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Research Paper

Nonlocal Critical Velocities of Fluid Conveying Clamped-Pinned Single-Walled Carbon Nanotubes Subjected to Axial Magnetic Field

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The problem of stability of fluid conveying carbon nanotubes clamped at one end and pinned at the other end and subjected to an axial magnetic field is investigated in this paper. Non-local continuum mechanics formulation is utilized to derive the governing fourth-order partial differential equations, which takes into consideration the small length scale effects and the axial magnetic field. Galerkin's technique is used to find the solution of the governing equation for the case of clamped-pinned boundary. Closed-form expressions for the critical flow velocity above which the system becomes unstable, of the fluid conveying carbon nanotubes, are obtained and numerical results for different values of axial magnetic field parameter are presented in this paper for use in industrial dynamic design of such devices. The results obtained from these simple and approximate expressions are compared with those existing in literature, wherever available and an excellent agreement is found between them. Along with extensive results on critical velocities new and interesting results are also reported for varying values of nonlocal length parameter. From the results presented in this paper, it is observed that the non-local length parameter along with axial magnetic field parameter are having considerable influence on the critical velocities of the fluid conveying nanotubes.

Key words: critical flow velocity, Single-Walled Carbon Nano-Tubes (SWCNT), non-local, axial magnetic field.

1. Introduction

In recent times, a number of researchers are focusing their research on various aspects of carbon nanotubes. Carbon nanotubes have very good mechanical, electrical and chemical properties, with potential applications as bio-sensors, nano-oscillators, fluid transporters, mass flow sensors, drug delivery systems, or as purely structural components in nano-devices. In the area of fluid transport, and particularly in the field of dynamics of fluid conveying carbon nanotubes, research has accelerated in the last five years.

Analysis of the dynamic behavior of fluid conveying Single-Walled Carbon Nano-Tubes (SWCNT) started around 2005. Researchers have applied theory of classical continuum mechanics and used the same equations developed for pipes conveying fluid to study the carbon nanotubes conveying fluid, see for example, Yoon *et al.* [1], Reddy *et al.* [2] and Chang and Lee [3].

The diameter of a single-walled carbon nanotube is in the range of 1–7 nm and the length, even for an aspect ratio of 20, is in the range of 20–140 nm. At such small length scales, the properties of the material at atomic level, such as lattice spacing or C-C bond length, may have an influence on the dynamic behaviour. Hence, application of classical continuum mechanics models to carbon nanotubes is questionable. To address this problem, researchers are increasingly using Eringen's non-local continuum mechanics theory [4, 5], wherein the stress at any point is defined to be a functional of the strain field at every point in the body. The first to apply non-local mechanics to study vibrations of a fluid conveying SWCNT were Lee and Chang [6]. Again Lee and Chang [7], studied the vibration behaviour of fluid conveying carbon nanotubes embedded in a Winkler type of elastic medium. Wang and Varadan [8] develpoed a non-local continuum mechanics model and applied to study the vibration of both single-walled nanotubes (SWNTs) and double-walled nanotubes (DWNTs) via elastic beam theories. However, Tounsi *et al.* [9] pointed out an error in the formulation of Lee and Chang [6] and derived the correct governing equation. Wang [10] also formulated a consistent model, perhaps independently of Tounsi, but did not consider any embedding elastic medium. Farshidianfar *et al.* [11], have considered a two-parameter elastic embedding medium in their analysis of fluid conveying carbon nanotubes. Both Pasternak and viscoelastic type two-parameter foundation models have been used. However, their formulation of the problem is based on classical continuum mechanics and does not include the most important nonlocal elasticity effects. Ghorbanpour Arani and Amir [12] studied nonlocal vibration of embedded coupled CNTs conveying fluid under thermo-magnetic fields via Ritz method. The results indicate that magnetic field has significant effect on stability of coupled system.

Very recently, including the small scale nonlocal and surface effects along with a viscoelastic sandwich-beam model, Liang and Bao [13] presented a stability analysis of a fluid conveying carbon nanotube which is embedded in a Kelvin-Voigt type of two-parameter foundation. However, in the formulation of governing equation for the fluid conveying pipe, while appropriately considering the nonlocal effects due to fluid mass and fluid velocity, the authors ignored the nonlocal effects that arise due to the presence of Kelvin-Voigt foundation parameters because of which results presented by them on the stability regions for the case of simply supported boundary condition may become incorrect predictions. Kiani [14] studied the vibration and instability of a single-walled carbon nanotube (SWCNT) under a general magnetic field, using nonlocal Rayleigh beam theory and Maxwell's equations. They have shown that the critical transverse magnetic field increases with the longitudinally induced magnetic field and its value decreases as the effect of the small-scale parameter increases.

