are described in Sec. 2.3.3.

Simplifications listed above originate mainly from the lack of knowledge that would be required to deal with more sophisticated models. For example, the thermal conductivity of the bone tissue as reported in numerous papers on the subject (see e.g. [8, 13, 29, 44]) varies from 0.26 to 0.60 W/mK and the specific heat varies from 1150 to 2370 J/kgK, etc. These measurements were performed on fresh and on the dry human cadaveric femur. For detailed report from mea-

measurements of variation of thermal properties along femur and its fluid content, the reader is referred to the work of BIYIKLI *et al.* [8].

Generally, the bone tissue material parameters depend on multiple factors, mainly the bone composition (cortical/spongy) and bone marrow and water content. These properties vary along the bone and are different for different individuals.

One way to cope with this difficulty is to construct mathematically tractable model of heat transfer of bone tissue on the basis of anatomical observation, see [23] for the description of bone structure. While being more realistic, this approach presents considerable difficulties, arising from the considerable scatter of measurements of bone properties and the fact that the very architecture of the bone changes with time [23].

Other possible way of approaching the problem is to use experimental results to obtain the distribution of values of average material properties, needed to solve the problem within the framework of the classical heat conduction model, e.g. [8]. While less prospective, this approach allows immediate construction of simple theoretical models. Such an approach was used in all the papers mentioned.

2.3.2. Formulation of the heat transfer problem. The temperature T throughout the considered domain Ω is an unknown function. In the models that exclude surrounding soft tissue, Ω is essentially reduced to the sum of geometrical domains of prosthesis stem, cement and bone domains, when intramedullar part is considered, or prosthesis head, acetabular cup, cement and pelvis fragment domains, when the acetabular part is studied.

The starting point in construction of a mathematical model is the energy balance equation, [10, 79]

$$(2.1) \quad \rho c \frac{\partial T}{\partial \tau} + \nabla \mathbf{q} = q_v,$$

where T is the temperature, τ denotes time, \mathbf{q} is the heat flux vector, q_v is the volumetric heat production rate, c and ρ denote the specific heat and density, respectively. Taking into account the constitutive equation (Fourier's law):

$$(2.2) \quad \mathbf{q} = -\lambda \nabla T,$$

we obtain the Fourier-Kirchhoff equation:

$$(2.3) \quad \rho c \frac{\partial T}{\partial \tau} = \nabla \cdot (\lambda \nabla T) + q_v.$$

Equation (2.3) is satisfied separately in the stem, cement, bone and muscle domains ($\Omega_s, \Omega_c, \Omega_b, \Omega_m$).

$$\begin{aligned}
 \rho_s c_s \frac{\partial T}{\partial \tau} &= \nabla \cdot (\lambda_s \nabla T), & \mathbf{x} \in \Omega_s, \\
 \rho_b c_b \frac{\partial T}{\partial \tau} &= \nabla \cdot (\lambda_b \nabla T), & \mathbf{x} \in \Omega_b, \\
 \rho_c c_c \frac{\partial T}{\partial \tau} &= \nabla \cdot (\lambda_c \nabla T) + q_v, & \mathbf{x} \in \Omega_c, \\
 \rho_m c_m \frac{\partial T}{\partial \tau} &= \mathcal{B}(\mathbf{x}, T, T_{,i}, T_{,ij}), & \mathbf{x} \in \Omega_m.
 \end{aligned}
 \tag{2.4}$$

The soft tissue domain is denoted by Ω_m . Since different bioheat equations can be used, the equation is not stated explicitly in the above formulation and instead the \mathcal{B} symbol is used. For a review of available bio-heat equations the reader is referred to [66]. The subscripts s, b, c, m denote the prosthesis stem, bone, cement and soft tissue properties, respectively. Additionally, the boundary conditions need to be imposed. The boundary of the considered domain is denoted

$$\Gamma = \partial(\bar{\Omega}_s \cup \bar{\Omega}_c \cup \bar{\Omega}_b \cup \bar{\Omega}_m),$$

where bar denotes the closure of the domain. Only the Fourier boundary condition (also called the *Newton* boundary condition) is considered:

$$\lambda \frac{\partial T}{\partial \mathbf{n}} = \alpha(T_f - T), \quad \mathbf{x} \in \Gamma. \tag{2.5}$$

Here T_f and α are functions prescribed on the boundary Γ representing ambient temperature and convection coefficient respectively.

To complete the formulation of the problem, thermal contact conditions have to be specified. These are the continuity of the heat fluxes across the contact surfaces. The temperature is discontinuous across these surfaces with the jump proportional to the normal heat flux across the interface. The interfaces are denoted by $\Gamma_{bc} = \bar{\Omega}_b \cap \bar{\Omega}_c$ (the interface between bone and cement domains), $\Gamma_{cs} = \bar{\Omega}_c \cap \bar{\Omega}_s$ (the interface between cement and stem domains). The constitutive equation on the bone-cement interface is given by:

$$\lambda_b \frac{\partial T_b}{\partial \mathbf{n}_b} = -\lambda_c \frac{\partial T_c}{\partial \mathbf{n}_c} = \beta_{bc}[[T]], \quad \mathbf{x} \in \Gamma_{bc}. \tag{2.6}$$

As usually, $[[f]]$ denotes the jump of f ; n_b and n_c are outer normal vectors of the boundaries of the bone and cement domains respectively. Assuming perfect contact between the bone and the cement means that $\beta_{bc} \rightarrow \infty$ i.e. that temperature jump across the interface must be equal to zero and nonzero heat flux across the interface can nevertheless be maintained.

In Table 2 the values of the material coefficients collected from the literature are provided [5, 8, 13, 29, 44, 68] and [82]. It should be noted that these values

correspond to a certain type of prosthesis considered here (metal stem, polyethylene distal plug, [44]). As it is well-known there is a wide variety of materials used for prosthetic heads, acetabula, stems etc.

Table 2. Values of material coefficients.

	stem (metal)	femur (bone)	bone cement	polyethylene
thermal conductivity λ [W/mK]	14	0.26–0.60	0.17–0.21	0.29–0.45
specific heat c [J/kgK]	460	1260–2370	1460–1700	2220
density ρ [kg/m ³]	7800	1000 – 2900	1100	960

In Table 3 possible values of interface conductivities are provided, [29, 44]. The values for thermal contact conductivity between polyethylene acetabular cup and cement and articulating surface of prosthesis head are estimated by HUISKES in [29] to be of the order of 500 W/m²K. One may suspect higher value at the latter interface due to the presence of the synovial fluid. In [65] different values of the thermal contact conductivity β were used, namely in the range 10²–10⁶ W/m²K and the study revealed no significant change in the process when the values exceeding $\beta = 2000$ W/m²K were used. So, according to the estimations provided in Table 3, the determination of the value of the thermal contact conductivity of the bone-cement interface may prove important. Since experimental data on this topic are scarce (see [21] for a review of the recent developments) such research would be very helpful. Currently, we are planning to perform measurements of the thermal contact conductivity of the interface *in vitro*, in order to estimate the magnitude of the temperature jump across the interface in the conditions experienced *in vivo*.

Table 3. Values of interface conductivities/convection film coefficients, the values are in [W/m²K], after [29].

	cement	ambient
metal stem	1000–10000	50–100
bone	100–1000	500–10000

The values of interface conductivities have a significant influence on both the temperature distribution and on the monomer leftover. In Fig. 2 the simulated dependence between the thermal conductivity and the overall volume-averaged

final polymerisation fraction is presented. Details of the numerically simulated model are given in [65].

