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The Initial Velocity of a Metal Plate Explosively-Launched from an Open-Faced Sandwich (OFS)

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Using analytic solutions for one-dimensional plane boundary value problems related to the launching of solids by detonation products (DPs) of high explosives, algebraic formulae which define the displacement and velocity of a metal plate launched by expanding DPs from an open-faced sandwich (OFS) are derived. In addition, the variations of the mechanical parameters of displacement, velocity and density versus the Lagrangian coordinate of expanding DPs gases during the launching of the metal plate from the OFS are defined. Variations of these mechanical parameters are also described by algebraic formulae.

From analysis of the obtained results, it follows that Gurney's assumptions of the linear velocity profile of constant – density DPs gases significantly deviates from conventional theory of gas dynamics. Gurney's assumptions introduce a serious error of estimated metal plate velocity (about several dozen percent).

Key words: metal launching, Gurney's formula, gas dynamics, open-faced sandwich.

1. Introduction

The problem of launching different solids by the detonation products (DPs) of high explosives (HE) has been investigated by many researchers. A review of the literature connected with this problem is given in the fourth chapter of Walters and Zukas' monograph [1].

One of the methods of estimation of the initial velocity of fragments from ammunition casing is the R.W. Gurney method [2–7].

One of the configuration considered by Gurney is the so-called open-faced sandwich (OFS) (Fig. 1). It consists of a slab of explosive confined by a metal plate on one side only. The other side of the slab is a free surface. This configuration is often used in experiments to obtain constitutive properties of materials or when it is necessary to impact a large target surface. It is also used as a physical model of shaped charges. The interaction of the DPs and the liner of the

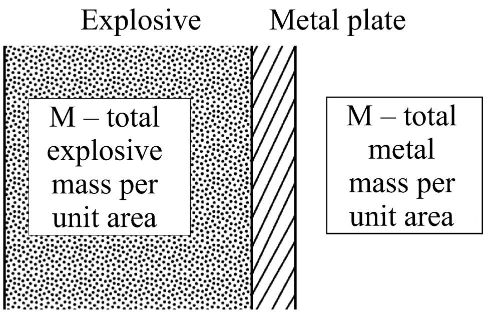

FIG. 1. Open-faced sandwich [1].

shaped charge influences the initial velocity of the liner elements and, therefore the characteristics of jet [1, 8–10].

Using conservation laws of momentum and energy, Gurney defined the initial velocity of the metal plate launching from an OFS by means of simple algebraic formula:

$$v = \sqrt{2E}\left[\frac{\left(1+2\dfrac{M}{C}\right)^3+1}{6\left(1+\dfrac{M}{C}\right)}+\frac{M}{C}\right]^{-1/2}, \tag{1.1}$$

where C and M represent masses of unit area of explosive and metal, respectively, and quantity $\sqrt{2E}$ is the Gurney characteristic velocity for a given explosive.

It follows from Eq. (1.1) that final plate velocity depends only on the M/C ratio and the Gurney characteristic velocity $\sqrt{2E}$, which characterizes considered explosive. This formula does not define the distribution of velocity in space and time.

In order to obtain the simple algebraic formula, Gurney assumed far-reaching simplifications, namely:

- the distribution of the particle velocity of the expanding DPs is a linear function of the Lagrangian coordinate and retains this form at any given time (Fig. 2);
- the gas products of detonation are assumed to expand uniformly with constant density, which is equal to the initial density of the explosive, ρ_e.

In order to verify these assumptions, the following one-dimensional boundary value problem needs to be solved: the launching rigid piston with mass, M, by HE charge in infinite non-deformable pipe with a unit cross-section area. The charge is confined by the piston on one side only. The second charge side is the

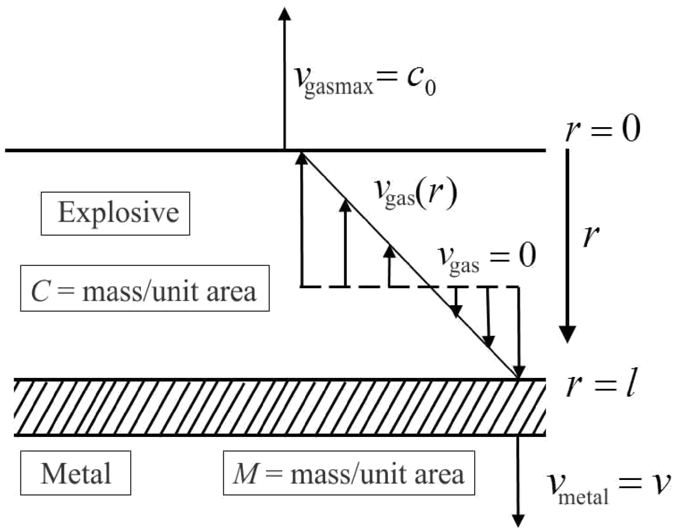

FIG. 2. The linear profile of DPs velocity versus Lagrangian coordinate, r, for an OFS.