Ponnusamy and Amuthalakshmi [15] studied the effect of constant axial force due to thermal effects and the longitudinal magnetic field on the vibration analysis of a fluid conveying double walled carbon nanotube using nonlocal elasticity theory and Euler-Bernoulli beam equation. It is concluded that the frequencies of fluid conveying DWCNT embedded in an elastic medium under thermal and longitudinal magnetic field is lower than the frequencies of fluid conveying SWCNT embedded in an elastic medium under thermal and longitudinal magnetic field. Milan *et al.* [16] studied the vibration of a nanobeam under axial magnetic field using Finite Element Method. The exerted magnetic field increases the natural frequency of the system due to the increase in overall stiffness of the system. An increase of nonlocal parameter, which represents the nonlocal effects on nanoscale level, leads to a decrease of frequency compared to the classical local elasticity models. Very recently, Hosseini *et al.* [17] presented a differential transformation method (DTM) of solution for the problem vibration and instability of fluid conveying carbon nanotubes.

From the literature survey, it is observed that many researchers have addressed various aspects of the dynamic behaviour of fluid conveying carbon nanotubes but none have considered the combined effect of non-local elasticity effects due to presence of axial magnetic field in a mathematically consistent and exact manner. In this paper, it is proposed to study the influence of axial magnetic field on the critical velocities of fluid conveying single walled carbon nanotubes using assumed modes method of solution wherein the mode shape of a beam for the specific boundary conditions without considering the effects of conveying fluid mass, fluid velocity and any other effects also is used as an approximate solution for solving the governing differential equation of motion. The objective is to obtain simple closed-form expression for computing fundamental frequencies of fluid conveying pipe having one end clamped and the other end pinned. The governing equations are derived for a fluid conveying single-walled carbon nanotube (SWCNT) modelled as an Euler-Bernoulli beam including the effect of axial magnetic field, using the consistent formulation of Tounsi *et al.* [9]. The solution for the clamped-pinned end condition is obtained by utilizing the Galerkin's method. Closed-form expressions are obtained for the critical velocity for clamped-pinned boundary conditions considered here and are solved for different values of non-local length parameter and the axial magnetic field parameters. Extensive data on critical velocities is presented in this paper both in numerical as well as graphical form for use in design as well as to highlight the effects of both the non-local parameter and also the axial magnetic field parameters. Excellent comparison is found between the results obtained for critical velocities in this paper with those reported in the available literature.

The critical flow velocity V_{cr} is an important parameter for the study of stability of fluid conveying SWCNTs. At the critical flow velocity, the natural frequency becomes zero, leading to divergence instability of the SWCNT. The lowest root of V in Eq. (2.14) is the critical flow velocity, V_{cr}. In view of the excellent comparison obtained, as shown in Table 1, the simple and approximate expression, Eq. (2.14), is derived for the critical velocity in this paper for the case of fluid conveying pipe having one end clamped and the other end pinned, for use in the industrial dynamic design of fluid conveying nanotubes.

2. Equations of motion and solution

2.1. Non-local relations

As discussed by Eringen [5], the non-local constitutive relations take the form

$$\left[1-(e_0a)^2\,\nabla^2\right]\sigma_{kl}=\tau_{kl}, \tag{2.1}$$

where the right hand side term $\tau_{kl}(x)$ denotes the classical stress, $\sigma_{kl}(x)$ is the non-local stress tensor at any point x, the local (classical) stress tensor at any point x' in the body is represented by $\sigma_{kl}(x')$, e_0 is a material constant which depends on the results of experiments, a is an internal characteristic length which could be the C-C bond length or the lattice parameter. Equation (2.1), for a one-dimensional structure, is transformed into the following non-local constitutive equation,

$$\sigma_{xx}-(e_0a)^2\,\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\varepsilon_{xx}, \tag{2.2}$$

where σ_{xx} is the axial stress, ε_{xx} is the axial strain and E is Young's modulus of the carbon nanotube.