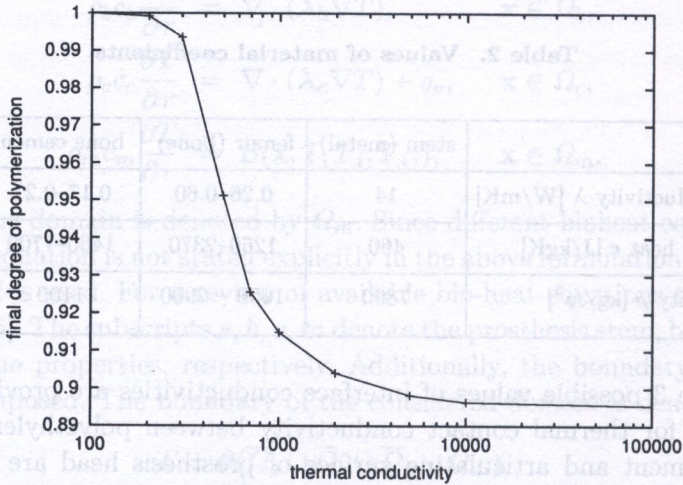


FIG. 2. Volume-averaged final polymerisation fraction dependence on the thermal contact conductivity [$\text{W}/\text{m}^2\text{K}$] taken uniform and the same for all the interfaces. Details of the model are given in [65].

2.3.3. Formulation of the polymerisation problem. Bone cement polymerisation is another physical phenomenon (apart from heat transfer) that requires mathematical description in the course of the problem formulation. Its modelling is undertaken for three basic purposes:

- 1) to gain information about the outcome of the polymerisation process in cement domain, i.e. resulting properties of the cement mantle (monomer leftover, porosity etc.),
- 2) to enable modelling of the temperature history throughout the cement,
- 3) to make modelling of the thermal stresses in the cement mantle possible.

It should be emphasized that the last reason mentioned receives recently more attention, [38, 39, 77].

To synthesize the PMMA polymerisation kinetics models existing in the literature in a systematic manner, we introduce the scalar variable called *polymerisation fraction* describing the progress of the polymerisation process at a given material point. This variable is defined in terms of the released heat fraction [3]:

$$(2.7) \quad w(\mathbf{x}, t) = \frac{1}{Q_{\text{total}}} \int_0^t q(\mathbf{x}, \tau) d\tau$$

where t denotes time, Q_{total} is the total heat of the polymerisation per unit volume and q denotes the local heat generation rate per unit volume. The above definition implies that at the time $t = 0$ the polymerisation fraction is zero.

Differentiating both sides of Eq. (2.7) one can see that the volumetric heat generation rate q is proportional to the rate of change of w i.e. the polymerisation rate. This quantity is described by the kinetics equation of the form

$$(2.8) \quad \frac{\partial w}{\partial \tau} = f(w, T, \tau),$$

where f is a prescribed function. Spatial dependence is not included since initial material homogeneity is assumed.

Different ways to approach the problem of the PMMA polymerisation coupled with heat transfer within the cement will be now presented in the order of increasing complexity.

Instantaneous polymerisation

This model assumes instantaneous polymerisation over the whole cement volume with the release of the heat equal to Q_{total} per unit mass, which uniformly heats up the cement domain. Only calculation of the temperature field is possible and therefore such model can not predict the monomer leftover. The function f on the r.h.s. of Eq. (2.8) for such a model takes the form of the Dirac delta function of time

$$f(w, T, \tau) = \delta(\tau).$$

Therefore modelling the heat generation in the cement is equivalent to imposing the appropriate initial condition:

$$(2.9) \quad T(\mathbf{x}) = \begin{cases} T_0 & \text{if } \mathbf{x} \in \Omega_s, \\ T_{\text{body}} & \text{if } \mathbf{x} \in \Omega_b, \\ T_0 + \frac{Q_{\text{total}}}{c_c} & \text{if } \mathbf{x} \in \Omega_c, \end{cases}$$

where T_0 denotes the ambient temperature, T_{body} is the normal body temperature and c_c is the specific heat of the cement. Then the transient equation of heat conduction is solved to obtain the maximum temperature rise on the bone domain boundary. Such a model of the acetabular implant was constructed and solved by JEFFERISS *et al.* [33] for the one-dimensional case. Figure 3 presents the results. Discontinuous initial condition (2.9) is distinguished by the thick line. As can be inferred from Fig. 3 the initial temperature of 70°C produces temperatures of the order of 50°C at the bone-cement interface in the first stage of the process of heat dissipation. JEFFERISS *et al.* [33] argue that the instantaneous polymerisation model will always yield higher tissue temperatures than

any other model, where the energy is released over the finite time period. Therefore the data presented in [33] are supposed always to reflect the "worst case" in terms of tissue damage. It should be observed, however, that this would only be the case if the tissue-damage criteria were based on the temperature only, not on the temperature history. The experimental data suggest that preheating lowers the ability of the tissue to withstand elevated temperatures. Therefore such a simplified model of heat release during polymerisation might not be satisfactory.

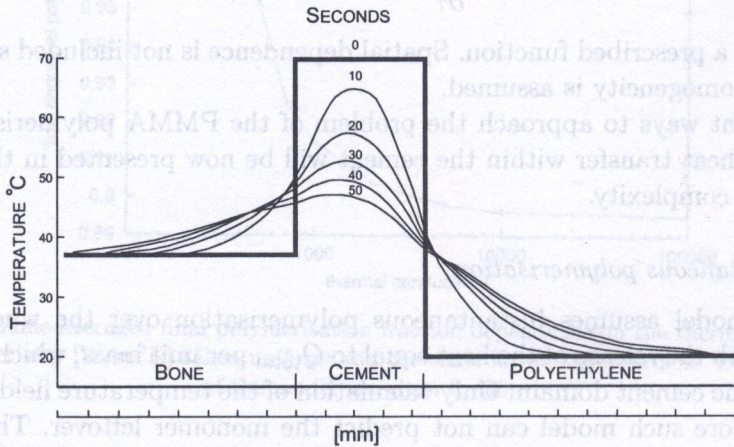


FIG. 3. Temperature field relaxation in the one-dimensional model of cemented implant with instantaneous polymerisation. Thick line denotes discontinuous initial condition, after [33].

Uniform, time-dependent polymerisation

In this model the polymerisation extends over the finite period of time and the heat generation rate function is assumed to have a prescribed form (obtained from the experiment). The function f on the r.h.s. of Eq. (2.8) is dependent neither on the temperature nor on the current polymerisation fraction and is thus given by

$$f(w, T, \tau) = p(\tau).$$

The heat generation rate is therefore uniform. The temperature field during polymerisation needs not to be uniform. This model gives also no information about the distribution of final monomer leftover but an attempt is undertaken to make the model resemble the real situation more closely. The properties of cement such as retardation time can be modelled here. Still only linear equation of heat conduction needs to be solved.

The function $p(\tau)$ is assumed to satisfy the requirement $\int_0^\infty p(\tau) d\tau = 1$ and apart from that is arbitrary (its shape is based on experimental data). Such

an approach was used by HUISKES [29] and by SWENSON *et al.* [68]. Figure 4 presents various polymerisation functions, i.e. integrals of $p(\tau)$, used in [29]. The rate of these functions determines the heat generation rate at any moment in time. It has been found that the choice of the polymerisation rate function affected mainly the temperature history, whereas the change in peak temperature was very small. In the finite element study of thermal bone necrosis RAMANIRAKA assumed constant cement heat power, [53].

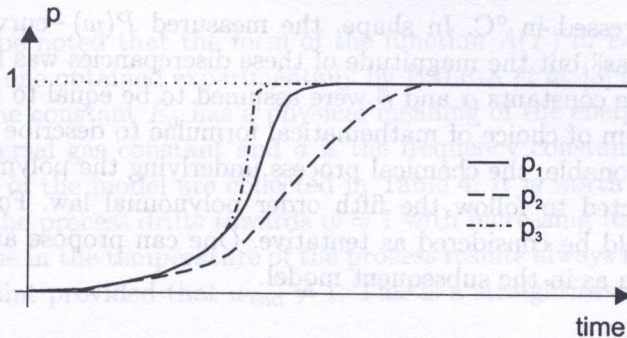


FIG. 4. Three polymerisation functions used by HUISKES [29].

