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ON THE MOVEMENT OF GRANULAR MATERIALS IN BINS PART II: THREE-DIMENSIONAL PROBLEMS

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The paper is devoted to commemoration of the contribution of Professor Jerzy Litwiniszyn to the development of the mechanics of granular media

A simple numerical procedure for determining three-dimensional displacements of granular media in bins is proposed. In the previous paper [6] such a numerical procedure was limited to the two-dimensional analysis only. The proposed procedure is based on the concept published by Professor Jerzy Litwiniszyn, who treated the gravity flow of granular media as a stochastic process.

1. Introduction

J. Litwiniszyn in his papers [1–3] indicated that the classical treatment of the mechanics of granular materials based on the slip-lines technique (see e.g. [4, 5]) not always corresponds to real fields of displacements, especially when the movement is caused by the gravity forces and when the so-called active pressure conditions are involved in the medium. More remarks concerning this problem may be found in the previous paper [6].

According to Litwiniszyn's approach [3], the movement of granular media caused by gravity forces is of the mass character of random changes in contacts between the particles, and consequently, the displacements of particles are random.

As the starting point for the two-dimensional analysis of the movement of granular media in bins, in the previous paper [6] a demonstrating device is described, known as Galton's board (cf. e.g. [7]), in which small metal balls falling downwards from a container are striking numerous regularly located pins and are randomly directed to the right or to the left with the same probability equal to 1/2.

Let us imagine now a demonstrating device analogous to Galton's board, composed of a number of plates (layers) resting one on the other, each with a regular array of square holes. A few upper plates of the whole arrangement are shown in Fig. 1 in an expanded form. Subsequent plates are arranged one with respect to the other in such a manner that central axes of the holes in each plate coincide with the common line of four corners of holes in the adjacent plate below or above.

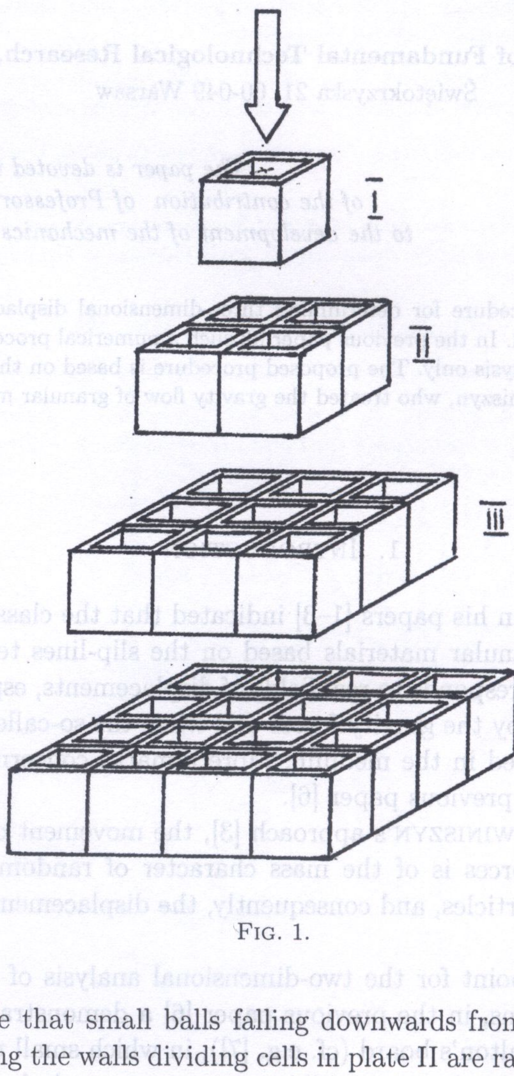


FIG. 1.

Let us assume that small balls falling downwards from a particular cell in plate I and striking the walls dividing cells in plate II are randomly directed into one of the four cells below with the same probability equal to 0.25. The same random path of the falling balls is repeated for each plate below and finally they fall at random into one of separate containers at the bottom of the device.

The probabilities that a ball migrating downwards will pass through a particular cell in a subsequent plate below are shown as an example in Fig. 2 for a few upper plates.