2.2. Effect of Axial Magnetic Field

Considering Maxwell's equations and assuming that the Lorentz force acts in the longitudinal direction of the fluid conveying carbon nanotube with one end clamped and the other end pinned as shown in the Fig. 1, the present paper aims at deriving a simple quadratic equation for critical value of fluid velocity V including the effects of nonlocal elasticity as detailed in Subsec. 2.1.

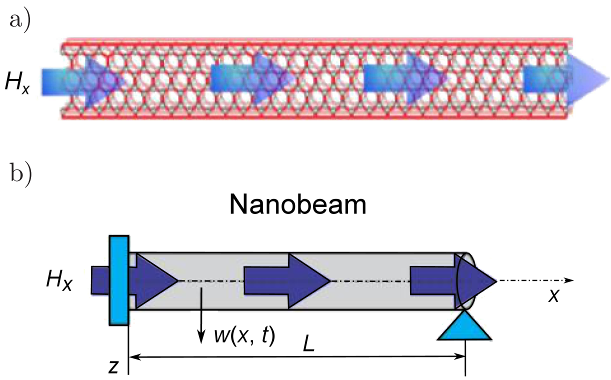

Fig. 1. Fluid conveying SWCNT as nanobeam under the influence of axial magnetic field H_x undergoing transverse displacement $w(x,t)$ at any point x: a) physical model, b) mechanical model of clamped-pinned nanobeam.

2.3. *Analytical model of the SWCNT conveying fluid*

To derive the equation of motion by the Newtonian approach, we first consider the beam equations [8]:

$$\varepsilon_{xx} = z\frac{\partial^2 w}{\partial x^2}, \quad M_b(x,t) = \int_A z\sigma_{xx}dA, \quad I = \int_A z^2 dA, \quad Q = -\frac{\partial M_b}{\partial x}, \tag{2.3}$$

where z-axis is vertical axis, I the moment of inertia of the cross section, M_b is the bending moment, Q is the shearing force.

Multiplying with zdA on either side of Eq. (2.2) we obtain

$$z\sigma_{xx}dA - (e_0a)^2\frac{\partial^2}{\partial x^2}(z\sigma_{xx}dA) = Ez\varepsilon_{xx}dA. \tag{2.4_1}$$

Substituting the expression for ε_{xx} in Eq. $(2.4)_1$ we get

$$z\sigma_{xx}dA - (e_0a)^2\frac{\partial^2}{\partial x^2}(z\sigma_{xx}dA) = E(z^2dA)\frac{\partial^2 w}{\partial x^2}. \tag{2.4_2}$$

Integrating Eq. $(2.4)_2$ over the area of cross-section of the carbon nanotube, we obtain

$$\int z\sigma_{xx}dA - (e_0a)^2\frac{\partial^2}{\partial x^2}\left(\int z\sigma_{xx}dA\right) = E\left(\int z^2dA\right)\frac{\partial^2 w}{\partial x^2}. \tag{2.4_3}$$

Rewriting the Eq. $(2.4)_3$ by moving the nonlocal term to the right side of equation, we get

$$(2.4)_4 \qquad \int z\sigma_{xx}dA = E\left(\int z^2 dA\right)\frac{\partial^2 w}{\partial x^2} + (e_0 a)^2 \frac{\partial^2}{\partial x^2}\left(\int z\sigma_{xx}dA\right).$$

Using the expression for moment of inertia I and bending moment M_b we get

$$(2.4)_5 \qquad M_b(x,t) = EI\frac{\partial^2 w}{\partial x^2} + (e_0 a)^2 \frac{\partial^2 M_b}{\partial x^2}.$$

Differentiating Eq. $(2.4)_5$ twice with respect to x we get

$$(2.5) \qquad \frac{\partial^2 M_b}{\partial x^2} = EI\frac{\partial^4 w}{\partial x^4} + (e_0 a)^2 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 M_b}{\partial x^2}\right).$$

The equation of motion of a fluid conveying SWCNT (which can be considered a pipe) subjected to axial magnetic field can be derived as follows [15]:

$$(2.6) \qquad -\frac{\partial^2 M_b}{\partial x^2} = -\mu A H_x^2 \frac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2} + m_f a_{fz} + m_c a_{cz}.$$

In the above equation, the axial force due to magnetic field considered is $\mu A H_x^2$, where μ is the magnetic field permeability, H_x is the magnitude of magnetic field in x direction and A is the area of cross section of the fluid conveying carbon nanotube. The inertial force due to the SWCNT element acceleration in z-direction is given as $m_c a_{cz}$. The inertia force due to the fluid acceleration in the z-direction is $m_f a_{fz}$. The fluid flow is considered to be a simple plug flow.