The model of the instantaneous polymerisation can be viewed as a specific case of the more general, time-dependent polymerisation model but its greater simplicity justifies distinguishing it as a separate case.

Temperature-dependent polymerisation without monomer leftover

Such models describe the fact that the polymerisation rate is dependent on the temperature and the instantaneous polymerisation fraction. Time-dependence is neglected and the function f on the r.h.s. of Eq. (2.8) has the form

$$(2.10) \quad f(w, T, \tau) = u(w, T).$$

It is convenient to describe this function in a factorized form

$$(2.11) \quad u(w, T) = A(T)P(w).$$

The function P can be identified from experiments in the isothermal regime, while A is a temperature-dependent scaling factor. The function $P(w)$ must satisfy the requirement $P(1) = 0$ and $P(w) \geq 0$ since polymerisation is an irreversible process. Many experimental data indicate that also $P(0) = 0$ must hold; see [3] and the references cited therein. BALIGA *et al.* [3] proposed the following form of the function $P(w)$

$$(2.12) \quad P(w) = w^\beta(1-w)^\gamma,$$

where β and γ are constants independent of temperature.

The temperature-dependent scaling factor was obtained by means of curve-fitting and was computed by BALIGA *et al.* [3] on the basis of earlier experiments:

$$A(T) = 10^3 \times (-105116 + 13056T - 595.307T^2 + 12.736944T^3 - 0.12349128T^4 + 4.443296 \times 10^{-4}T^5)$$

where T is expressed in °C. In shape, the measured $P(w)$ -curves resembled "skewed parabolas" but the magnitude of these discrepancies was found negligible [3], hence the constants α and β were assumed to be equal to unity.

While freedom of choice of mathematical formulae to describe experimental data is unquestionable, the chemical process underlying the polymerisation can be hardly expected to follow the fifth order polynomial law. For this reason this model should be considered as tentative. One can propose an exponential description, such as in the subsequent model.

Temperature-dependent polymerisation with monomer leftover

This model uses multiplicative decomposition of the polymerisation rate function as presented in Eq. (2.11). In this model the assumption $P(1) = 0$ is replaced by its more general form $P(w^*) = 0$ while the remaining assumptions are unchanged. The quantity $0 \leq 1 - w^* \leq 1$ is the temperature-dependent equilibrium monomer leftover, in general not equal to zero. One notes that both terms of the r.h.s. of Eq. (2.11) become dependent on temperature but the factorized form is retained for clarity.

MAZZULLO *et al.* [44] proposed the bilinear variation of the function w^* with absolute temperature:

$$(2.13) \quad w^*(T) = \begin{cases} \frac{T}{T_g}, & \text{if } T \leq T_g, \\ 1, & \text{if } T > T_g, \end{cases}$$

where T_g is the *glass transition temperature*.

Consequently, the function P takes the form, see Eq. (2.12),

$$(2.14) \quad P(T, w) = \begin{cases} \frac{\alpha}{w^*(T)} w^{1-1/\alpha} (w^*(T) - w)^{1+1/\alpha}, & \text{if } w < w^*(T), \\ 0, & \text{if } w \geq w^*(T). \end{cases}$$

In comparison with the previous model the two independent exponents β and γ are expressed by a single constant α where $|\alpha| > 1$.

MAZZULLO *et al.* [44] experimentally obtained $\alpha = 9, 2$ (Howmedica Simplex-P cement). For such a high value, the deviation from the parabolic form is minimal.

The temperature-dependent scaling factor used by MAZZULLO *et al.* [44] takes the form of the Arrhenius-type relation:

$$(2.15) \quad A(T) = a \exp\left(-\frac{E_a}{RT}\right).$$

It should be noted that the form of the function $A(T)$ in Eq. (2.15) is very similar to the one obtained experimentally by BALIGA *et al.* [3] and presented in Eq. (2.13). The constant E_a has a physical meaning of the energy of activation, R is the universal gas constant and a is the frequency constant. The values of the constants of the model are collected in Table 4. It is worth noting that the end point of the process drifts towards $w = 1$ with increasing temperature. Also a fixed increase in the temperature of the process results always in the same shift of this end point provided that $w_{\text{end}} \neq 1$. This is a straightforward consequence of Eq. (2.13).

Table 4. Polymerisation constants used in Eqs. (13) – (15), after MAZZULLO *et al.* [44]

Q [J/kg]	a [1/s]	E_a [J/mol]	α	T_g [K]	B [J/molK]
193.0×10^3	2.6397×10^8	62866.0	9.2	378.0	8.3143

Among the presented phenomenological models, the model proposed by MAZZULLO *et al.* in [44] seems to be the most suitable one. However, the experimental investigation conducted by the first author reveals that the assumption of bilinear variation of equilibrium polymerisation fraction (see Eq. (2.13)) leads to the phenomena that are experimentally not observed [64] and therefore the model by MAZZULLO *et al.* is most probably not exactly correct.

The polymerisation models presented are of phenomenological nature. While constants for the models presented here correspond to commercially available surgical cements, the actual behaviour of the cement is also dependent on its exact chemical formulation. For some results on this topic the reader is referred to [12].

Instead of describing the observed cumulative effect (heat generation) via the preset curves one can also construct a model of polymerisation using the knowledge of the detailed sequence of chemical events (initiation, monomer activation, chain growth and termination) and their kinetics. Recently, HANSEN

[27] proposed a model that considers the polymerisation process composed of the separate sub-processes of initiation and chain growth. The heat generation rate is then proportional to the rate of disappearance of monomers. Thus, the model presented in [27] makes use of three additional spatial variables – the concentrations of initiator $[I]$, monomer $[M]$ and activated monomer $[M^\bullet]$. The kinetics is then described by the set of equations:

$$\begin{aligned}
 (2.16) \quad & \frac{\partial [I]}{\partial t} = -k_d(T)[I], \\
 & \frac{\partial [M]}{\partial t} = -2fk_d(T)[I] - k_p(T)[M][M^\bullet], \\
 & \frac{\partial [M^\bullet]}{\partial t} = 2fk_d(T)[I] \left(1 - \frac{2\nu_0 k_t^0(T) [M^\bullet]}{k_p(T) [M]} \right).
 \end{aligned}$$

Here k_d , k_p and k_t^0 are the temperature-dependent rate constants of Arrhenius-type. By f the efficiency of the free radicals is denoted.

The kinetic equations are coupled to heat conduction problem by means of the definition of the heat generation rate:

$$(2.17) \quad q_v = -Q_c \frac{\partial [M]}{\partial t}.$$

A more complex model was also proposed in [64].

We have not discussed here the modelling of the heat transfer in soft tissue layer. A variety of such models is available, [66]. Including them in a complete formulation is an important step in constructing the mathematical model of heat exchange in human limb or extremity [61, 80, 83].

2.4. Comparison with experimental data and comments

The ultimate goal of calculation of temperature/monomer leftover distribution throughout the cement mantle is to assess the quality of the chosen prosthesis fixation technique, estimate danger to the living tissue and to predict the probability of possible loosening of the prosthesis as an effect of the resulting damage. To this end one needs criteria of thermal/chemical damage to the bone tissue (bone necrosis) that one could compare with the computed values of the temperature and monomer leftover.

In general, the use of polymethylmethacrylate (PMMA) bone cement in the orthopaedic surgery may lead to threats. These can be classified as follows, [24, 33],

- 1) vascular disturbance at the site of implantation,
- 2) disruption of the cortical and marrow circulation,
- 3) thermal necrosis during polymerisation,
- 4) chemical necrosis during and after polymerisation,
- 5) introduction of residual stresses (thermal stresses) to the cement mantle.