free surface. The length and mass of the charge are represented by l and C, respectively. At the moment $t = 0$, the HE charge is instantaneously detonated (the hypothesis of instantaneous detonation is used) [10–13]. The compressed gaseous DPs expand and launch the rigid piston. The thermodynamic properties of the DPs are described by the ideal gas model. The friction forces between the pipe and the piston and the DPs are neglected.

The above problem was solved analytically in the closed form. With the aid of the algebraic formulae derived in this paper, it will be possible to estimate how much the Gurney model differs from the exact solution.

2. Mathematical formulation of the problem

The one-dimensional plane motion of the DPs in the above-presented sandwich is described by the following set of equations:

$$\rho_0 = \rho(1 + u,_r), \tag{2.1}$$

$$u,_{tt} = -\frac{1}{\rho_0} p,_r , \tag{2.2}$$

where the symbols u, p and ρ refer to: the displacement, pressure and density of the DPs, respectively; r is the Lagrangian coordinate and t is time. The subscript 0 marks the initial values of the suitable parameters of the DPs.

Equations (2.1) and (2.2) are completed with an isentropic equation of the DPs in the following form:

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}, \tag{2.3}$$

where γ is the isentropic exponent.

Equations (2.1) and (2.2) can be reduced to the nonlinear differential hyperbolic equation of the second order:

$$u_{,tt} = a^2(u_{,r})u_{,rr}, \tag{2.4}$$

or replaced by two ordinary differential equations:

$$dv = \pm a(\varepsilon)d\varepsilon. \tag{2.5}$$

These equations are satisfied along the following characteristics, respectively:

$$dr = \pm a(\varepsilon)dt, \tag{2.6}$$

where $v = u_{,t}$ is the flow velocity of the DPs and $\varepsilon = u_{,r} = (\rho_0/\rho) - 1$ is the relative measure of the DPs expansion (analogues to a strain in solids).

The quantity $a(u_{,r}) = a(\varepsilon)$ is the velocity of the propagation of disturbances, expressed in the Lagrangian coordinate (r).

For the polytropic curve (2.3), the quantity $a(u_{,r})$ is described by the following series of relationships:

$$a(u_{,r}) = c_0(1+u_{,r})^{-\frac{\gamma+1}{2}} = c_0\left(\frac{\rho_0}{\rho}\right)^{-\frac{\gamma+1}{2}} = c_0\left(\frac{p_0}{p}\right)^{-\frac{\gamma+1}{2\gamma}}, \tag{2.7}$$

where c_0 is the initial local speed of sound in the DPs, i.e.,

$$c_0 = \left(\gamma\frac{p_0}{\rho_0}\right)^{1/2}.$$

After substitution of Eq. (2.7) into Eq. (2.5) and integration, we obtain:

$$u_{,t} = v = -\frac{2c_0}{\gamma-1}(1+u_{,r})^{-\frac{\gamma-1}{2}} + J_+ = -\frac{2c_0}{\gamma-1}\left(\frac{p_0}{p}\right)^{-\frac{\gamma+1}{2\gamma}} + J_+, \tag{2.8}$$

$$\text{if} \quad dr = a(u_{,r})dt,$$

and

$$u_{,t} = v = \frac{2c_0}{\gamma-1}(1+u_{,r})^{-\frac{\gamma-1}{2}} + J_- = \frac{2c_0}{\gamma-1}\left(\frac{p_0}{p}\right)^{-\frac{\gamma+1}{2\gamma}} + J_-, \tag{2.9}$$

$$\text{if} \quad dr = -a(u_{,r})dt,$$

where J_+ and J_- are quantities preserving constant value along the suitable characteristics. They are calculated from the boundary conditions of the problem.

The boundary conditions in the investigated problem have the following form:

$$p(0,t) \equiv 0, \tag{2.10}$$

$$p(l,t) \equiv M\frac{dv}{dt}. \tag{2.11}$$

The above-mentioned boundary conditions are completed by suitable relationships on the characteristics.