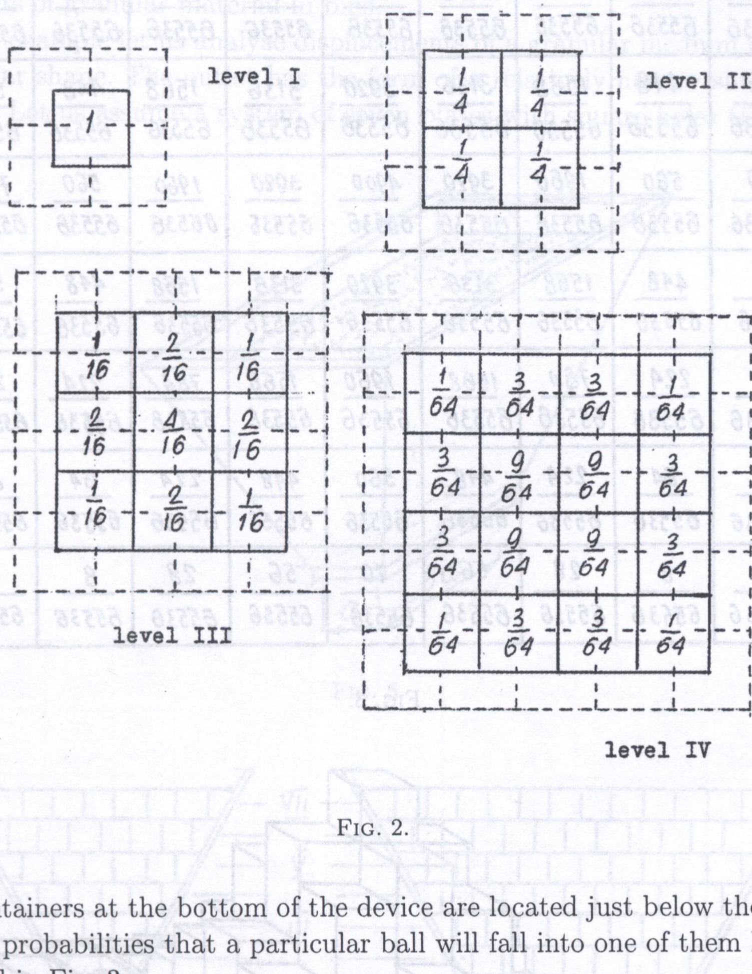


FIG. 2.

If containers at the bottom of the device are located just below the plate IX then the probabilities that a particular ball will fall into one of them have been presented in Fig. 3.

These calculated values of probabilities are shown in the form of a histogram – Fig. 4. It can be seen that the larger is the number of plates in our experimental device, the more the calculated probability distribution approaches the circular normal distribution.

Let us notice that an analogous demonstrating device may be composed of a number of plates with hexagonal holes like a honeycomb. Details are described in [8].

1/65536	8/65536	28/65536	56/65536	70/65536	56/65536	28/65536	8/65536	1/65536
8/65536	64/65536	224/65536	448/65536	560/65536	448/65536	224/65536	64/65536	8/65536
28/65536	224/65536	784/65536	1568/65536	1960/65536	1568/65536	784/65536	224/65536	28/65536
56/65536	448/65536	1568/65536	3136/65536	3920/65536	3136/65536	1568/65536	448/65536	56/65536
70/65536	560/65536	1960/65536	3920/65536	4900/65536	3920/65536	1960/65536	560/65536	70/65536
56/65536	448/65536	1568/65536	3136/65536	3920/65536	3136/65536	1568/65536	448/65536	56/65536
28/65536	224/65536	784/65536	1568/65536	1960/65536	1568/65536	784/65536	224/65536	28/65536
8/65536	64/65536	224/65536	448/65536	560/65536	448/65536	224/65536	64/65536	8/65536
1/65536	8/65536	28/65536	56/65536	70/65536	56/65536	28/65536	8/65536	1/65536

Fig. 3.