Only points 2.3 and 2.5 are considered here, although the formulations presented encompass also the phenomena of incomplete polymerisation and allow, in principle to predict chemical damage to the tissue. For the experimental investigation of chemical trauma the reader is referred to LINDER [40], where cement dough was implanted into rabbit tibia and care was taken to minimise the mechanical and thermal injury. The reported chemical trauma was very limited, giving rise to speculation that the primary injury mechanism during implantation is of mechanical nature, i.e. vascular injury, blocking of Haversian canals by cement or intramedullary fat particles during drilling, etc.

The presented models can form a basis for a thermomechanical analysis. Thermally-induced residual stress can potentially lead to crack growth in the cement and can cause debonding and prosthesis loosening. The significant factor is also the cement porosity which causes local stress concentrations. KUSY [35] distinguishes two kinds of porosity: intrinsic and acquired, the former being the result of cement curing whereas the latter results from leaking out of the water-soluble components of the cement (e.g. antibiotics) over the period of months or years. Thus the overall porosity of the cement depends on its composition.

One of the most important factors contributing to the level of residual stress is the fact, that stress-locking occurs in the cement at the highest temperature attained during the course of polymerisation. This phenomena was observed during simultaneous measurements of the strain and temperature in a polymerising cement block [77].

LENNON and PRENDERGAST [38] performed experimental investigations of the model of the cement mantle and appropriate numerical analysis using the polymerisation model due to BALIGA *et. al.* [3]. The main finding is that cracks induced in the cement mantle during polymerisation are often perpendicular to the computed principal directions of stress. This supports the view that thermal stress induced by the heat of the polymerisation may significantly affect the fatigue life and mechanical properties of the cement mantle.

NUÑO and AVANZOLINI [48] performed a numerical analysis of the cemented implant system to determine if potential residual stresses at the cement-stem interface changes the stress distribution that arises in response to external loads. Commercial FEM package ANSYS 5.4 was used. The residual stresses were prescribed, basing on the earlier experimental and numerical evidence. No polymeri-

sation simulation was performed and the bone-cement was assumed to be fully bonded while, conversely, the stem-cement interface was assumed fully debonded (no adhesion) with press-fit. Details of the numerical simulation are given in [48]. The results showed a significant increase of the stress resulting from loading typical for the implant system due to the residual stress at the stem-cement interface. It should be noted, however, that the analysis was purely mechanical and the residual stresses were introduced by means of the press-fit, not by the thermal effects.

The issue of thermal bone necrosis has been investigated by many researchers. The results suggest two basic mechanisms. One is the collagen protein denaturation. According to SWENSON *et al.* [68] it takes place at temperature range 56–70° C. The second mechanism is caused by cellular death, which occurs at lower temperatures and is therefore more important. The results presented in [5, 6, 7, 29, 44, 68] and [70] point out the time-temperature dependence inherent in thermal necrosis criteria. For example, the temperature of 70°C is believed to kill cells instantly, 50°C needs to be maintained for 30 seconds and 45°C – for 5 hours. Higher temperatures (of order 70°C) are needed to destroy the regenerative capacity of the bone tissue, [29, 68].

From the available data a straightforward mathematical criteria, based on the 'additivity rule' has been constructed by MAZZULLO *et al.* [44]. Assume that the time necessary to cause thermal bone necrosis at a given temperature T is

$$(2.18) \quad \tau_c = M \exp\left(\frac{\mu}{R(T - T_{\text{ref}})}\right), \quad T > T_{\text{ref}}.$$

Obviously, as $T \rightarrow T_{\text{ref}}$, the time to necrosis τ_c becomes infinite and below the reference temperature T_{ref} no thermal damage processes take place. M and μ are model constants. In prescribed non-isothermal conditions the local measure of thermal bone tissue damage η can be constructed as an integral of fractions of exposure time at given temperatures over time

$$(2.19) \quad \eta(\mathbf{x}, \tau) = \int_0^\tau \frac{dt}{\tau_c(T(\mathbf{x}, t))}.$$

The values of η equal to or in excess of unity indicate local bone necrosis. FUKUSHIMA *et al.* [22] assumed the criterion similar to (2.19) but they took the function under the integral to be an arbitrary polynomial of the third order. They identified it via the least-squares method as:

$$(2.20) \quad \eta(\mathbf{x}, \tau) = \int_0^\tau (0.0127272T^3 - 1.92419T^2 + 97.20029T - 1640.088)dt,$$

where the temperature is expressed in degrees centigrade.

This criterion is analogous to Palmgren–Miner hypothesis of linear damage accumulation in fatigue mechanics and has similar drawbacks. For example, it does not take into account the succession of the different stages of thermal load; thus various intensity heating periods will produce the same ‘damage’, irrespective of their relative order.

The necessary constants for the model were obtained by MAZZULLO *et al.* in [44] by means of linear regression analysis and are given in Table 5.

Table 5. Experimental constants for thermal bone necrosis criteria appearing in Eq. (2.18), after MAZZULLO *et al.* [44].

M [s]	μ [J/mol]	T_{ref} [K]
1/27.4	1000.0	310.0

While it seems to be a good measure for a single heat shock cellular damage, this criterion may be insufficient when dealing with repeated thermal bone loading as the living tissue can probably adapt to higher temperatures by means of producing ‘heat shock’ proteins. It is not known whether they also exist in normal human joint, but this is feasible as natural joints heat up by ca 2.5° C during walking and probably more during more intensive activities, [71].

Nowadays researchers tend to believe that heat-induced bone necrosis is the secondary mechanism of tissue damage during implantation of cemented endoprostheses.

RECKLING and DILLON [54] concluded their measurements of temperature at the bone-cement interface in acetabular component with the statement that temperatures high enough to cause bone necrosis are not attained during the polymerisation of the cement. Conversely, SCHATZKER *et al.* [24] measured temperatures of order of 75–95°C at the bone-cement interface *in vivo*. This value is high enough to cause thermal bone necrosis.

The toxic leftovers of the polymerisation process are considered to be a more challenging problem. The negative influence of residual monomer, which is a powerful fat solvent and free radicals released from the cement dough, is substantially prolonged when compared with pure high-temperature damage. WILLERT *et al.* cit. by SWENSON *et al.* [68] reports 3 mm necrotic zone in 3-weeks postoperative specimens. Possible damage mechanisms were identified to be PMMA polymerisation heat, free radical release and vascular damage. GOODMAN *et al.* [24] investigated the influence of acrylic cement on proximal humeral and proximal tibial methaphysis of the dog in the minimally loaded state. The animals were

killed at 2, 4 and 5 months after the operation and the histological sections were done. Apart from other observations made by GOODMAN *et al.* [24], it is worth noting that the small quantities of the cement were found in the marrow spaces and in the Haversian canals indicating finger-like penetration outside the cement domain. The cement plugging of the Haversian canals resulted in localized areas of necrosis. Measurements of the remodelling activity were also made and the periprosthetic bone was significantly less active. The study presented in [17] suggests that after implantation into the site characterized by low level of mechanical loading the cement is encapsulated by thin connective tissue membrane containing scattered histiocytes and giant cells. Inflammatory cells were seldom observed. Unlike the case of weight-bearing implants the synovial-like fluid and fibrocartilage were absent. Although marrow necrosis did occur in areas surrounding the cement implant, viable marrow has been found also in the nearest vicinity of the implant. This effect suggests that the blocking of Haversian canals and not, or at least not only, the toxic influence of monomer leftover is the factor that contributes to the bone tissue necrosis.

It seems therefore important to model the polymerisation process as well as temperature distribution and to develop criteria similar to (2.19) for assessing the tissue damage due to polymerisation leftover.

The eventual effect of bone necrosis is bone resorption at the bone-cement interface. The mantle of fibrous connective tissue is developed and the mechanical load-carrying capacity of the interface is seriously compromised, which usually leads to implant loosening and the need for reoperation.