Bearing in mind the values of the isentropic exponent γ for the majority of the condensed explosives (see Table 1), the formulated problem has been solved analytically in the closed form for $\gamma = 3$.

Table 1. Isentropic effective exponents of the DPs of some condensed explosives [11].

No	Explosive	γ	No	Explosive	γ	No	Explosive	γ
1	BTF	2.933	11	LX-04-1	3.068	21	RX-08-DR	2.998
2	Comp. B Grade A	2.965	12	LX-07	3.034	22	RX-04-DS	2.921
3	Cyclotol	2.938	13	LX-09	3.016	23	RX-06-AF	2.927
4	Dipam	2.837	14	LX-10	2.998	24	RX-08-AC	2.967
5	EL-506 A	2.951	15	LX-11	3.112	25	RX-08-BV	2.961
6	EL-506 C	3.038	16	LX-14	3.017	26	RX-08-DW	3.041
7	HNS	2.874	17	Octol	2.991	27	XTX-8003	3.025
8	HMX	3.223	18	PBX 9010	3.189	28	Tetryl	3.018
9	HMX-TNT Inert	3.015	19	PBX 9011	3.049	29	TNT	2.959
10	HMX Inert	2.962	20	PBX 9404-3	3.064	30	C4	3.033

3. Solutions of the problem in regions 0, I, II and III

This problem has been solved using the method of characteristics.

Part of the wave picture of the solution of the problem is presented as a network of the limiting characteristics in Fig. 3. The backward rarefaction waves propagate in the space between the rigid piston and the free surface, filled with the DPs. These waves are reflected from the piston and free surface and interact with each other. As a result of this refraction set of the rarefaction waves, separable regions are created in the plane $(r,\, t)$, where the suitable initial and boundary value problems to be solved are the Cauchy initial value problem in region 0, the Picard boundary value problem in regions I and II, the Darboux value problem in region III.

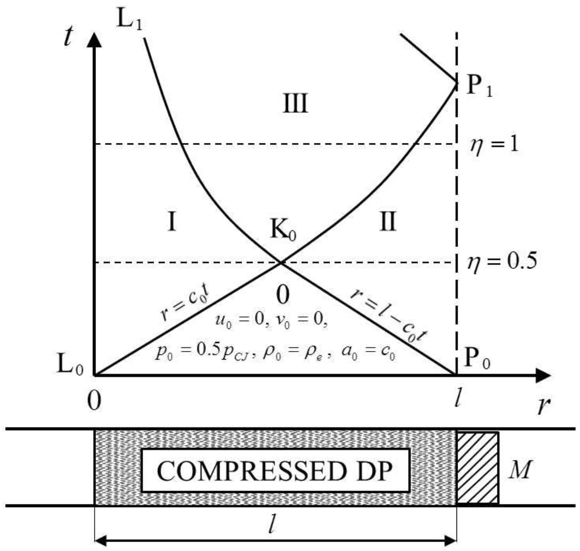

FIG. 3. Part of the network of limiting characteristics in the space between piston and free surface.

Let us introduce the following dimensionless quantities:

$$\text{(3.1)}\qquad \begin{aligned} &\xi = \frac{r}{l}, \qquad \zeta = \frac{x}{l}, \qquad \eta = \frac{c_0 t}{l}, \qquad \overline{u}_i = \frac{u_i}{l}, \qquad \overline{v}_i = \frac{v_i}{c_0}, \\ &\overline{\rho}_i = \frac{\rho_i}{\rho_0}, \qquad p_0 = \frac{1}{2} p_{\mathrm{CJ}}, \qquad M_c = \frac{M}{C}, \qquad C = l\rho_{\mathrm{e}}, \end{aligned}$$

where

$$\text{(3.2)}\qquad c_0 = \sqrt{\gamma \frac{p_0}{\rho_0}} = \sqrt{\frac{\gamma}{2} \frac{p_{\mathrm{CJ}}}{\rho_e}} = \sqrt{\frac{\gamma}{2(\gamma+1)}} D = \sqrt{\frac{3}{8}} D.$$

Symbols x, l, C, D and γ refer to: the Eulerian coordinate, the length of explosive charge and its mass per unit area, the detonation velocity and the isentropic effective exponent of the DPs, respectively. Symbol ρ_e stands for the density of the explosive, p_{CJ} is the pressure in the Chapman-Jouguet point.

