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A FUZZY-SET APPROACH TO BUCKLING ANALYSIS OF COMPOSITE STRUCTURES

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A fuzzy-set approach conjugated with finite element analysis is used to investigate the influence of the variability (random field) of geometric and material properties on buckling loads understood as one of possible failure modes for composite structures. The uncertainty (the scatter) of buckling loads is created by the prescribed variations of thickness and in Young's and Kirchhoff's moduli. The α-cut and vertex methods are utilized to study the sensitivity of buckling loads to fuzzy parameters variations. Numerical results are presented for axially compressed angle ply-plates and shallow cylindrical panels.

1. Introduction

The use of fibre-reinforced composite materials in modern engineering structural design has become a common practice. However, since more design variables typically exist when composite materials are employed and the manufacturing processes for producing composites are more complex, more variability can exist in a design produced with composites compared to conventional materials. Thus, the variability of properties that occurs in composite structures leads directly to a random field of variables describing constructions. A scatter of properties has a different origin, but, in general, it may be divided and classified in the following manner: (1) geometric properties (imperfections), (2) physical and mechanical properties, (3) environmental effects (exploitation), (4) technology (understood in the sense of geometric dimensions but as an origin of local defects, a scatter of fibre directions etc.). Therefore, there is a fundamental question: how many and which of the above factors can (or should) be incorporated in the design process and how can we manage to take into account the existing variability of parameters.

Although the majority of available references in literature discussing the design problems of composite structures is devoted to the analysis conducted in a pure deterministic way, the variety of methods exists that have been adopted in order to take into considerations random field of parameters describing the composites. Among them one can distinguish the following approaches: (a) statistical methods: starting from the BOLOTIN method [1], the Monte-Carlo simulation – ELISHAKOFF [2] or using the statistical distributions (see e.g. YANG *et al.* [3] or LEE *et al.* [4]), (b) probabilistic finite element methods that intend to incorporate micromechanics in the FE (linear or non-linear) analysis – see e.g. Refs. [5–8], (c) stochastic description – see e.g. JUSZANOV, BOGDANOVITCH [9, 10] or (d) non-stochastic methods – a fuzzy set approach. The fuzzy set approach has been used mainly in various optimisation problems solved for composite structures – see ADALI [11] or KIM *et al.* [12]. NOOR *et al.* [13] have utilized the fuzzy set approach to the FE analysis of failure problems. MUC *et al.* [14] explained a theoretical background of the fuzzy set theory and demonstrated the application of the fuzzy set approach in various mechanical problems (the use of the fuzzy set approach in engineering, mechanics and mechanics of composites) such as, e.g. definitions of material properties for unidirectional and textile composites (application of micromechanical models), damage analysis of the limit load carrying capacity of composite structures including buckling response, the first-ply-failure (FPF), fatigue problems and stacking sequence (topology) optimization of composite structures in a fuzzy environment.

The aim of the present work is an attempt to analyse the influence of scatter of the material and mechanical properties on the values of buckling loads and on the optimal fibre orientations for angle-ply laminates. In the first part of the work we discuss briefly the fundamental problems connected with the fuzzy set analysis. Then, we use these methods in the numerical (FE) or theoretical analysis of buckling problems for composite, multilayered compressed plates and cylindrical panels. The presented examples illustrate the advantages and disadvantages of the proposed fuzzy set approach to the design problems of composite structures. The proposed methodology can be easily extended to the FE examination of other than buckling failure modes, e.g. the first- and the last-ply failure, delaminations etc.

2. FOUNDATIONS OF THE FUZZY SET THEORY

This section deals with foundations of the fuzzy set theory. First of all we present the notation of a fuzzy set and discuss forms of their possible representations. Then, we discuss some aspect of their fuzzy sets.