Nanotube and fluid acceleration terms in Eq. (2.6) have been derived by many researchers as detailed in [15], and the acceleration terms are given by

$$(2.7) \qquad a_{cz} = \frac{\partial^2 w}{\partial t^2}, \qquad a_{fz} = \left(\frac{\partial^2 w}{\partial t^2} + U^2 \frac{\partial^2 w}{\partial x^2} + 2U\frac{\partial^2 w}{\partial x \partial t}\right).$$

Substituting Eq. (2.5) in Eq. (2.6), using Eqs. (2.7) and Eq. (2.2) and rearranging, the final non-local equation of motion for a fluid conveying SWCNT including, the effect of axial magnetic field is obtained as

$$(2.8) \qquad EI\frac{\partial^4 w}{\partial x^4} + M\frac{\partial^2 w}{\partial t^2} + \left(m_f U^2 - \mu A H_x^2\right)\frac{\partial^2 w}{\partial x^2} + 2m_f U\frac{\partial^2 w}{\partial x \partial t} - (e_0 a)^2 \left\{M\frac{\partial^4 w}{\partial x^2 \partial t^2} + \left(m_f U^2 - \mu A H_x^2\right)\frac{\partial^4 w}{\partial x^4} + 2m_f U\frac{\partial^3 w}{\partial x^3 \partial t}\right\} = 0,$$

where $M = m_f + m_c$; m_c is the mass of SWCNT and m_f is the mass of the nanofluid flowing inside.

Solution for clamped-pinned case:

The boundary conditions for the fluid conveying carbon nanotubes clamped at one end and pinned at the other end are as follows:

$$(2.9)_1 \qquad \text{at} \quad x = 0, \quad \text{the transverse displacement} \quad w(0) = 0 \quad \text{and the corresponding slope} \quad \frac{\partial w}{\partial x} = 0,$$

$$(2.9)_2 \qquad \text{at} \quad x = L, \quad \text{the transverse displacement} \quad w(L) = 0 \quad \text{and the bending moment} \quad \frac{\partial^2 w}{\partial x^2} = 0.$$

Again, the procedure given in [15, 18] is adopted to obtain results for clamped-pinned case. The solution of Eq. (2.8) is taken to be

$$(2.9)_3 \qquad w(x,t) = \Re\left[\phi_n\left(\frac{x}{L}\right)\mathrm{e}^{\mathrm{i}\omega t}\right].$$

In Eq. (2.9), $\Re$ denotes the real part, $\phi_n\left(\frac{x}{L}\right)$ is a series of beam eigen-functions. $\psi_r(\xi)$ is given by [21]:

$$(2.10) \qquad \begin{aligned} \phi_n(\xi) &= \sum_{r=1}^{n} a_r \psi_r(\xi), \\ \xi &= \left(\frac{x}{L}\right), \\ \psi_r &= \cosh(\lambda_r \xi) - \cos(\lambda_r \xi) - \sigma_r (\sinh(\lambda_r \xi) - \sin(\lambda_r \xi)), \\ r &= 1, 2, 3, 4, 5, \ldots, n. \end{aligned}$$

In the above equation, λ_r is frequency parameter of the SWCNT without fluid flow, which is considered as a beam, and its values for $r = 1, 2$ [19] are: $\lambda_1 = 3.926602$ and $\lambda_2 = 7.068583$ for clamped-pinned case.

Substituting Eq. (2.9) in the equation of motion Eq. (2.11) gives

$$(2.11) \qquad L_n(\phi) = L_n\left(\sum_{r=1}^{n} a_r \psi_r(\xi)\right) = 0,$$

where L_n is a differential operator given by

$$L_n = \left[EI - e_n^2 L^2 V^2 \frac{EI}{L^2} + e_n^2 L^2 \delta_m \frac{EI}{L^2}\right] \frac{\partial^4}{\partial x^4} - \left[2e_n^2 L^2 M\beta \frac{V}{L}\sqrt{\frac{EI}{M\beta}}\right] i\omega \frac{\partial^3}{\partial x^3} + \left[2M\beta \frac{V}{L}\sqrt{\frac{EI}{M\beta}}\right] i\omega \frac{\partial}{\partial x} + \left[\frac{V^2 EI}{L^2} - \delta_m \frac{EI}{L^2}\right] \frac{\partial^2}{\partial x^2} + \left[e_n^2 L^2 M\right] \omega^2 \frac{\partial^2}{\partial x^2} - M\omega^2. \tag{2.12}$$