The above-mentioned boundary value problems have been considered in [15, 16]. The formulae describing the parameters of the hydrodynamic state of the DPs in the suitable regions considered in these papers have been stated. The variables and parameters assigned to each region will be denoted by a subscript corresponding to the number of the given region.

The analysis of the problem has been limited to the following functions: displacement velocity and density of the DPs in the selected regions.

Region 0

Region 0 is bounded by the axis $0r$ and the characteristics K_0L_0 and K_0P_0, which overlap the trajectories of the fronts of the rarefaction waves propagating from the free surface and from the movable piston, respectively (Fig. 3). The variables ξ, ζ and η along the characteristics delimitating the given regions are denoted by a subscript which corresponds to the number of the region, for instance, symbol ξ_{01} marks the variation of variable ξ along the characteristic K_0L_0 (Fig. 3), delimitating regions 0 and I, and so on.

The characteristics K_0L_0 and K_0P_0 are defined by the formulae:

$$\begin{aligned} &\xi_{01} = \zeta_{01} = \eta_{01} && \text{– along the characteristic } K_0L_0, \\ &\xi_{02} = \zeta_{02} = 1 - \eta_{02} && \text{– along the characteristic } K_0P_0. \end{aligned} \tag{3.3}$$

The values of the variables ξ_0 and η_0 are contained within the intervals:

$$\eta_{01} \le \xi_0 \le 1 - \eta_{02}, \qquad 0 \le \eta_0 \le 0.5.$$

The DPs are motionless in region 0. Their state parameters are defined by the hypothesis of instantaneous detonation [14, 15]. These parameters are described by the following formulae:

$$\begin{gathered} u_0 = 0, \qquad v_0 = 0, \qquad \rho_0 = \rho_{\mathrm{e}} = \text{const}, \\ c_0 = \sqrt{\frac{3}{8}} D = \text{const}, \qquad p_0 = \frac{\rho_e D^2}{8} = \text{const}. \end{gathered} \tag{3.4}$$

The parameters of the state of the DPs in the remaining regions will be compared with the quantities c_0, ρ_0, p_0 (see dimensionless variables (3.1)).

Region I

Region I is bounded by the axis $0t$ and the characteristics K_0L_0 and K_0L_1 (Fig. 3). In this region, the Picard boundary value problem has been solved for Eq. (2.4) with homogenous conditions on the characteristic K_0L_0 and with the boundary condition (2.10). The formulae describing the relative parameters of the DPs in region I have the following form:

$$\overline{u}_1(\xi_1, \eta_1) = 2\sqrt{\xi_1 \eta_1} - (\xi_1 + \eta_1), \tag{3.5}$$

$$\overline{v}_1(\xi_1, \eta_1) = -1 + \sqrt{\frac{\xi_1}{\eta_1}}, \tag{3.6}$$

$$\overline{\rho}_1(\xi_1, \eta_1) = \sqrt{\frac{\xi_1}{\eta_1}} = \overline{v}_1(\xi_1, \eta_1) + 1. \tag{3.7}$$

The relationship between the Eulerian variable, ζ_1, and the Lagrangian variable, ξ_1, has the following form in region I:

$$\xi_1 = \frac{(\zeta_1 + \eta_1)^2}{4\eta_1} \qquad \text{or} \qquad \zeta_1 = 2\sqrt{\xi_1 \eta_1} - \eta_1. \tag{3.8}$$

After substitution of relationship (3.8) into formulae (3.5)–(3.7) we obtain:

$$\overline{u}_1(\zeta_1, \eta_1) = \frac{1}{2}\left[2\zeta_1 - \frac{(\zeta_1 + \eta_1)^2}{2\eta_1}\right], \tag{3.9}$$

$$\overline{v}_1(\zeta_1, \eta_1) = \frac{\zeta_1 + \eta_1}{2\eta_1} - 1 = \frac{\zeta_1 - \eta_1}{2\eta_1}, \tag{3.10}$$

$$\overline{\rho}_1(\zeta_1, \eta_1) = \frac{\zeta_1 + \eta_1}{2\eta_1} = \overline{v}_1(\zeta_1, \eta_1) + 1. \tag{3.11}$$

Note that the velocity and density are the linear functions of the Eulerian coordinate ζ_1, but contrarily, these parameters are nonlinear functions of the Lagrangian coordinate, ξ_1.