If we consider a certain space, for instance the set $\mathbb{N}$ of all integers, we generally describe, data by defining subsets of the given space. In the space $\mathbb{N}$ the feature "less than 10" is characterized by the set:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \subset \mathbb{N}. \tag{2.1}$$

Another representation of the data "integer less than 10" is the definition of the characteristic function Π_A in the following way:

$$\Pi_A : \mathbb{N} \to \{0,1\}$$

$$\Pi_A(\eta) = \begin{cases} 1, & \text{if less than 10} \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

that yields the value 1 for each element of the space $\mathbb{N}$ that belongs to the set A and the value 0 for each element that does not. The above representation is commonly called a crisp set. However, this concept can not be used directly since we intend to characterise the typical property for composite materials as e.g.: the failure of CFRP under tension occurs as the tensile strain ε_x is equal to the ultimate value 0.015. The characteristic function of this set is depicted in Fig. 1a. A problem arises if the linguistic term "the failure under tension" has to be described. It is well known that from the micromechanical point of view, the failure starts from microcracks in the matrix for the strain values much lower than 0.015. In addition, the value 0.015 is usually an average value characterizing rather a scatter of random values of macrocracks appearing at the strain level 0.015. Therefore, for some specimens one can observe the final (macroscale) failure as ε_x is equal to 0.0159 or to 0.0141. A possible solution to this problem is generalization of the definition of the characteristic function in a way that it should yield values from the interval [0, 1] and not just the two values of the set {0, 1}. This leads to the notion of a fuzzy set.

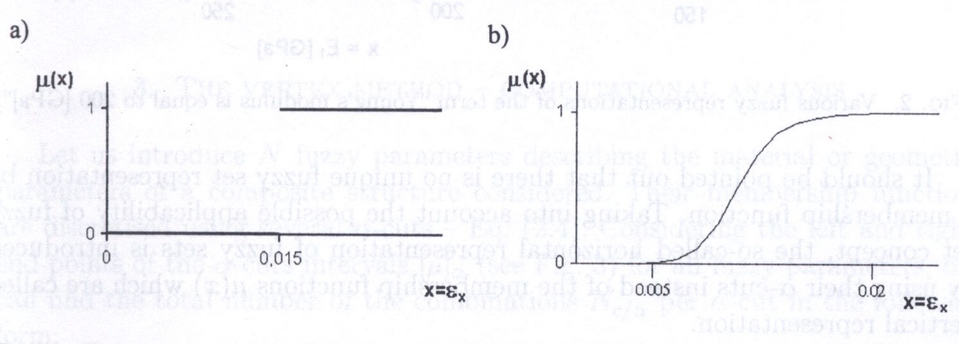

FIG. 1. The representation of the term "the failure occurs ..." a) a crisp set, b) a fuzzy set.

A fuzzy set μ of X is a function that maps the space X onto the unit interval, i.e.:

$$\mu : X \to [0,1]. \tag{2.3}$$

The value $\mu(x)$ denotes the membership function of x in the fuzzy set μ. Figure 1b shows (subjectively defined) a membership function of the fuzzy set

μ describing the linguistic meaning of the term "the failure under tension". The use of fuzzy sets to formally represent vague data is often done in an intuitive way because in many applications, there is no model that provides a clear interpretation of the membership degrees, although we want or we try to base on various experimental data.

Of course, there are different possibilities to determine and represent the membership functions characterizing a fuzzy set. If the subspace X contains only a finite number of elements, a fuzzy set μ of X will be defined by specifying for each element $x \in X$ its membership degree $\mu(x)$. If the number of elements is very large or a continuum is chosen for X, then $\mu(x)$ can be better defined by a function that can use parameters which are adapted to the actual modelling problem. For instance, if we want to represent the term "Young's modulus is equal to 200 [GPa]" in the sense of a fuzzy set having a finite amount of the experimental data, we can select one of different representations given in Fig. 2 and Ref. [14].