Different measures are taken or considered to remedy this problem. These are:

- 1) implanting cementless prostheses,
- 2) development of new low-temperature and bioactive bone cements, [58, 60, 75] and the discussion below,
- 3) water cooling (acetabular parts of total hip implants, [81]),
- 4) shielding layers, [29, 44],
- 5) lowering thermal contact between cement mantle and the bone which can also have a beneficial effect of lowering monomer leftover, [65],
- 6) pre-cooling or pre-heating the prosthesis stem (possibly the bone and cement mixture),
- 7) using as little cement as possible.

The most straightforward method is to refrain from using cemented implants at all, in favor of the cementless ones. There are, however, extensive clinical data from postoperative followups indicating that percentage of failures, marked by the necessity of revision, is significantly lower in the case of

cemented prostheses, see [52] and the references cited therein. Also, cementless implants are avoided in the case of weaker bones, like in the rheumatoid patients.

The most promising method is to develop new cements that polymerise at low temperatures leaving no monomer leftover. Another advantage of lowering the maximum temperature in the system (not necessarily bone temperature) is that some heat-labile cement components (e.g. antibiotics) are not deactivated during the cement setting and also the potential problem of MMA boiling and cement porosity induced in this manner is avoided.

Lowering the peak temperatures during the polymerisation process reduces also the overall shrinkage of the cement during the cooling phase. As calculated by HOLM [28] the final cooling of the cement is responsible for 40–50% of the final contraction (the coefficient of linear expansion is $\alpha = 2.27 \times 10^{-4}$ [1/K], see [28] and the references therein). It is well known that the PMMA cement shrinkage is an undesirable effect that contributes to the loosening of the prosthesis.

The task of lowering polymerisation temperature can be in part accomplished by means of adding "heat sink" additives to the PMMA powder and/or changing initial P/L ratio of cement mixture. Unfavourable outcome of these actions is the lower mechanical strength of the cement since its porosity increases. These issues are discussed in detail in [29].

Pre-cooling the prosthesis stem was proved to be ineffective [7, 68, 72]; furthermore it was shown by BISHOP *et al.* [7] that the low stem temperature substantially deteriorates cement-stem interface quality and that pre-heating should be used instead of pre-cooling. These results were confirmed by the study of IESAKA *et al.* [31] and BALEANI *et al.* [2].

Pre-cooling of the cement mixture also prolongs the setting period of the cement. In that period any motion of the installed prosthesis usually causes fixation failure. Implant has to be extracted from the femur, the cement has to be removed and new implant installed. Water cooling is possible only in acetabular components fixation and is reported to have a positive effect on the bone temperature [81].

The possibility of lowering the thermal contact between cement and bone was also investigated, [29, 44]. MAZZULLO *et al.* [44] showed that introducing thin layer of rubber-like material would have a beneficial effect on both the conversion of the monomer and temperature of the bone. Such a barrier could also protect the tissue against the diffusion of toxic substances from the cement dough. Unfortunately, in view of the efforts to create the best possible environment for the bone tissue to grow into and 'interlock' with the cement, such a solution should be considered impractical.

Another issue connected with the implantation of cemented prostheses that is worth mentioning is the fact that the mechanical properties of cement depend

on the temperature. Therefore results of the tests conducted in the room temperature are not immediately applicable to the *in vivo* temperatures. LEE *et al.* [37] cited in [57] reported a 4% decrease in the Young's modulus (in compression) and 10% drop in the ultimate compression strength at 37°C when compared to the properties at room temperature. LAUTENSCHLAGER and MARSHALL [36] measured the compressive strength of the plain Palacos-R cement at 0, 23 and 37°C and obtained the following values 125.7 ± 1.8 , 90.5 ± 1.1 and 86.9 ± 2.5 MPa. JAFEE *et al.* [32] estimated the compression yield stress drop to be equal to $0.3 \left[\text{MPa}/^\circ\text{C} \right]$. The work by SAHA and PAL [57] contains a comprehensive review of the research on this topic. Also the work by KUSY [35] presents a comparison of chemical and physical properties of four commercially available bone cements.

3. BONE DRILLING AND SAWING DURING ORTHOPAEDIC OPERATIONS

3.1. Aim of the study

As outlined in Table 1, temperature increase during orthopaedic operations is of short duration; however considerable temperature values may be reached. Two processes may lead to frictional bone heat-up during operation: sawing (e.g. during bone preparation for endoprosthesis implantation) and drilling (e.g. during preparation for screw fixation). The resulting thermal bone necrosis at the screw site leads to instability and consequently invalidates any benefits from all stabilising devices fixed to the bone. The correct choice of drilling/sawing parameters is therefore of importance. The parameters under consideration are:

- rate of rotation of the drill, speed of the saw,
- force on the drill or saw in the direction of drilling/sawing,
- the kind of tool, permitted level of wear,
- possible pre-drilling parameters,
- cooling(irrigation) parameters.

3.2. Experimental methods

The influence of above factors was measured experimentally, see [34, 43, 73]. The outcome can be summarized as follows:

- The rotational speed of the drill is reported to have no marked influence on the spatial temperature distribution in bone but it affects the duration of the exposure to high temperature – higher drill speeds produced high temperature for shorter periods [43]. Similar measurements made by KRAUSE *et al.* [34] for high-speed (20.000 rpm and 100.000 rpm) cutting burs confirmed that that there's no general correlation between rotational

speed of the bur and bone temperature, tendencies for different kinds of burs being different.

- The increase in force measured in the direction of drilling results in significant decrease of temperature in the vicinity of drill. MATTHEWS and HIRSCH [43] report approximately 15°C decrease (from ca 82°C) in location 0,5 mm from the drill when the force is changed from 2 to 6 kG and further 17°C decrease with force changed to 12 kG, see Fig. 5.

Similar effect was reported by KRAUSE *et al.* [34] for high-speed cutting burs; higher feed rates, and therefore cutting forces, cause lower temperature elevations. Strength of this tendency varied with the kind of bur used.

Effect of applied force magnitude is much more pronounced when duration of exposure to temperatures over 50°C is compared. For forces 2, 6 and 12 kG these durations are 35, 8 and ca 1 second respectively at the mentioned location.

In case of the dental burs and smooth pins used as drilling tools (as opposed to twist drills) the force is reported not to influence the temperatures in a way described above, Those tools have no means of eliminating the bone debris and therefore milling occurs rather than cutting and furthermore the debris gets compacted between the tool and the hole walls thus greatly increasing friction.

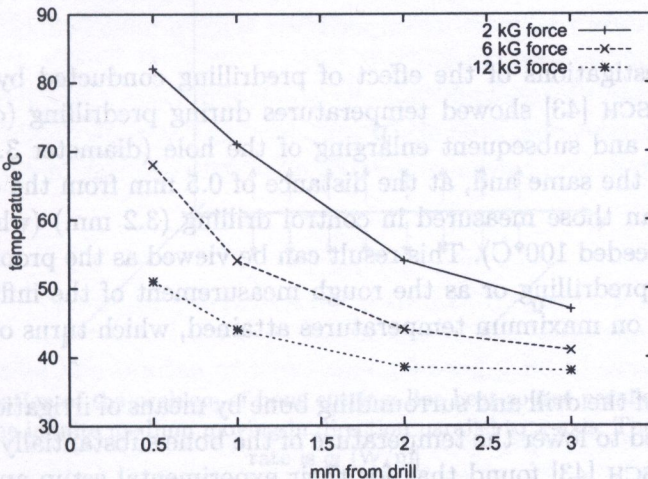


FIG. 5. Temperatures recorded in bone for different axial forces during drilling, after [43].

- Study of the influence of drill wear conducted in [43] showed that worn drills (used to drill 200 holes before) could produce temperatures over

20 K higher in the immediate neighborhood than the new ones. Also time of exposure was significantly prolonged. As can be inferred from this one and other experimental investigations, the shape of the tool has a marked influence on bone temperatures during drilling [34, 73]. The same applies to the saw, [74]. Figure 6 presents a comparison of temperatures measured by KRAUSE *et al.* [34] for two different saw blades.