The characteristics $\mathrm{K_0L_0}$ and $\mathrm{K_0L_1}$ (Fig. 3) are defined by the following relationships:

- in the Lagrangian variables (ξ, η)

$$\begin{aligned} &\xi_{01} = \eta_{01} && \text{– along the characteristic } \mathrm{K_0L_0}, \\ &\xi_{13} = \frac{1}{4\eta_{13}} && \text{– along the characteristic } \mathrm{K_0L_1}; \end{aligned} \tag{3.12}$$

- in the Eulerian variables (ζ, η)

$$\begin{aligned} &\zeta_{01} = \eta_{01} && \text{– along the characteristic } \mathrm{K_0L_0}, \\ &\zeta_{13} = 1 - \eta_{13} && \text{– along the characteristic } \mathrm{K_0L_1}. \end{aligned} \tag{3.13}$$

The values of the variables ξ_1 and η_1 are contained within the intervals:

$$\begin{aligned} &0 \le \xi_1 \le \eta_{01}, \qquad \text{if} \qquad 0 \le \eta_{01} \le 0.5, \\ \text{and} \qquad &0 \le \xi_1 \le \frac{1}{4\eta_{13}}, \qquad \text{if} \qquad 0.5 \le \eta_{13} < \infty. \end{aligned} \tag{3.14}$$

Region II

Region II is bounded by the line $\xi = 1$ and the characteristics $\mathrm{K_0P_0}$ and $\mathrm{K_0P_1}$ (Fig. 3). Similarly to region I, we obtain:

$$
\begin{aligned}
&\overline{u}_2(\xi_2,\eta_2) = -\sqrt{(2\eta_2+3\overline{M})\left[2(1-\xi_2)+3\overline{M}\right]} - \xi_2 + \eta_2 + 3\overline{M} + 1,\\
&\zeta_2(\xi_2,\eta_2) = \overline{u}_2(\xi_2,\eta_2) + \xi_2,
\end{aligned}
\tag{3.15}
$$

$$
\overline{v}_2(\xi_2,\eta_2) = 1 - \sqrt{\frac{2(1-\xi_2)+3\overline{M}}{2\eta_2+3\overline{M}}},
\tag{3.16}
$$

$$
\overline{\rho}_2(\xi_2,\eta_2) = \sqrt{\frac{2(1-\xi_2)+3\overline{M}}{2\eta_2+3\overline{M}}} = 1 - \overline{v}_2\,(\xi_2,\eta_2).
\tag{3.17}
$$

The relationship between the Eulerian variable ζ_2 and the Lagrangian variable ξ_2 in region II has the form:

$$
\begin{aligned}
2(1-\xi_2)+2\overline{M} &= \frac{(\eta_2-\zeta_2+3\overline{M}+1)^2}{2\eta_2+3\overline{M}}\\
\text{or}\qquad \zeta_2 &= \eta_2+3\overline{M}+1-\sqrt{(2\eta_2+3\overline{M})\left[2(1-\xi_2+\overline{M})\right]}.
\end{aligned}
\tag{3.18}
$$

After substitution of relationship (3.18) into formulae (3.15)–(3.17), we obtain:

$$
\overline{u}_2(\zeta_2,\eta_2) = \zeta_2 + \frac{(\eta_2-\zeta_2+3\overline{M}+1)^2}{2(2\eta_2+3\overline{M})} - \frac{3}{2}\overline{M} - 1,
\tag{3.19}
$$

$$
\overline{v}_2(\zeta_2,\eta_2) = 1 + \frac{\zeta_2-\eta_2-3\overline{M}-1}{2\eta_2+3\overline{M}} = \frac{\zeta_2+\eta_2-1}{2\eta_2+3\overline{M}},
\tag{3.20}
$$

$$
\overline{\rho}_2(\zeta_2,\eta_2) = \frac{\eta_2-\zeta_2+3\overline{M}+1}{2\eta_2+3\overline{M}} = 1-\overline{v}_2(\zeta_2,\eta_2).
\tag{3.21}
$$

The characteristics K_0P_0 and K_0P_1 are described by the following formulae:

- in the Lagrangian coordinates (ξ,η):

$$
\begin{aligned}
&\xi_{02} = 1-\eta_{02},\\
&\xi_{23} = \frac{1}{2}\left[2+3\overline{M} - \frac{(1+3\overline{M})^2}{2\eta_{23}+3\overline{M}}\right];
\end{aligned}
\tag{3.22}
$$