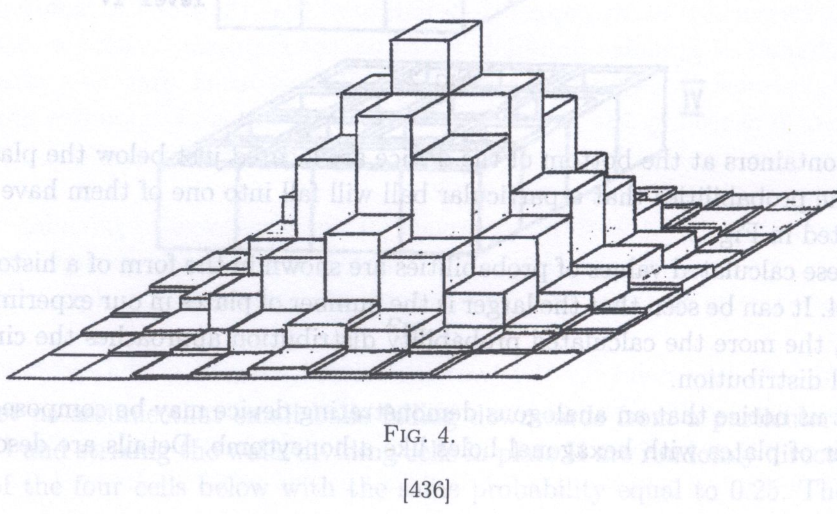


Fig. 4.

2. THE METHOD OF FINITE CELLS

As stated in Part I of the paper [6], LITWINISZYN analysed an inverse problem in which the cavities existing in the bulk of a loose medium migrate randomly upwards from the bottom – cf. [1, 3]. His approach may be used as the basis for the numerical finite cells procedure for the analysis of three-dimensional movements of granular material in bins.

As an example let us analyse displacements of a granular medium in a bin of rectangular shape. The outlet has the form of a relatively narrow slit as shown in Fig. 5. Let us assume a system of seven plates with square holes (Fig. 6).

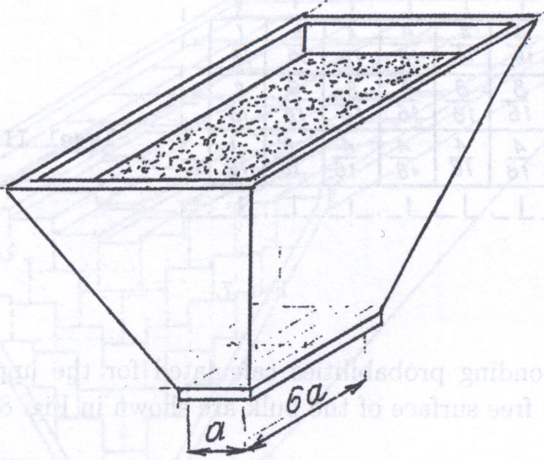


FIG. 5.

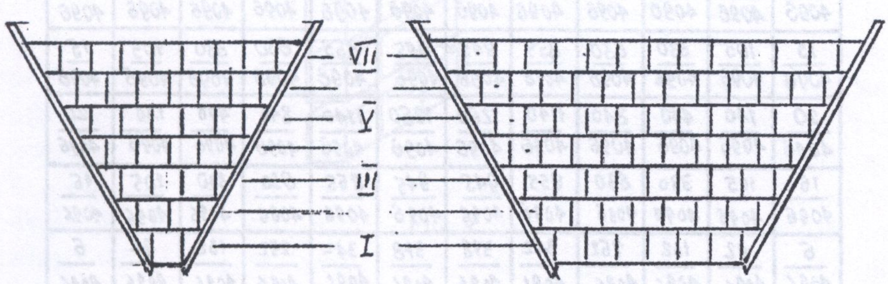


FIG. 6.

When a portion of six elementary volumes falls down from the bin through the outlet at the bottom, the elementary cavities formed in this manner begin to migrate upwards. They migrate according to the stochastic distribution described in the previous section. The probabilities that they pass during the migration through a particular cell in three lower plates are shown in Fig. 7.