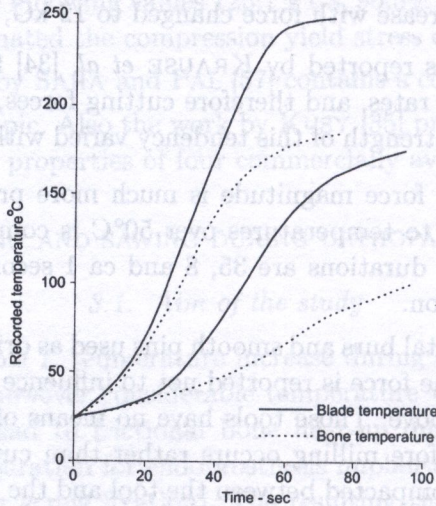


FIG. 6. Recorded temperatures for two saw blades, after [34].

- The investigations of the effect of predrilling conducted by MATTHEWS and HIRSCH [43] showed temperatures during predrilling (drill diameter 2.2 mm) and subsequent enlarging of the hole (diameter 3.2 mm) to be virtually the same and, at the distance of 0.5 mm from the drill, ca 60 K lower than those measured in control drilling (3.2 mm) (where temperatures exceeded 100°C). This result can be viewed as the proof of beneficial effect of predrilling or as the rough measurement of the influence of drill diameter on maximum temperatures attained, which turns out to be very high.
- Cooling of the drill and surrounding bone by means of irrigation with water is reported to lower the temperature of the bone substantially. MATTHEWS and HIRSCH [43] found that for their experimental setup and the diameter of drill they used (32 mm), no significant advantage was gained when raising the coolant flow above 500 ml/minute. The coolant they used was water at room temperature. Using pre-cooled water would probably allow less coolant to be used. The results of measurements conducted by KRAUSE *et al.* [34] for different kinds of burs and reciprocating saws con-

firmed the conclusion that cooling may significantly reduce the bone and tool temperature. Studies by TOKSVIG-LARSEN *et al.* [73] with the prototype oscillating-blade saw showed that with adequate coolant flow, the temperature elevations are negligible and well below the values usually associated with bone necrosis.

As can be seen from the above experimental results, appropriate choice of tools and cooling methods permits to avoid the danger of thermal bone damage during preparation for orthopaedic operation entirely.

3.3. Modelling

The problem of bone heating-up during sawing can mathematically be stated in the first approximation as finding the temperature distribution in the infinite solid with moving line heat source. The geometry is depicted in Fig. 7. For steady conditions the solution is given in [10] as

$$(3.1) \quad asT(x, z) = T_0 + \frac{q_l}{2\pi\lambda_b} \exp\left(\frac{Ux}{2a_b}\right) K_0\left(\frac{U\sqrt{x^2 + z^2}}{2a_b}\right).$$

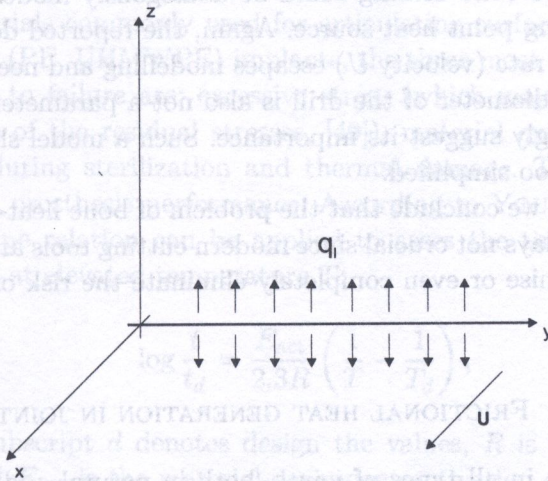


FIG. 7. Idealisation of the problem of bone cutting: line heat source parallel to the y axis is embedded in the infinite medium moving in direction parallel to x -axis. The heat generation rate is q_l [W/m]

It is assumed here that infinite line source located at y -axis moves with the velocity U (sawing feed rate) in direction of x -axis. K_0 is the modified Bessel function of the second kind of order zero whilst λ_b and a_b are the thermal conductivity and thermal diffusivity of the bone respectively; q_l is the rate of heat

generation expressed in $[W/m]$. The problem can also be visualized as the distribution of smoke in a medium, which flows past the line emitting smoke, [10].

The closed-form solution (3.1) can be obtained due to the simplicity of the model. The assumptions not reflecting the physical situation are:

- Treating the bone specimen as an infinite solid. That assumption should result in under-estimating the temperature as the heat is allowed to escape the real geometrical domain of the bone without any boundary resistance. This is thought to have no significant influence as long as the sawing process is fast.
- Treating the saw as a one-dimensional entity. This is reasonable for steady-state situations (deep cut), when the temperature of the saw does not change anymore. The heat flow through the blade in its direction and the blade heat capacity are thus neglected.

In such a formulation the crucial parameter q_l needs to be obtained experimentally by means of temperature or calorimetric measurements.

Alternatively to this formulation, Rosenthal (1946), cit. by KRAUSE *et al.* [34] proposed solution for an infinite solid with moving plane heat source instead of line source.

The problem of bone drilling could be analogously modelled by an infinite solid with a moving point heat source. Again, the reported dependence of heat generated on feed rate (velocity U) escapes modelling and needs to be supplied. Furthermore, the diameter of the drill is also not a parameter, whereas experimental data strongly suggest its importance. Such a model should be therefore considered to be too simplified.

To summarise, we conclude that the problem of bone heat-up during drilling or sawing is nowadays not crucial since modern cutting tools are designed in such a way as to minimise or even completely eliminate the risk of thermal damage to tissue, [73].

4. FRICTIONAL HEAT GENERATION IN JOINTS

Friction occurs in all types of joints, both in natural and in artificial ones. The heat generation and dissipation is therefore a process that takes place every time the joint is used. In normal human hip joints the measured temperature elevation is of the order of $+2.5^\circ\text{C}$ during walking and probably more during running, [71].

The artificial joints are less efficient and one can expect that temperatures attained in such a joint can be higher. BERGMANN *et al.* [4] reported a 3.5°C temperature increase in the case of titanium alloy hip prosthesis after 45 min. of normal walking. The temperature was measured inside the neck of the pros-

thesis. Its value at the bearing surface can be considerably higher, see [20, 42] for the finite elements estimation of this temperature for different articulating pairs: the zirconia, cobalt-chromium and alumina implant heads articulating on polyethylene cups.

4.1. Motivation for the research

Investigation of the problem of temperature distribution in an artificial joint, when the joint is being used is done for two specific reasons:

- To establish if thermal damage to the tissue can take place. Such a damage may lead to prosthesis, usually hip acetabular cup or knee prosthesis, loosening via the mechanism outlined in Sec. 2.4. The issue of criteria for thermal damage is more difficult here than in the case of bone cutting or cement polymerisation because the thermal loading is now cyclic and this may result in "thermal bone damage accumulation". On the other hand it is suggested that bone tissue can develop "heat shock" proteins and adapt to elevated temperature, [11, 46]. In fact, no adequate criterion is known to the authors of the present paper.
- To assess the working temperatures and their influence on wear and creep rate of materials commonly used for articulating surfaces. In the case of polyethelene (PE, UHMWPE) implants, the three most important factors contributing to failure are: excessive stress (which may be magnified by the presence of the residual stresses, [49]), material oxidation due to γ -irradiation during sterilization and thermal damage. The latter is vital to long-term prosthesis performance. According to YOUNG *et al.* [82] the Arrhenius-type relation can be applied to asses the time to failure t of polyethylene at elevated temperature T :

$$\log \frac{t}{t_d} = \frac{E_{\text{act}}}{2.3R} \left(\frac{1}{T} - \frac{1}{T_d} \right),$$

where the subscript d denotes design the values, R is the universal gas constant and E_{act} is the material-dependent activation energy. When the equation is evaluated for values of constant appropriate for PE ($E_{\text{act}} = 235.2$ kJ/mol) and we select $T_d = 310$ K, it appears that prosthesis service life is shortened almost by half when the temperature increases by 2K over the design value.