- in the Eulerian coordinates (ζ,η):

$$
\begin{aligned}
&\zeta_{02} = 1-\eta_{02},\\
&\zeta_{23} = \eta_{23}.
\end{aligned}
\tag{3.23}
$$

The values of the variables ξ_2 and η_2 are contained within the intervals:

$$\begin{aligned} &1-\eta_{02} \le \xi_2 \le 1, && \text{if} \quad 0 \le \eta_{02} \le 0.5, \\ &\frac{1}{2}\left[2+3\overline{M}-\frac{(1+3\overline{M})^2}{2\eta_{23}+3\overline{M}}\right] \le \xi_2 \le 1, && \text{if} \quad 0.5 \le \eta_{23} \le 1+\frac{1}{6\overline{M}}. \end{aligned} \tag{3.24}$$

Region III

From the solution of the Darboux boundary value problem in region III [16], we obtain:

$$\overline{u}_3(\zeta_3,\eta_3) = \zeta_3(\eta_3) - \xi_3, \tag{3.25}$$

$$\overline{v}_3(\zeta_3,\eta_3) = \frac{4\eta_3+3\overline{M}}{2\eta_3(2\eta_3+3\overline{M})}\zeta_3 - \frac{3\overline{M}+2}{2(2\eta_3+3\overline{M})}, \tag{3.26}$$

$$\overline{\rho}_3(\zeta_3,\eta_3) = \frac{3\overline{M}}{2\eta_3(2\eta_3+3\overline{M})}\zeta_3 + \frac{3\overline{M}+2}{2(2\eta_3+3\overline{M})}. \tag{3.27}$$

The equation of the trajectory of the particles of the DPs presented by the Lagrangian coordinate $\xi_{13}(\eta_{13})$ in region III has the form:

$$\zeta\left(\eta_3\right) = \frac{1}{3\overline{M}}\left[-\left(2+3\overline{M}\right)\eta_3 + \left(2\eta_{13}+3\overline{M}\right)\sqrt{\frac{\eta_3\left(2\eta_3+3\overline{M}\right)}{\eta_{13}\left(2\eta_{13}+3\overline{M}\right)}}\right], \tag{3.28}$$

where

$$\eta_{13} = \frac{1}{4\xi_{13}}. \tag{3.29}$$

The value of parameter η_{13} and the value of the suitable Lagrangian coordinate ξ_{13} are contained within the intervals:

$$0.5 \le \eta_{13} < \infty, \qquad 0 \le \xi_{13} \le 0.5. \tag{3.30}$$

Analogous relationships have been obtained for the second part of region III, namely:

$$\zeta_3(\eta_3) = \frac{1}{3\overline{M}}\left[-(2+3\overline{M})\eta_3 + 2\eta_{23}(3\overline{M}+1)\sqrt{\frac{\eta_3(2\eta_3+3\overline{M})}{\eta_{23}(2\eta_{23}+3\overline{M})}}\right], \tag{3.31}$$

where

$$\eta_{23} = \frac{1+6\overline{M}\xi_{23}}{2(2+3\overline{M}-2\xi_{23})}, \tag{3.32}$$

$$0.5 \le \eta_{23} \le 1+\frac{1}{6\overline{M}}, \qquad 0.5 \le \xi_{23} \le 1. \tag{3.33}$$

In this way, we obtain the algebraic relationships which characterize the exact parameters of the expanding DPs in the initial period of launching the metal plate in the OFS. By means of these expressions, it is possible to estimate the degree of accuracy of the results obtained from the Gurney formula (1.1).

4. Quantitative analysis of the parameters of the OFS during the launching of the metal plate

In order to quantitatively analyze the parameters of the considered sandwich, we assumed explosive HMX. This explosive is characterized by the following parameters:

$$p_{\mathrm{CJ}} = 42 \text{ GPa}, \quad \rho_0 = \rho_{\mathrm{e}} = 1891 \text{ kg/m}^3, \quad p_0 = 0.5 p_{\mathrm{CJ}} = 21 \text{ GPa},$$

$$D = 9110 \text{ m/s}, \quad \sqrt{2E} = 2970 \text{ m/s}, \quad \sqrt{6E} = 5144 \text{ m/s}, \quad c_0 = 5774 \text{ m/s}, \quad \gamma \approx 3.$$

On the basis of the algebraic formulae which define the parameters of the expanding DPs in regions I, II and III (Fig. 3), we performed calculations and the obtained results are presented as graphs in the following figures.