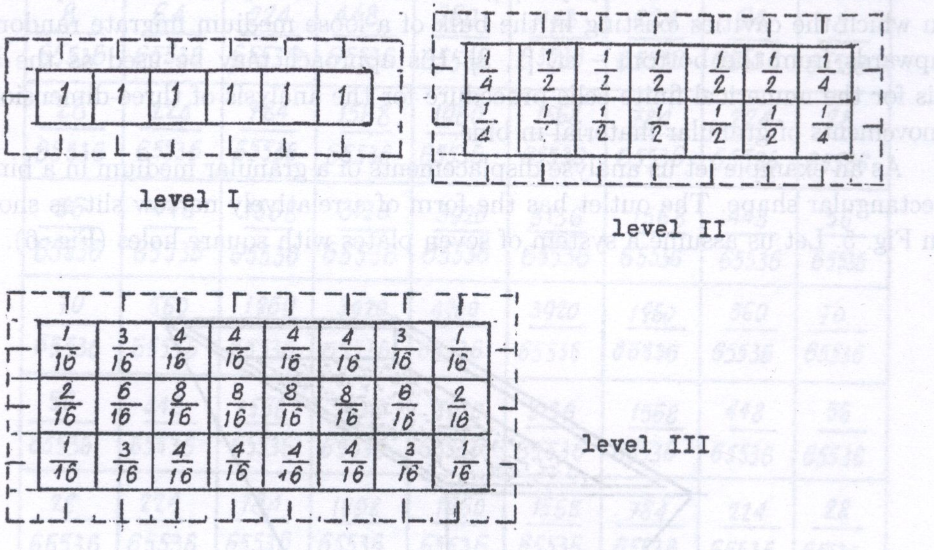
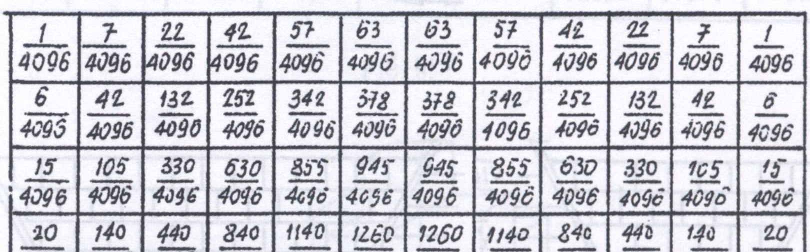


FIG. 7.

The corresponding probabilities calculated for the upper plate VII corresponding to the free surface of the bulk are shown in Fig. 8.



level VII

FIG. 8.

Using the calculated data for the upper layer VII we can construct, in the manner analogous to that described in Part I for a two-dimensional flow, the step-wise deformation pattern of the upper surface of the bulk contained in the bin. One half of the deformed surface for the particular case when three portions of the unit volume left the bin through the outlet at the bottom is presented in Fig. 9. The step-wise image of the surface may be treated as the first approximation resulting from the distribution of the probabilities shown in Fig. 8. In the next section, a simple method allowing to calculate the vectors of displacements will be proposed. With the use of this method, a smooth shape of the deformed surface can be calculated.