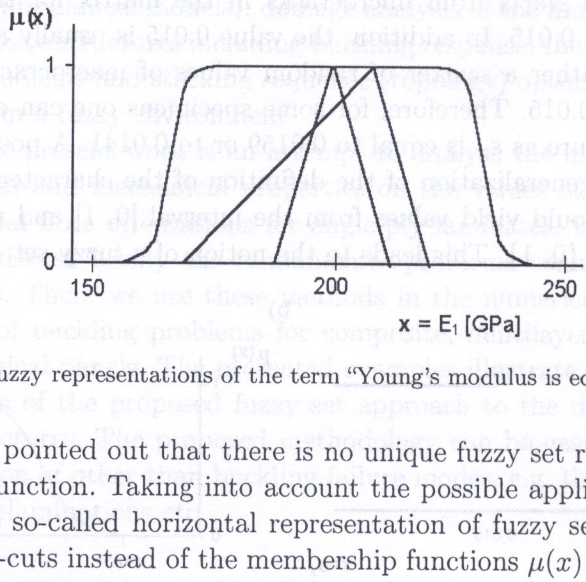

FIG. 2. Various fuzzy representations of the term "Young's modulus is equal to 200 [GPa]".

It should be pointed out that there is no unique fuzzy set representation by a membership function. Taking into account the possible applicability of fuzzy set concept, the so-called horizontal representation of fuzzy sets is introduced by using their α-cuts instead of the membership functions $\mu(x)$ which are called vertical representation.

Let $\mu \in F(x)$ and $\alpha \in [0, 1]$. The set

$$[\mu]_\alpha = \{x \in X | \mu(x) \geq \alpha\} \tag{2.4}$$

is called the α-cuts of μ.

Let μ be the triangular function on $\mathbb{R}$ given in Fig. 3. The α-cuts of μ are in this case defined as follows:

$$[\mu]_\alpha = \begin{cases} [a + \alpha \cdot (m - a), b - \alpha \cdot (b - m)] & \text{if} \quad 0 < \alpha \leq 1 \\ \mathbb{R} & \text{if} \quad \alpha = 0. \end{cases} \tag{2.5}$$

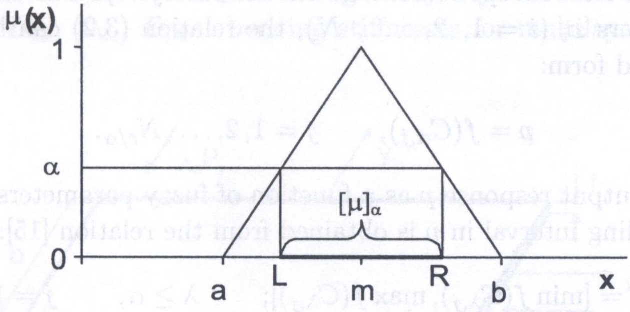

FIG. 3. Definition of α-cuts.

In the present study it is assumed that the membership functions of the fuzzy parameters are triangular as shown schematically in Fig. 3 (see also Eq. (2.5)) where:

$$m = \frac{a+b}{2}, \qquad a = 0.9 \cdot m, \qquad b = 1.1 \cdot m \tag{2.6}$$

and m is an average value for each of the fuzzy parameters, for instance it can be evaluated from the experimental data. As it may be seen from the relation (2.6), the variability (fuzziness) is taken to be equal to $\pm 10\%$, which falls within the typical ranges of scatter in experimental data for static tests.

3. The vertex method – computational analysis

Let us introduce N fuzzy parameters describing the material or geometric parameters of a composite structure considered. Their membership functions are discretised using several α-cuts – Eq. (2.4). Considering the left and right-end points of the α-cuts intervals $[\mu]_\alpha$ (see Fig. 3) for all fuzzy parameters, one can find the total number of the combinations $N_{c/\alpha}$ per α-cut in the following form:

$$N_{c/\alpha} = \begin{cases} 2^N & \text{for} \quad 0 \le \alpha < 1, \\ 1 & \text{for} \quad \alpha = 1. \end{cases} \tag{3.1}$$