Figure 4 shows the algebraic variations of the relative velocity $\overline{v}$ of the DPs in the Lagrangian coordinate for the selected values of parameters η and $\overline{M}$. As the comparative background, the linear Gurney approximations of this velocity are also presented in this figure. It can be noticed that the accuracy of this approximation depends on time η and mass $\overline{M}$. The difference between the algebraic curve and the approximation line reaches several dozen percent.

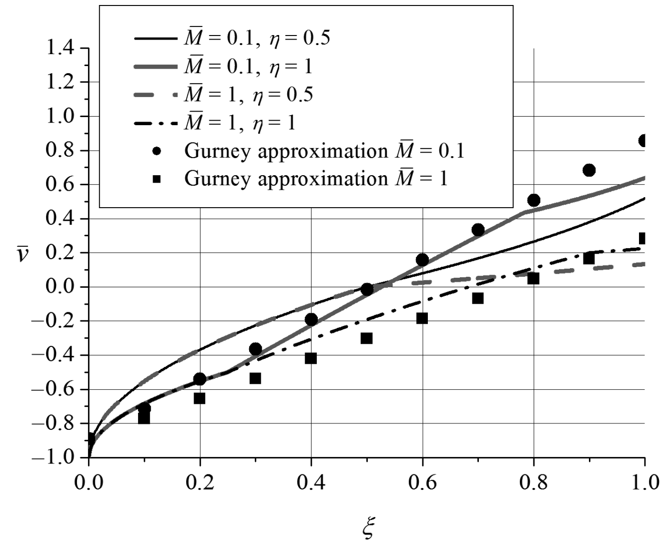

FIG. 4. Variations of the relative velocity, $\overline{v}$, of the DPs versus Lagrangian coordinate, ξ, for selected values of η and $\overline{M}$ (on the background linear Gurney approximation).

Figure 5 shows graphs of the density in the expanding DPs during the launching of the metal plate of the considered sandwich for selected values of η and $\overline{M}$. As can be seen, the Gurney approximation of this function ($\overline{\rho} = 1$) at all launching times largely differs from the exact algebraic results.

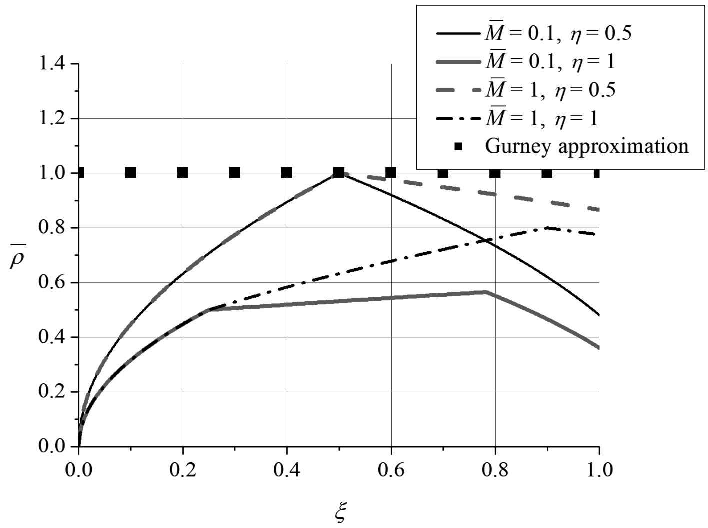

FIG. 5. Variation of the relative density, $\overline{\rho}$, of the DPs versus Lagrangian coordinate, ξ, for selected values of η and $\overline{M}$.

Figure 6 represents the variations of the relative displacement and velocity of the metal plate in terms of time in the initial period of its launching for se-

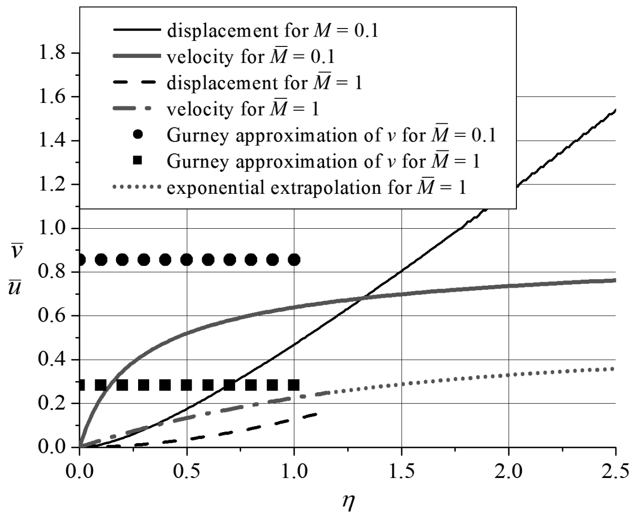

FIG. 6. Variation of the relative displacement and velocity of metal plate versus relative time η, for selected values of $\overline{M}$.