On the other hand, as pointed out by LU and MCKELLOP [42], at elevated temperatures the proteins present in the lubricant fluid in the experiments *in vitro* may precipitate forming a cushion shielding the bearing surfaces from wear and reducing the effect of the adhesive transfer of the polyethylene

to metal surface. Unfortunately, with the soluble components precipitating, the lubricating quality of the fluid deteriorates.

4.2. Experimental techniques

Frictional heat generation has been investigated by various experimentalists in two distinct ways:

- *In vivo* measurements in patients. BERGMANN *et al.* measured the temperature distribution and forces acting on the head of the implant by means of instrumented endoprosthesis, [6]. In Fig. 8 a schematic drawing of this prosthesis is provided after GRAICHEN *et al.* [25].

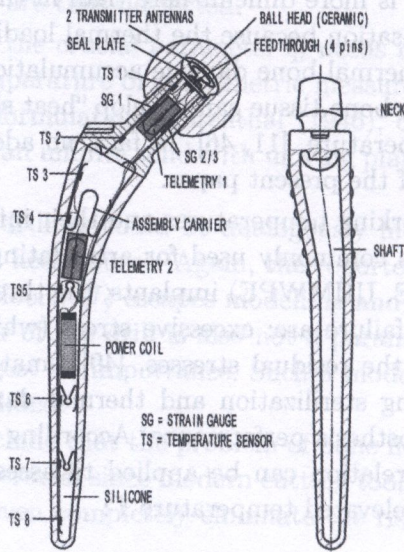


FIG. 8. Schematic view of the instrumented hip endoprosthesis used for measurements of temperature and strain *in vivo*, after [25].

- *In vitro* measurements of frictional torque, temperatures and material wear on a laboratory set-up over a prescribed range of joint motions. Such measurements were made by DAVIDSON *et al.* [14, 15] and by LU and MCKELLOP [42]. Influence of the choice of material for articulating surfaces and a number of other parameters was studied. Some of the results are reviewed below.

In [14] the authors describe the so-called "simulated *in vivo*" test where movement of the prosthesis is replaced by resistance heater embedded in

the prosthetic head and the system is assembled in such a way as to resemble the real situation as much as possible (the joint capsule is simulated by pieces of bovine muscle tissue), therefore reproducing mechanisms of heat transport that occur *in vivo*. However, the effect of vascularity and blood flow is not reproduced in such a setup.

Below some of the important results reported in the literature are reviewed.

4.2.1. Biomaterials used in artificial joints. Material of articulating surfaces has a marked influence on the heat production and rate of wear. In [14] and [15] the authors provide an extensive study of these issues. Three kinds of articulating pairs are taken into consideration (mentioned here in stem-acetabular cup order):

- 1) Co-Cr-Mo steel on UHMWPE (Ultra High Molecular Weight PolyEthylene),
- 2) alumina (Al_2O_3) on UHMWPE,
- 3) alumina on alumina.

Some of the results of *in vitro* tests are displayed in Fig. 9, after [14]. Specialized test setup was used to produce rocking motion with variable hip loading. The profile of the loading during a single cycle is depicted in Fig. 10 after [14]. The load-time history was selected to reflect natural hip loading during walking. The results cover the experiments conducted with two values of maximum force applied to femoral head, namely 2500 N and 5000 N [15] and [20].

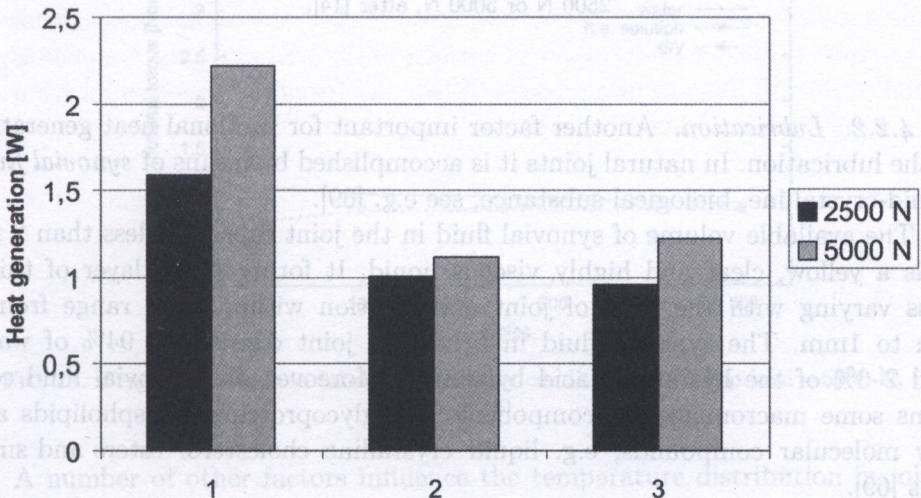


FIG. 9. Heat generation for different articulating surfaces, after [14].

The results presented in [42] show superior performance of alumina-UHMWPE, when compared to metal-UHMWPE pair *in vitro*. BERGMANN *et al.* [6] performed experiments which showed the superior performance of alumina-alumina articulation *in vivo*.

The differences in heat production for various pairs come from different friction coefficients and can greatly influence wear performance of artificial joint. More detailed information about the rate of wear for different kinds of surfaces can be found in [15] and [20].

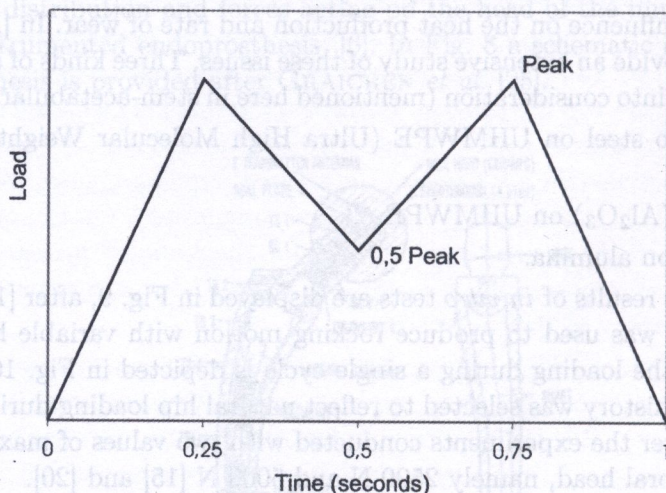


FIG. 10. Walking load history, used in *in-vitro* frictional heating tests. Peak value was set to 2500 N or 5000 N, after [14].

4.2.2. Lubrication. Another factor important for frictional heat generation is the lubrication. In natural joints it is accomplished by means of *synovial fluid*, liquid-crystalline, biological substance, see e.g. [69].

The available volume of synovial fluid in the joint capsule is less than 2 ml. It is a yellow, clear and highly viscous liquid. It forms a film layer of thickness varying with the type of joint and location within it, in range from 6 μm to 1mm. The synovial fluid in a healthy joint consists to 94% of water and 2–3% of the hyaluronic acid by weight. Moreover, the synovial fluid contains some macromolecular components like glycoproteins, phospholipids and low molecular compounds, e.g. liquid crystalline cholesterol esters and small ions [69].

The main purpose of the synovial fluid is the lubrication of the joint. Furthermore, it provides the necessary nutrients for the cartilage and protects it from enzyme activity.

Properties of the synovial fluid are affected by pathological processes. The shear viscosity coefficient is smaller for synovial fluid from degenerated joints.