The output response denoted by p is an unknown function of the input fuzzy parameters x_i $(i = 1,\ 2,\ \ldots,\ N)$, so that:

$$p = f(x_1, \ldots, x_N). \tag{3.2}$$

Using the α-cut concept combined with the binary representation (3.1) of the fuzzy parameters x_i $(i = 1, 2, \ldots, N)$, the relation (3.2) can be rewritten in the abbreviated form:

$$p = f(C_{\alpha,j}), \qquad j = 1, 2, \ldots, N_{c/\alpha}. \tag{3.3}$$

Since the output response p as a function of fuzzy parameters is a fuzzy set, the corresponding interval in p is obtained from the relation [15]:

$$[p^L_\alpha, p^R_\alpha] = [\min_{\lambda,j} f(C_{\lambda,j}), \max_{\lambda,j} f(C_{\lambda,j})]; \qquad \lambda \geq \alpha, \qquad j = 1, 2, ..., N_{c/\alpha}, \tag{3.4}$$

As it may be seen, the relation (3.4) allows to obtain a scatter of the output parameters an then to build the appropriate probability distributions and reliability functions by sweep of α-cut at different possibility levels.

In order to conduct the computations and to evaluate the upper and lower bounds of the output response (3.4), it is necessary to determine the deterministic method of definition of the function f given in Eq. (3.2). It can be defined in a pure analytical way or alternatively – in a pure numerical way. As it may be noticed, the vertex method resembles here the Monte Carlo simulation method where the output response has also the deterministic, unique form.

The function f existing in Eq. (3.2) may describe an arbitrary failure criterion for composites, e.g. buckling, delamination, the first-ply-failure etc., whereas symbol p denotes the corresponding value of the failure load.

4. Results

4.1. *The influence of variations of fibre orientations*

The following analysis deals with the evaluation of variations of fibre orientations. Figure 4 shows a compressed composite rectangular plate. Compressive loads are given by forces P_x and P_y. The parameter of buckling λ_b has been derived by Whitney and Leissa [16], λ_b is defined in the following way:

$$\lambda_b = \frac{(m_b/\pi \cdot a)^2}{P_x \cdot (1 + k \cdot \beta^2_m)} \cdot T_{33} \tag{4.1}$$

where:

$$T_{33} = D_{11} + 2 \cdot (D_{12} + 2 \cdot D_{66}) \cdot \beta^2_m + D_{22} \cdot \beta^4_m,$$

$$\beta_m = \frac{n_b \cdot a}{m_b \cdot b}, \qquad k = \frac{P_y}{P_x}, \tag{4.2}$$

a, b – dimensions of sides of the plate – Fig. 4, n_b, m_b – number of half-wave of buckling, D_{11}, D_{12}, D_{22}, D_{66} – bending stiffnesses for multilayered composite structures.

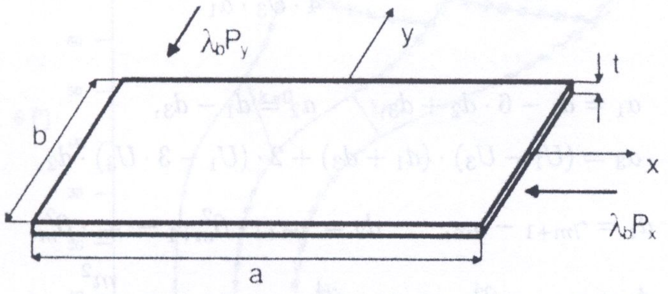

FIG. 4. Geometry of a compressed multilayered rectangular plate.

In this analysis, the value of n_b is equal to 1 and the value of m_b is taken as 1, 2 or 3. Using the relationship (4.1), MUC [17] determined the optimal value of fibre orientations as follows:

$$\theta = 0^\circ \quad \text{or} \quad \theta = 90^\circ \quad \text{or} \quad \cos(2\cdot\theta) = \varphi \cdot \frac{\beta_m^4 - 1}{-6\cdot\beta_m^2 + \beta_m^4 + 1}, \tag{4.3}$$

where:

$$\varphi = \frac{U_2}{4\cdot U_3} = \frac{1 - \dfrac{E_2}{E_1}}{1 + \dfrac{E_2}{E_1} - 2\cdot\nu_{12}\cdot\dfrac{E_2}{E_1} - 4\cdot(1-\nu_{12}\cdot\nu_{21})\cdot\dfrac{G_{12}}{E_1}}, \tag{4.4}$$

$$U_2 = \frac{1}{2}\cdot(Q_{11} - Q_{22}), \qquad U_3 = \frac{1}{8}\cdot(Q_{11} + Q_{22} - 2\cdot Q_{12} - 4\cdot Q_{66}), \tag{4.5}$$