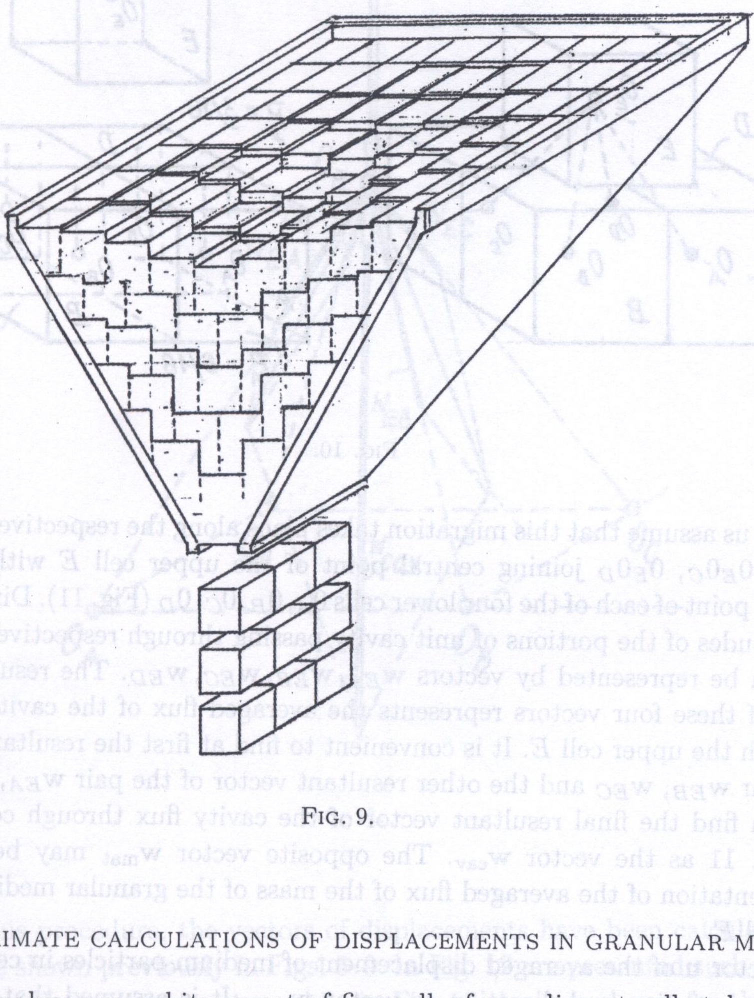


FIG. 9.

3. APPROXIMATE CALCULATIONS OF DISPLACEMENTS IN GRANULAR MEDIUM

Let us analyse any arbitrary set of five cells, four adjacent cells taken from a particular layer and one cell taken from the layer just above the previous one (Fig. 10).

Let, for example, the four lower cells be taken from layer III, and let the single cell above them be taken from layer IV (cf. Fig. 7). The corresponding values of probabilities that the migrating upwards cavity of unit volume passes through a particular cell are indicated in Fig. 10b. According to the finite cells technique described above, only one quarter of the unit cavity migrated from each cell A, B, C, D to the upper cell E.

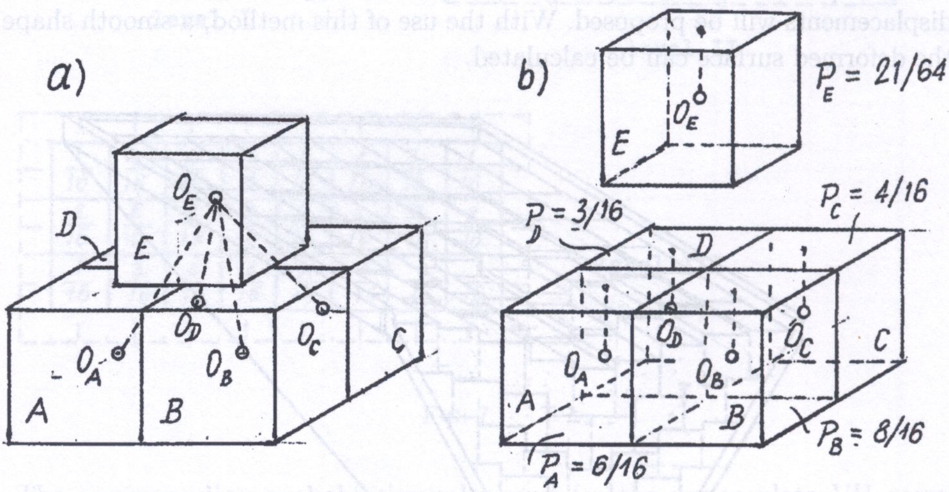


FIG. 10.

Let us assume that this migration takes place along the respective lines 0_E0_A, 0_E0_B, 0_E0_C, 0_E0_D joining central point of the upper cell E with respective central point of each of the four lower cells $0_A, 0_B, 0_C, 0_D$ (Fig. 11). Directions and magnitudes of the portions of unit cavity passing through respective cells of our set can be represented by vectors $\mathbf{w}_{EA}, \mathbf{w}_{EB}, \mathbf{w}_{EC}, \mathbf{w}_{ED}$. The resultant vector $\mathbf{w}_{\text{cav}}$ of these four vectors represents the averaged flux of the cavity migrating through the upper cell E. It is convenient to find at first the resultant, vector of the pair $\mathbf{w}_{EB}$, $\mathbf{w}_{EC}$ and the other resultant vector of the pair $\mathbf{w}_{EA}, \mathbf{w}_{ED}$. Then we can find the final resultant vector of the cavity flux through cell E shown in Fig. 11 as the vector $\mathbf{w}_{\text{cav}}$. The opposite vector $\mathbf{w}_{\text{mat}}$ may be treated as representation of the averaged flux of the mass of the granular medium through the cell E.