lected values of $\overline{M}$. The horizontal lines mark the values of the relative velocities estimated by means of the Gurney formula (1.1). The data depicted in Fig. 6 show that there are considerable differences between these velocities.

5. Final conclusions

- In this paper, the algebraic formulae to define the exact velocity of a metal plate launched from an OFS have been derived. Furthermore, the distributions of the mechanical parameters of displacement, velocity and density in the expanding DPs gases during the launching of the metal plate from the OFS have been defined. Distributions of these mechanical properties have also been described by algebraic formulae.
- It seems that distributions of the velocity and density in expanding DPs during the metal plate launching are the linear functions of the Eulerian coordinate, but contrarily, these parameters are represented by nonlinear functions of the Lagrangian coordinate – inversely to Gurney's assumptions.
- Gurney's assumptions of the linear velocity profile and the constant density of DPs during the entire period of the plate launching, significantly deviate from conventional gas dynamics theory (see Fig. 4 and Fig. 5). These assumptions introduce the largest error in configuration involving a free explosive surface such as an OFS. The range of applicability of the Gurney formula is restricted due to the above – mentioned simplifications in the derivation.
- The algebraic formulae, listed in this paper, derived in Eulerian coordinate on the basis of laws of conventional gas dynamics theory, seem to be more precise than the Gurney formula and can be applied in engineering calculations.

References

1. Walters W.P., Zukas J.A., *Fundamentals of shaped charges*, John Wiley and Sons, New York – Chichester – Brisbane – Toronto – Singapore, 1989.
2. Gurney R.W., *The initial velocities of fragments from bombs, shells and grenades*, BRL Report No 405, Aberdeen Providing Ground, pp. 1–11, 1943.
3. Gurney R.W., *Fragmentation of bombs, shells and grenades*, BRL Report 635, March 1947.
4. Henry J.G., *The Gurney formula and related approximations for the high-explosive deployment of fragments*, Hughes Aircraft Company, Culver City, CA, Report No. PUB-189, April (AD 813398), 1967.

5. Hirsch E., *Improved Gurney formulas for exploding cylinders and spheres using hard core approximation*, Propellants, Explosives, Pyrotechnics, **11**(3): 81–84, 1986.

6. Jones G.E., Kennedy J.E., Berthof L.D., *Ballistics calculations of R.W. Gurney*, American Journal of Physics, **48**(4): 264–269, 1980.

7. Kennedy J.E., *Gurney energy of explosives: estimation of the velocity and impulse imparted to driven metal*, Sandia National Laboratories, Report No. SC-RR-70-790, December 1970.

8. Włodarczyk E., *On hydrodynamic stationary theory of jet formation*, Journal of Technical Physics, **35**(3): 241–252, 1994.

9. Włodarczyk E., *The impact of direction and velocity of the detonation wave and the liner apex angle on parameters of a shaped charge jet and slug as well as an explosively formed penetrator*, Journal of Technical Physics, **35**(4): 393–413, 1994.

10. Włodarczyk E., *Influence of casing and liner inertia of wedge cumulative war-head on displacement velocity of their explosively driven elements, and on active mass of explosive*, Journal of Technical Physics, **43**(1): 19–33, 2002.

11. Włodarczyk E., *Fundamentals of explosion mechanics* [in Polish], WN PWN, Warszawa, 1994.

12. Cook M.A., *The science of high explosives*, Reinhold Publ. Corp., London, 1958.

13. Ficket W., Davis W.C., *Detonation*, University of California Press, Berkeley, 1979.

14. Cheret R., *Detonation of condensed explosives*, Springer-Verlag, New York, 1993.

15. Włodarczyk E., *The backward launching of solids by products of detonation*, Journal of Technical Physics, **41**(2): 101–119, 2000.

16. Włodarczyk E., *The closed form solution of the Darboux boundary value problem for backward driving of solids by the high explosive*, Journal of Technical Physics, **42**(1): 5–21, 2001.

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