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-\nu_{12}\cdot\nu_{21}}, & Q_{12} &= \frac{\nu_{12}\cdot E_2}{1-\nu_{12}\cdot\nu_{21}}, \\ Q_{22} &= \frac{E_2}{1-\nu_{12}\cdot\nu_{21}}, & Q_{66} &= G_{12}. \end{aligned} \tag{4.6}$$

If parameter β_m is located in this interval

$$\frac{-6 \pm \sqrt{36 + 4\cdot(\varphi^2 - 1)}}{2\cdot(\varphi - 1)} \le \beta_m^2 \le \frac{6 \pm \sqrt{36 + 4\cdot(\varphi^2 - 1)}}{2\cdot(\varphi - 1)} \tag{4.7}$$

then optimal fibre orientations is obtained from the bimodal condition in the following form

$$\cos(2\cdot\theta) = \frac{-U_2\cdot a_2 \pm \sqrt{U_2^2\cdot a_2^2 - 8\cdot U_3\cdot a_1\cdot a_1}}{4\cdot U_3\cdot a_1} \tag{4.8}$$

where:

$$\begin{aligned} &a_1 = d_1 - 6\cdot d_2 + d_3, \qquad a_2 = d_1 - d_3,\\ &a_3 = (U_1 - U_3)\cdot(d_1+d_3) + 2\cdot(U_1 - 3\cdot U_3)\cdot d_2, \end{aligned} \tag{4.9}$$

$$\begin{aligned} &d_1 = \gamma_{m+1} - \gamma_m, \qquad d_2 = \gamma_{m+1}\cdot\beta_{m+1}^2 - \gamma_m\cdot\beta_m^2,\\ &d_3 = \gamma_{m+1}\cdot\beta_{m+1}^4 - \gamma_m\cdot\beta_m^4, \qquad \gamma_m = \frac{m^2}{1+k\cdot\beta_m^2}. \end{aligned} \tag{4.10}$$

To demonstrate and compare the results two composite materials, are considered herein, presented in Table 1. Three parameters have been considered as fuzzy variables: the Young moduli E_1, E_2 and the Kirchhoff modulus G_{12}. The calculations have been carried out for mechanical properties taken from Table 1 (a crisp set, i.e. $\alpha = 1$) and fuzziness properties for $\alpha = 0.5$.

Table 1. Mechanical properties of composite materials.

Material	Fibre	Resin	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12}	φ
1	Glass E	Polyester	28	8.2	2.8	0.29	0.9649
2	Boron	Epoxy	204	18.5	5.59	0.23	0.9675

Figures 5 and 6 demonstrate the influence of fuzziness in composite mechanical properties on the scatter of optimal fibre orientations. The solid line represents the results of deterministic approach corresponding to $\alpha = 1$ whereas broken lines present the scatter of the optimal results for $\alpha = 0.5$. As it may be seen, the mechanical properties of composites affect the magnitude of scatter of fibre orientations as well as the scatter of buckling loads – see Table 2. However, it is worth to note that the scatter of buckling loads is almost identical for different types of material, although buckling loads are not linearly dependent on Young's modulus E_1 – see Eqs. (4.1) and (4.2).

Table 2. The fuzziness of fibre orientations θ.