According to the Galerkin's method, minimizing the mean square of the function in Eq. (2.11) over the length of the SWCNT, we have

$$\int_0^L L\left(\sum_{r=1}^{N} a_r \psi_r(\xi)\right) \psi_s(\xi)\, dx = 0, \qquad s = 1, 2, 3, 4, 5, \ldots N. \tag{2.13}$$

Following the procedure detailed in [8], substituting Eq. (2.10) in the above equation, using orthogonality relations and retaining only the first two terms, the following equation for critical velocity, above which the system becomes unstable, for a fluid conveying SWCNT having one end clamped and the other end pinned is obtained as:

$$\begin{aligned} &\left[(C_{11}C_{22} - C_{12}C_{21}) - e_n^2\left(\lambda_1^4 C_{22} + \lambda_2^4 C_{11}\right) + \lambda_1^4 \lambda_2^4 e_n^2\right] V^4 \\ &\quad + \left[\left\{\lambda_1^4 C_{22} + \lambda_2^4 C_{11} - 2C_{11}C_{22}\delta_m\right\}\right. \\ &\quad \left. + 2e_n^2\left\{\delta_m\left(\lambda_1^4 C_{22} + \lambda_2^4 C_{11}\right) - \lambda_1^4\lambda_2^4\right\} - 2\lambda_1^4\lambda_2^4 \lambda e_n^4\right] V^2 \\ &\quad + \left[\left\{\lambda_1^4\lambda_2^4 - \left(\lambda_1^4 C_{22} + \lambda_2^4 C_{11}\right)\delta_m + C_{11}C_{22}\delta_m^2\right\}\right. \\ &\quad \left. + e_n^2\delta_m\left\{2\lambda_1^4\lambda_2^4 - \left(\lambda_1^4 C_{22} + \lambda_2^4 C_{11}\right)\lambda\right\} + \lambda_1^4\lambda_2^4\delta_m^2 e_n^2\right] = 0, \end{aligned} \tag{2.14}$$

where

$$V = UL\sqrt{\frac{m_f}{EI}}, \qquad e_n = \frac{e_o a}{L}, \qquad \delta_m = \sqrt{\frac{\mu A H_x^2 L^2}{EI}}. \tag{2.15}$$

Equation (2.14) is quadratic in V^2 which can be easily solved for critical value of non-dimensional fluid velocity parameter V. The constants C_{11} and C_{22} in Eq. (2.14) are integral values taken from Felgar [20] which are also reproduced in Rao and Simha [21].

3. Results and discussions

The critical flow velocity V_{cr} is an important parameter for the study of stability of fluid conveying SWCNTs. At the critical flow velocity, the natural frequency becomes zero, leading to divergence instability of the SWCNT. In Eq. (2.14), the lowest root of V is the critical flow velocity, V_{cr}. For the clamped-pinned boundary conditions, this parameter has been evaluated for different values of the axial magnetic field parameter, δ_m and the non-local parameter e_n by using a specifically written MATLAB computer program for this case. In the formulation of Lee and Chang [6], critical velocity is shown to be not dependent on the non-local parameter due to the inherent error in their formulation. However, it can be noticed from the results presented here, that the non-local parameter has a considerable influence on the critical velocity, which is more pronounced at low values of axial magnetic field parameter. In Table 1, the results obtained from the analysis presented in this paper (for the case where the nonlocal elasticity effect ($e_n = 0$) and axial magnetic field effect ($\delta_m = 0$) are neglected) are compared with those obtained by using differential transformation method (DTM) [22], differential quadrature method (DQM) [17] and the exact solution [23]. One can easily observe that the results obtained from the present analysis method are in good agreement with those obtained from different numerical methods and the exact solution showing the accuracy of the present method.

Table 1. Critical velocities of a clamped-pinned supported pipe conveying fluid neglecting nonlocal and axial magnetic field effects ($e_n = 0$ and $\delta_m = 0$).