Vector $\mathbf{u}$ of the averaged displacement of medium particles in cell E has the same orientation and direction as vector $\mathbf{w}_{\text{mat}}$. It is assumed that the vertical component of the vector $\mathbf{u}$ is the same as the vertical displacement of the medium in the cell found previously in the manner described in Sec. 2.

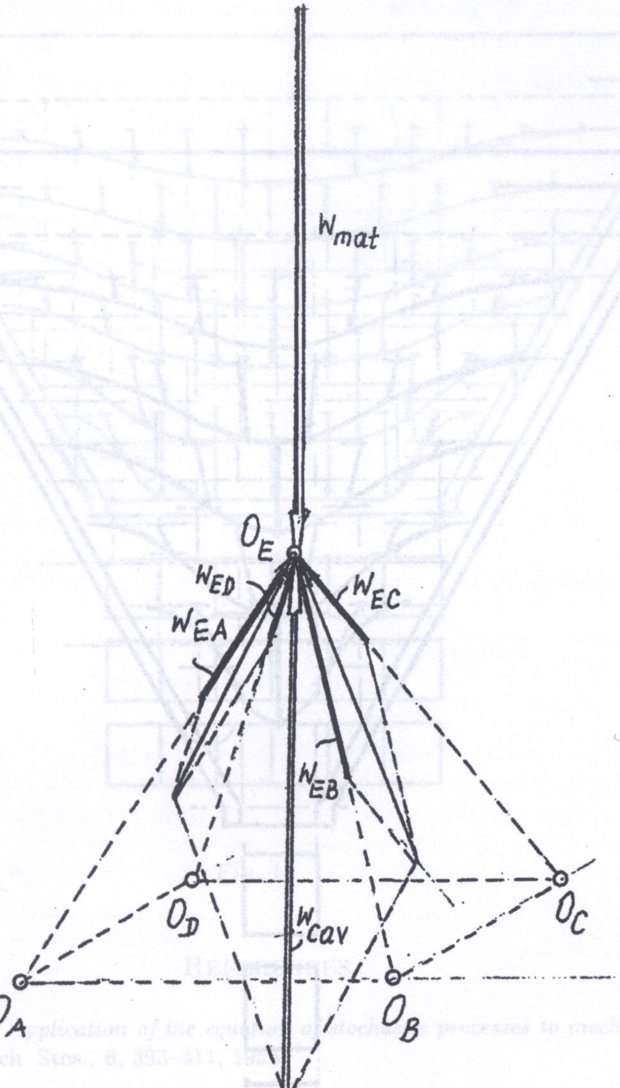


FIG. 11.

Using this procedure, the vectors of displacements have been calculated for the example shown previously in Figs. 5–9. In Fig. 12 are presented such vectors in one of the planes of symmetry, while in Fig. 13 they are shown for the other plane of symmetry. The method allows us to find the displacement vector for any point inside the bulk of the medium contained in the bin.