Material	$\Delta\theta = \theta_{\max} - \theta_{\min}$ [°]		Buckling loads	
	Half-wave of buckling	Bimodal condition	$\Delta\lambda_b$	$\dfrac{\Delta\lambda_b\cdot 100\%}{\Delta\lambda_{b\,\max\,\mathrm{det.}}}$
1	13.64	10.01	23.7	9.78%
2	5.94	0.98	16.6	9.91%

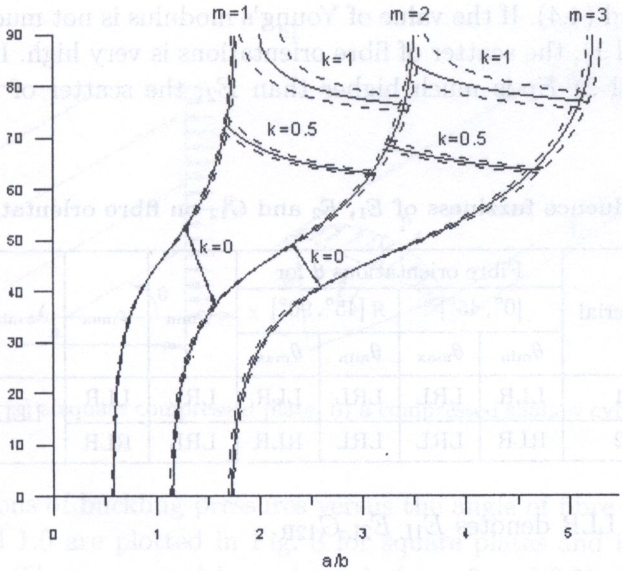

FIG. 5. Optimal fibre orientations θ for material 1, $\alpha = 1.0$ (solid line) and $\alpha = 0.5$ (broken line).

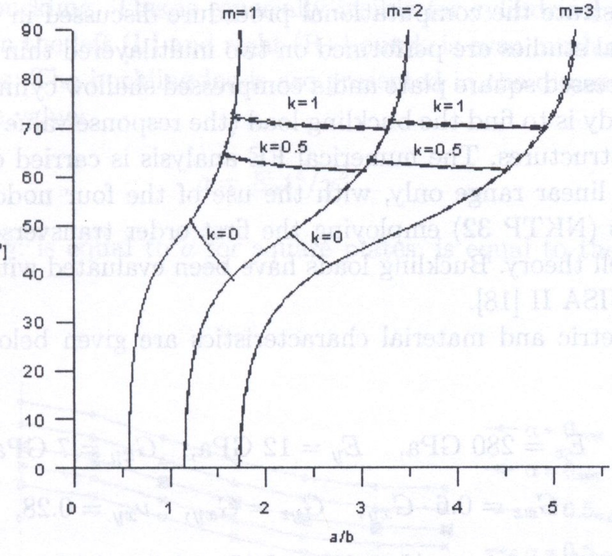

FIG. 6. Optimal fibre orientations θ for material 2, $\alpha = 1.0$ (solid line) and $\alpha = 0.5$ (broken line).

This effect is also presented in Table 3 since for $\alpha = 0.5$ the fuzziness in values of optimal fibre orientations is caused by different combinations of mechanical factors. In our opinion, the results are very sensitive to the variations of

Eqs. (4.3) and (4.4). If the value of Young's modulus is not much higher than E_2 (the material 1), the scatter of fibre orientations is very high. In the second case (the material 2) E_1 is much higher than E_2, the scatter of fibre orientations is low.

Table 3. Influence fuzziness of E_1, E_2 and G_{12} on fibre orientations θ for $\alpha = 0.5$.

<table>
<tr><th rowspan="3">Material</th><th colspan="4">Fibre orientations θ for</th><th rowspan="3">$\varphi_{\min}$</th><th rowspan="3">$\varphi_{\max}$</th><th rowspan="3">$\lambda_{b\,\min}$</th><th rowspan="3">$\lambda_{b\,\max}$</th></tr>
<tr><th colspan="2">$[0^\circ, 45^\circ]$</th><th colspan="2">$[45^\circ, 90^\circ]$</th></tr>
<tr><th>$\theta_{\min}$</th><th>$\theta_{\max}$</th><th>$\theta_{\min}$</th><th>$\theta_{\max}$</th></tr>
<tr><td>1</td><td>LLR</td><td>LRL</td><td>LRL</td><td>LLR</td><td>LRL</td><td>LLR</td><td rowspan="2">LLL</td><td rowspan="2">RRR</td></tr>
<tr><td>2</td><td>RLR</td><td>LRL</td><td>LRL</td><td>RLR</td><td>LRL</td><td>RLR</td></tr>
</table>

Here e.g. LLR denotes $E_{1\mathrm{L}}\, E_{2\mathrm{L}}\, G_{12\mathrm{R}}$.