	DTM [22]	DQM [17]	Païdoussis [23]	Present
1st mode	4.4934	4.4937	≈ 4.49	4.4998

Clamped-pinned case

Critical flow velocities are found by solving Eq. (2.14) for V_{cr}, for the case of fluid conveying pipe clamped at one end and pinned at the other end. Tables 2, 3 and 4 show the numerical results in the tabular format. Tables 2, 3 and 4 show the values of V_{cr} for different values of axial magnetic field parameter $\left(\delta_m = 0 - 10^5\right)$ and non-local parameter ($e_n = 0$ to 1.0). It is seen from the Tables 2–4 that as the value of the non-local parameter e_n increases, the values of the critical flow velocity parameter V_{cr} decreases. This decrease is more pronounced for lower values of the axial magnetic field parameter, δ_m. The selected values of axial magnetic field parameters (δ_m) provide a wide range of characteristics varying from lower to higher values of magnetic fields in this study.

Table 2. Values of the critical flow velocity parameter, V_{cr}, for different values of the non-local parameter, e_n, and the magnetic field parameter, δ_m, for a clamped-pinned fluid conveying SWCNT.

$e_n = 0.00$		0.05	0.10	0.15	0.20	0.25	0.30
δ_m	V_{cr}						
0.0	4.4998	4.3900	4.1035	3.7297	3.3447	2.9896	2.6785
0.01	4.5009	4.3912	4.1047	3.7310	3.3462	2.9912	2.6803
0.05	4.5053	4.3957	4.1095	3.7364	3.3522	2.9979	2.6878
0.075	4.5081	4.3985	4.1126	3.7397	3.3559	3.0021	2.6924
0.10	4.5108	4.4014	4.1156	3.7431	3.3596	3.0062	2.6971
0.50	4.5550	4.4466	4.1639	3.7961	3.4186	3.0721	2.7702
0.75	4.5823	4.4746	4.1938	3.8289	3.4550	3.1125	2.8150
1.00	4.6095	4.5025	4.2235	3.8614	3.4910	3.1524	2.8591
5.00	5.0247	4.9267	4.6732	4.3486	4.0233	3.7333	3.4892
7.50	5.2676	5.1742	4.9334	4.6271	4.3229	4.0543	3.8307
10.0	5.4998	5.4104	5.1806	4.8898	4.6030	4.3517	4.1442
50.0	8.3814	8.3230	8.1755	7.9944	7.8222	7.6771	7.5614
75.0	9.7595	9.7094	9.5832	9.4292	9.2837	9.1617	9.0650
10^2	10.9658	10.9212	10.8092	10.6729	10.5445	10.4373	10.3525
10^3	31.9413	31.9260	31.8879	31.8420	31.7992	31.7638	31.7360
10^4	100.1012	100.0963	100.0842	100.0695	100.0559	100.0447	100.0359
10^5	316.2598	316.2582	316.2544	316.2498	316.2455	316.2419	316.2391

Table 3. Values of the critical flow velocity parameter, V_{cr}, for different values of the non-local parameter, e_n, and the magnetic field parameter, δ_m, for a clamped-pinned fluid conveying SWCNT.

$e_n = 0.35$		0.40	0.45	0.50	0.55	0.60	0.65
δ_m	V_{cr}						
0.0	2.4120	2.1854	1.9925	1.8276	1.6858	1.5629	1.4557
0.01	2.4141	2.1877	1.9950	1.8303	1.6887	1.5661	1.4592
0.05	2.4223	2.1968	2.0050	1.8412	1.7005	1.5788	1.4728
0.075	2.4275	2.2025	2.0112	1.8480	1.7079	1.5867	1.4813
0.10	2.4326	2.2081	2.0174	1.8548	1.7152	1.5946	1.4897
0.50	2.5135	2.2969	2.1142	1.9596	1.8281	1.7154	1.6184
0.75	2.5628	2.3507	2.1726	2.0224	1.8952	1.7868	1.6939
1.00	2.6111	2.4033	2.2294	2.0833	1.9601	1.8554	1.7661
5.00	3.2890	3.1266	2.9950	2.8879	2.8003	2.7281	2.6682
7.50	3.6493	3.5037	3.3867	3.2924	3.2159	3.1532	3.1015
10.0	3.9772	3.8439	3.7376	3.6524	3.5835	3.5274	3.4813
50.0	7.4711	7.4011	7.3464	7.3034	7.2692	7.2417	7.2194
75.0	8.9899	8.9317	8.8865	8.8510	8.8228	8.8002	8.7818
10^2	10.2868	10.2360	10.1966	10.1656	10.1411	10.1214	10.1054
10^3	31.7146	31.6982	31.6855	31.6755	31.6677	31.6614	31.6563
10^4	100.0291	100.0239	100.0198	100.0167	100.0142	100.0122	100.0106
10^5	316.2370	316.2353	316.2340	316.2330	316.2323	316.2316	316.2311

Table 4. Values of the critical flow velocity parameter, V_{cr}, for different values of the nonlocal parameter, e_n, the magnetic field parameter, δ_m, for a clamped-pinned fluid conveying SWCNT.