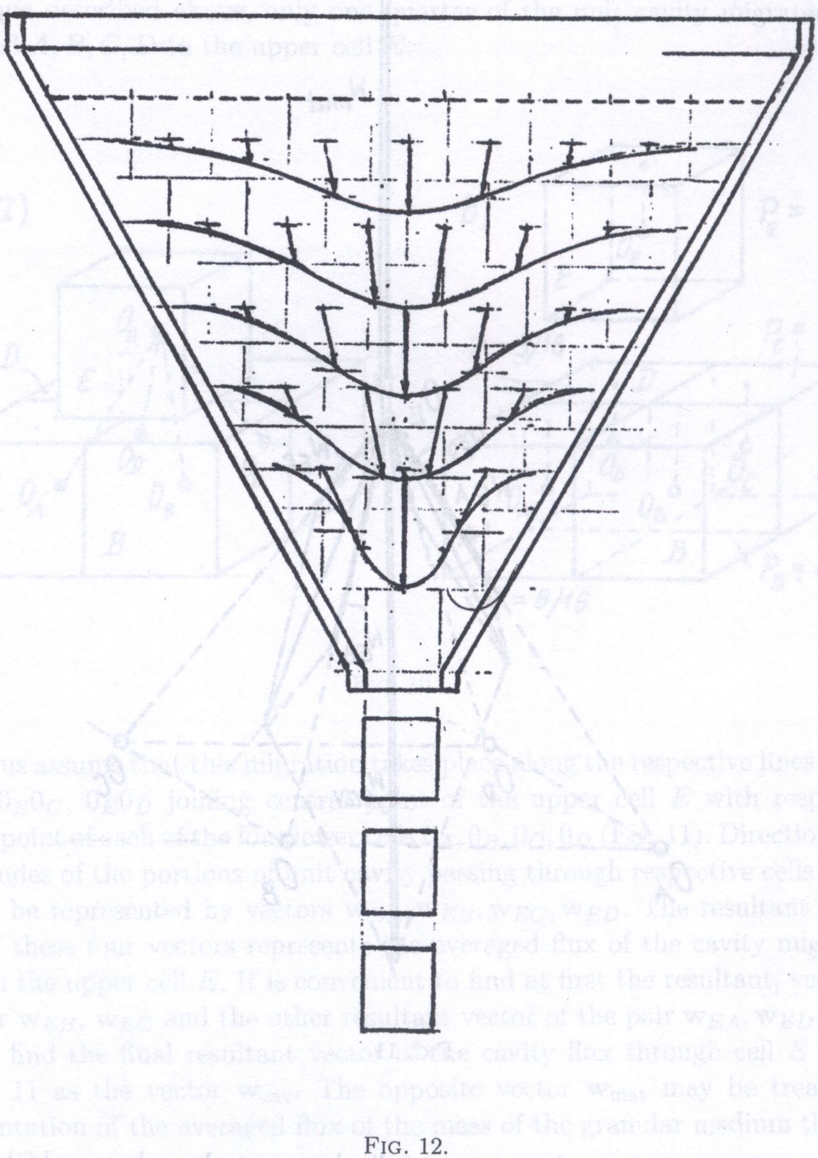


FIG. 12.

Using this procedure, the vectors of displacements have been calculated for the example shown previously in Figs. 5–9. In Fig. 12 are presented such vectors in one of the planes of symmetry, while in Fig. 13 they are shown for the other plane of symmetry. The method allows us to find the displacement vector for any point inside the bulk of the material contained in the bin.

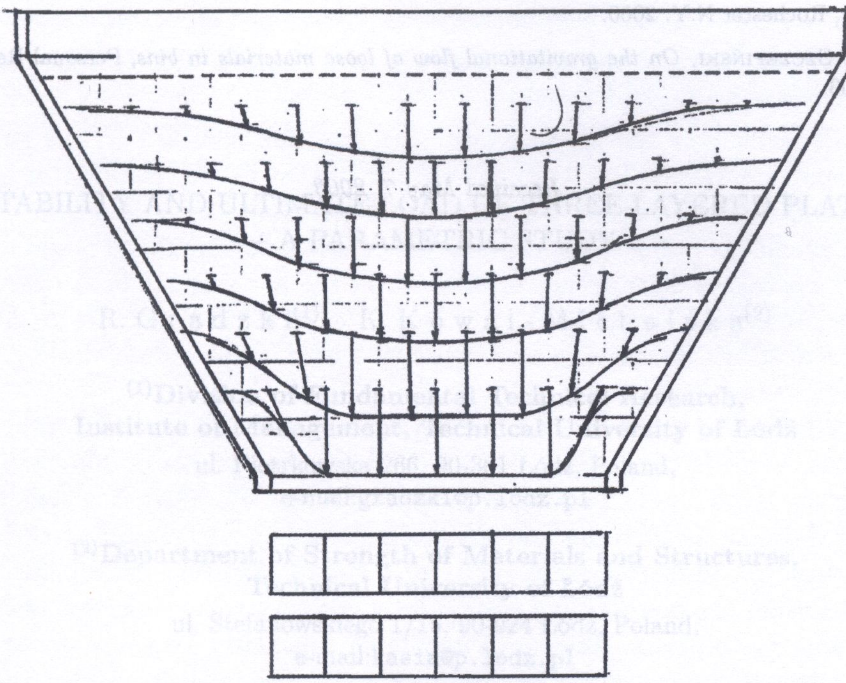


FIG. 13.

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