4.2. The effects of scatter of mechanical properties

To demonstrate the computational procedure discussed in the previous section, numerical studies are performed on two multilayered thin composite structures: a compressed square plate and a compressed shallow cylindrical panel. The aim of the study is to find the buckling load (the response value p – see Eqs. (3.2) or (3.3)) for structures. The numerical FE analysis is carried out in the elastic geometrically linear range only, with the use of the four nodded quadrilateral shell elements (NKTP 32) employing the first order transverse shear deformation plate/shell theory. Buckling loads have been evaluated with the help of the FE package NISA II [18].

The geometric and material characteristics are given below and shown in Fig. 7.

$$E_x = 280 \text{ GPa}, \quad E_y = 12 \text{ GPa}, \quad G_{xy} = 7 \text{ GPa},$$

$$G_{xz} = 0.6 \cdot G_{xy}, \quad G_{yz} = G_{xy}, \quad \nu_{xy} = 0.28, \tag{4.11}$$

$$t/a = 0.1, \quad t/L = 0.1, \quad L/R = 0.1, \quad f/L = 0.1,$$

where t is the plate/shell thickness, a is the plate length, L is the cylinder length, whereas f and R denote the shallowness parameter and the radius of a cylinder, respectively.

Four parameters have been considered as fuzzy variables (i.e. $N = 4$): the total thickness t, the Young moduli E_x, E_y and the Kirchhoff modulus G_{xy}.

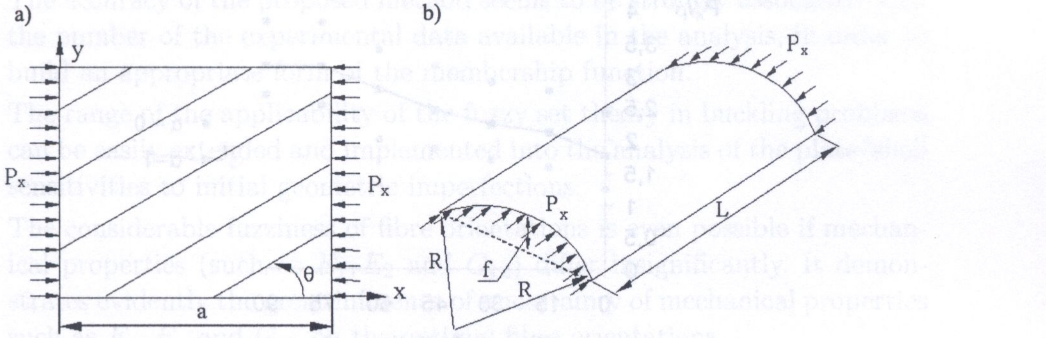

FIG. 7. Geometry of: a) a square compressed plate, b) a compressed shallow cylindrical panel.

The distributions of buckling pressures versus the angle of fibre orientations at $\alpha = 0$, 0.5 and 1.0 are plotted in Fig. 8 for square plates and in Fig. 9 for cylindrical panels. The upper and lower bounds ($\alpha = 0$ and 0.5) of the curves drawn for $\alpha = 1.0$ are not symmetric as it is demonstrated in Table 4. The interval (3.4) is strongly dependent on the fibre orientations as well as on the wave-number in buckling. This is especially visible for cylindrical panels where the unsymmetry in the left (L) and right (R) bounds is even much more striking than for the plates. The buckling loads are presented in the dimensionless form and related to the value:

$$\rho = E_x t^2/\gamma^2, \tag{4.12}$$

where parameter γ: is equal to a for square plates, is equal to the length L of cylindrical panels.