$e_n = 0.70$		0.75	0.80	0.85	0.90	0.95	1.00
δ_m	V_{cr}						
0.0	1.3616	1.2784	1.2044	1.1382	1.0787	1.0250	0.9762
0.01	1.3653	1.2823	1.2085	1.1426	1.0833	1.0298	0.9813
0.05	1.3798	1.2978	1.2250	1.1600	1.1016	1.0491	1.0015
0.075	1.3889	1.3074	1.2351	1.1707	1.1129	1.0609	1.0139
0.10	1.3978	1.3169	1.2452	1.1813	1.1241	1.0726	1.0261
0.50	1.5343	1.4609	1.3966	1.3400	1.2898	1.2452	1.2054
0.75	1.6137	1.5441	1.4834	1.4302	1.3833	1.3418	1.3050
1.00	1.6894	1.6230	1.5654	1.5151	1.4709	1.4320	1.3975
5.00	2.6180	2.5757	2.5398	2.5091	2.4827	2.4598	2.4399
7.50	3.0584	3.0223	2.9917	2.9657	2.9434	2.9241	2.9074
10.0	3.4430	3.4109	3.3839	3.3609	3.3412	3.3242	3.3095
50.0	7.2010	7.1857	7.1729	7.1621	7.1529	7.1450	7.1381
75.0	8.7666	8.7541	8.7436	8.7347	8.7272	8.7207	8.7151
10^2	10.0923	10.0814	10.0723	10.0646	10.0580	10.0524	10.0475
10^3	31.6521	31.6486	31.6457	31.6433	31.6412	31.6394	31.6378
10^4	100.0093	100.0082	100.0073	100.0065	100.0058	100.0053	100.0048
10^5	316.2307	316.2304	316.2301	316.2298	316.2296	316.2294	316.2293

Figure 2 presents a plot of critical velocity ratio dependence on the variation in nonlocal parameter e_n and the magnetic field parameter δ_m. The ordinate in Fig. 2 is plotted as a critical velocity ratio, as defined by Eq. (3.1).

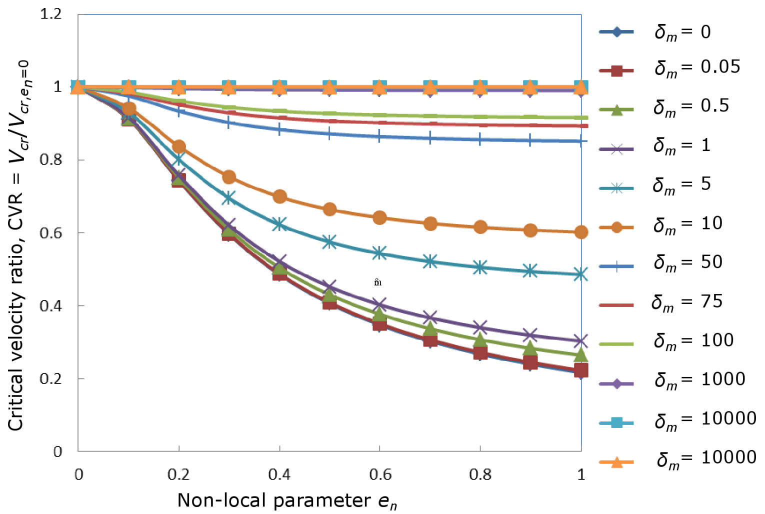

Fig. 2. Variation of the critical velocity ratio with non-local parameter e_n for different values of the magnetic field parameter δ_m for a clamped-pinned fluid conveying SWCNT.

Critical Velocity Ratio,

$$\text{CVR} = \left[\frac{(V_{cr})_{e_n}}{(V_{cr})_{e_n=0}} \right]. \tag{3.1}$$

It is clearly seen from the Fig. 2 that as the non-local parameter increases, the critical velocity decreases. The effect of the axial magnetic field parameter δ_m is also brought out clearly in the Fig. 2. It is observed from the Fig. 2, that the critical velocity ratio decreases with increasing values of the non-local parameter, (e_n) for a given values of axial magnetic field parameter ($\delta_m = 0, 0.5, 1, 5, 10, 50, 75, 10^2, 10^3, 10^4, 10^5$). The higher the axial magnetic field parameter, the lower is the rate of decrease in the critical flow velocity as the non-local parameter (e_n) increases.