In natural joints the quality of the lubrication is strongly linked with functioning of the articular cartilage which can be treated as a multiphase porous material. For details the reader is referred to [30, 47].

In the *in vitro* tests [15] the lubrication was attained by means of water or hyaluronic acid in different concentrations. The hyaluronic acid was chosen since it is the primary lubricant component of the synovial fluid. Additionally, friction in dry conditions as well as in the presence of 2 mg bone cement powder was investigated. No significant difference between water and hyaluronic acid lubrication was reported, while friction in dry conditions was, as expected, substantially greater. In Fig.11 illustrative experimental relation is depicted between frictional torque in joint and lubrication conditions for steel-UHMWPE articulating pair [15]. To obtain these results, a hip simulator was used to create a rocking motion over a 46° range. The axial load variation was chosen to reflect the walking loads history, see [15] for details.

The frictional torque rises by an order of magnitude when bone cement is present in the joint, even in a small quantity. As the results presented in [15] indicate, there is an almost linear relation between frictional torque and equilibrium temperature rise.

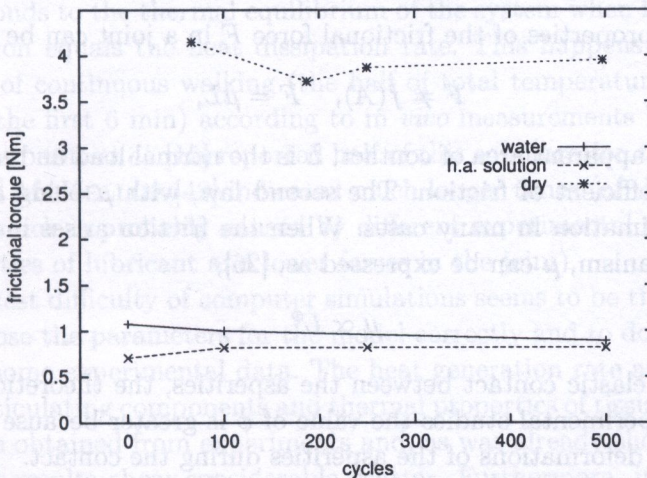


FIG. 11. Frictional torque in lubricated and dry conditions. Peak hip load = 5000 N, after [15].

A number of other factors influence the temperature distribution in joints. These are:

- Stride length and step frequency and consequently the flexion-extension angle and angular velocity, [62].

- Adaptation. BERGMANN *et al.* [6] reported that two patients with low measured temperatures in implanted joints had high body weight but were very active. This effect is assumed to have been caused by physiological adaptation of vascularity to higher temperatures (higher perfusion rates). It is, however, not always present.
- Possible head-acetabular cup separation during the joint movement. This effect is not present *in vivo*, where various supporting structures exist to restrain the femur head (fibrous capsule, acetabular labrum, ligament of the head of the femur and the iliofemoral, ischiofemoral, pubofemoral and transverse acetabular ligaments). During the total hip arthroplasty some of those structures may get surgically removed or resected to facilitate surgical exposure. The kinematics of the artificial joint is therefore different from the natural one. Measurements performed by DENNIS *et al.* [17] prove that articulating surfaces separation occurs *in vivo*. The influence of this effect on joint temperatures is not known but it is suspected to be beneficial [6]. The gap that opens during the separation would be filled with synovial fluid, which would cool the articulating surfaces and the lubricating film would be renewed.

4.3. Modelling and simulation

Two basic properties of the frictional force F in a joint can be stated, [26]:

$$F \neq f(A), \quad F = \mu L,$$

where A is the apparent area of contact, L is the normal load and μ is a number, termed the coefficient of friction. The second law, with μ being a constant, is a good approximation in many cases. When the friction arises mainly from an adhesive mechanism, μ can be expressed as, [26],

$$\mu \propto L^\phi.$$

For purely elastic contact between the asperities, the theoretical value of ϕ is $-1/3$. In experimental studies the value of ϕ is greater because of plastic, as well as elastic, deformations of the asperities during the contact.

For the system with lubrication, the conditions of friction are altered by the presence of fluid. Two regimes are generally distinguished here, [26].

- *Full fluid film lubrication.* The direct interaction between articulating surfaces is negligible. The frictional force arises only from the shear within the fluid. For a Newtonian fluid HALL and UNSWORTH [26] propose the following expression for μ :

$$\mu = \alpha \frac{\eta u}{L h_c}.$$

where u is the sliding velocity, η is the viscosity of the fluid and h_c denotes the central film thickness between articulating surfaces. The parameter α characterizes the contact regime.

- *Mixed lubrication.* The pressure in the film of lubricant is not sufficient to support the load and contact between the opposing surfaces occurs.

The simple engineering criterion for full fluid film lubrication to occur is that the surface separation ratio h_c/σ should exceed 3, where σ is the combined root mean square (RMS) surface roughness of the articulating surfaces [26].

In the hip joint the precise distribution of synovial fluid film thickness is not known and the frictional forces have contributions at different radii. Thus μ cannot be determined accurately from experimental data. Instead, a friction factor is defined by the following equation:

$$T = fRL.$$

where T is the total frictional torque and L is the radius of the femoral head. The torque T is the driving force for the heat generation in the joint.

The computer simulations of friction-induced heat dissipation are always considered in close connection with the experiment. Unlike the case of bone cement polymerisation the stationary temperature field is usually considered, see [5]. This corresponds to the thermal equilibrium of the system when heat generation rate by friction equals the heat dissipation rate. This happens approximately after 1 hour of continuous walking (the half of total temperature increase taking place in the first 6 min) according to *in vivo* measurements by BERGMANN *et al.* [6]. DAVIDSON *et al.* [15] reported half of this equilibration time for tests *in vitro*. LU and MCKELLOP [42] indicated much longer times (of the order of several hours) which is probably caused by different experimental protocol (much larger quantities of lubricant and lower forces in the joint).

The greatest difficulty of computer simulations seems to be the fact that one needs to choose the parameters for the model correctly and to do this one needs to resort to some experimental data. The heat generation rate at the surface of contact of articulating components and thermal properties of tissues and synovial fluid must be obtained from experiments and, as was already shown in Sec. 2.4, experimental results show considerable scatter. Furthermore, if only ordinary Fourier-Kirchhoff equation is used, the effect of vascularity is neglected or can be only roughly approximated.

In [5] BERGMANN *et al.* conducted a finite element analysis of a stationary temperature field in the implant system. The geometry of the FEM model was reconstructed by the use of CT scans. Uniform heat generation rate at the implant head-acetabular cup interface was assumed and its value was chosen in such a way that the temperature profiles resembled as closely as possible the

ones obtained experimentally. Then, the influence of the following factors was numerically investigated:

- the volume of synovial fluid,
- the perfusion rate in surrounding tissue,
- the possibility of separation between the femoral head and the acetabulum,
- friction conditions,
- thermal properties of individual elements of the system,
- the fixation technique (with or without cement),
- the size of the contact area between head and the cup.

Detailed results are described in [5].

5. FINAL REMARKS

The issues presented in the present paper are, from the point of view of heat transfer, a standard conduction and convection problems. Their distinctive feature is that they involve living hard tissue and the boundary and initial conditions are clinically relevant. The presented modelling and solution techniques are usually the standard engineering approach. The material, in most cases, is treated as a homogeneous, isotropic solid with no internal degrees of freedom (with the exception of bone cement). The obstacles to modelling and experimental validation are those common in biomechanics – the difficulties in performing the reliable *in vivo* tests and sheer complexity of the living tissues with many important biophysical, biochemical and biomechanical processes taking place simultaneously.

With modern analysis tools, such as computed tomography, finite element method and, above all, fast and efficient computers, one can be sure there will be a progress in the field of modelling the heat transfer in orthopaedics, but there are more questions than definite answers as yet.

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