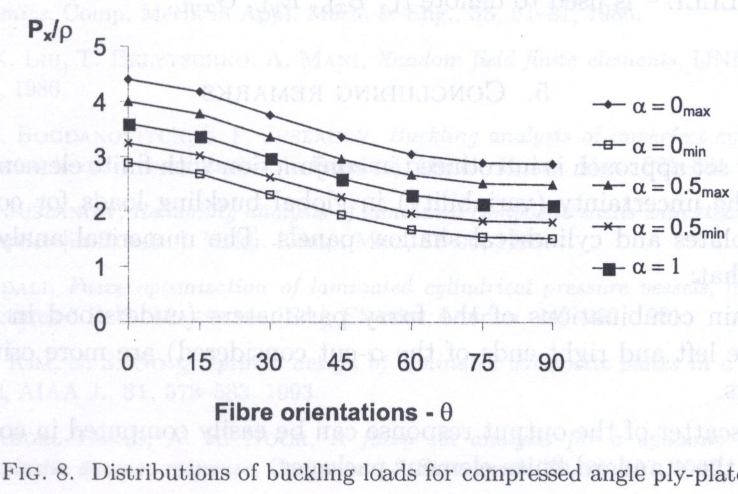

FIG. 8. Distributions of buckling loads for compressed angle ply-plates.

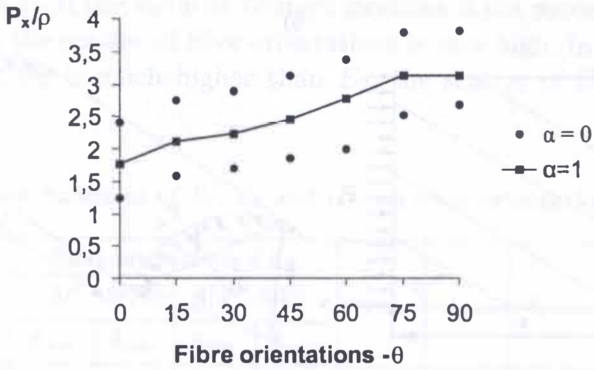

FIG. 9. Variations of buckling pressures with fibre orientations for compressed shallow cylindrical shells.

Table 4. Buckling pressures for compressed shallow cylindrical shells for $\alpha = 0°$ and $\theta = 75°$

Combination (t, E_x, E_y, G_{xy})	LLLL	LLLR	LLRL	LLRR	LRLL	LRLR	LRRL	LRRR
Dimensional buckling load [MPa]	429.300	459.344	486.979	524.699	429.344	459.352	487.027	524.700
Combination (t, E_x, E_y, G_{xy})	RLLL	RLLR	RLRL	RLRR	RRLL	RRLR	RRRL	RRRR
Dimensional buckling load [MPa]	659.152	712.814	738.685	805.717	659.134	712.735	738.675	805.630

where e.g. LLLL – is used to denote t_{L}, $E_{x\mathrm{L}}$, $E_{y\mathrm{L}}$, $G_{xy\mathrm{L}}$.

5. Concluding remarks

A fuzzy set approach is introduced in conjunction with finite element analysis to study the uncertainty (variability) in global buckling loads for compressed angle ply-plates and cylindrical shallow panels. The numerical analysis shows evidently that:

1. Certain combinations of the fuzzy parameters (understood in the sense of the left and right ends of the α-cut considered) are more critical than others.
2. The scatter of the output response can be easily computed in conjunction with the standard finite element packages.

3. The accuracy of the proposed method seems to be strongly associated with the number of the experimental data available in the analysis, in order to build an appropriate form of the membership function.
4. The range of the applicability of the fuzzy set theory in buckling problems can be easily extended and implemented into the analysis of the plate/shell sensitivities to initial geometric imperfections.
5. The considerable fuzziness of fibre orientations is even possible if mechanical properties (such as E_1, E_2 and G_{12}) differ insignificantly. It demonstrates evidently the great influence of uncertainty of mechanical properties such as E_x, E_y and G_{xy} on the optimal fibre orientations.

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