It is also observed from Fig. 2 that the critical velocity increases as magnetic field increases for a given value of non-local parameter. This effect diverges as non-local parameter increases. The variation of percentage change in the critical velocity with the magnetic field as non-local parameter changes from 0 to 1.0 is shown in Fig. 3. This is the dependence of percentage change in critical velocity from $e_n = 0$ to 1 on the magnetic field. It is noticed that the percentage change in the critical velocity decreases from 78.31% to 0.0096% with increase in magnetic field parameter as non-local parameter changes from 0 to 1.0. Hence, the non-local parameter has considerable influence on the stability of the SWCNT in the presence of magnetic field. As the magnetic field parameter δ_m attains a highest value such as 10^5, the effect of non-local parameter e_n on the critical flow velocity gradually becomes negligibly small.

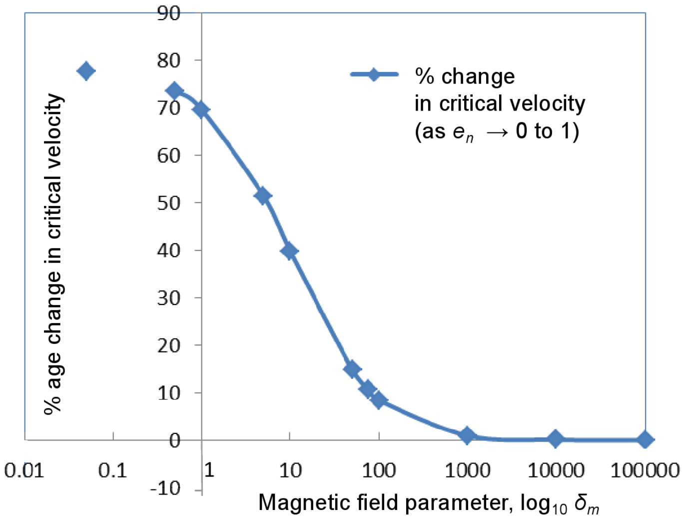

FIG. 3. Variation of the percentage of critical velocity with the magnetic field parameter δ_m as non-local parameter e_n changes from 0 to 1.0 for a clamped-pinned fluid conveying SWCNT.

But, the critical flow velocity converges to a constant value as the value of magnetic field parameter become higher and higher. It can also be seen that the magnetic field parameter in general acts similar to an axial tensile force and thus adding to the stiffness of the SWCNT conveying fluid. In the formulation of Lee and Chang [6], critical velocity is shown to be not dependent on the non-local parameter due to the inherent error in their formulation. However, it can be noticed from the results presented here, that the non-local parameter has a considerable influence on the critical velocity which is more pronounced at low values of axial magnetic field parameter.

4. Conclusions

This study has attempted to address the gaps in the literature by presenting numerical results for the stability of a fluid conveying SWCNT subjected to axial magnetic field. The governing equations have been formulated in the present paper duly taking into account the concept of non-local mechanics and the most important problem formulation errors made by Lee and Chang [7] in their study, as pointed out by Tounsi *et al.* [9]. A two-term Galerkin's procedure has been used for the case of clamped-pinned boundary conditions. It has been very well established that when the flow velocity reaches a certain value, called the critical flow velocity, the frequency becomes zero leading to instability. Simple quadratic equations in the critical velocity parameter are derived and presented here for the first time. The real values of the critical flow velocity parameter obtained for the clamped-pinned fluid conveying SWCNT subjected to axial magnetic field are presented in numerical as well as graphical form. In summary, it can be said that, higher the values of the non-local parameter the higher are the effects of reducing the stability of the system, which is the outcome of the consistent formulation presented here. One can easily see that in the Lee and Chang's analysis [6, 7], the non-local parameter does not have any effect on the critical flow velocity, which is actually not the case as can be seen from the results presented in this paper. It can be observed that even a two-term solution using Galerkin's methods gives almost accurate results as can be seen from the excellent agreement obtained between the results with those results presented by Wang [10], who also used a correct non-local formulation.

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