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Analysis of the Nonstability States During Bending Processes of Metallic Tubes at Bending Machines Part II. Examples of Calculations and Analysis of Some Unstability States

Zdzisław ŚLODERBACH

Faculty of Applications of Chemistry and Mechanics
Opole University of Technology
Luboszycka 7, 45-036 Opole, Poland
e-mail: z.sloderbach@po.opole.pl

In this part of the paper, applying expressions derived in Part I, the exemplary calculations of some selected states of loss of stability during cold bending of thin-walled metal tubes at bending machines are presented. The conditions of the dispersed and localised loss of stability together with formation of the plane state of deformation (PSD) under the plane stress state (PSS) and the cracking criterion based on the technological index A_5 as the criteria of instability were assumed. The majority of calculations were performed for a generalised model of strain and simplification of the 3rd type. It appears that they are two extreme cases: one provides the greatest strains and greatest bending angles and the other provides the lowest strains and lowest bending angles [1]. For simplifications of types 1 and 2, values of the bending angles are included between these extreme values. The calculations were performed for the top points of the elbow where strains are the greatest and the wall thickness is the smallest. The calculation results were presented as suitable graphs being useful nomograms.

Key words: allowable strains and stresses, bending angles, neutral layer, wall thickness of elbows.

1. Introduction

The second part of work makes a further development of the first part. First computational values of equivalent strains depending on the rate of the hardening coefficient n and depending on the rate of the normal anisotropy coefficient r are presented. The calculations were performed for the case of formation of the plane state of the deformation (PSD) in the conditions of the plane state of stress (PSS). The formation of PSD in PSS conditions results from initiating of located loss of the stability (e.g., in the form of the appearance of the local groove). Material of the pipe was assumed as rigid-plastic with the isotropic hardening.

The increase of r causes that thinning of the wall of the bent pipe is smaller, which means resistance to thinning; so materials with higher values of r coefficient exhibit slightly better properties for bending. The increase of hardening coefficient n improves significantly bending properties of material, since this exponent is numerically equal to the value of uniform elongation in a test of uniaxial stretching and is partly related to the texture and the grain size of steel of a given sort.

The next part of the paper presents exemplary calculations of some selected states of loss of stability during cold bending of thin-walled metal tubes at bending machines. The conditions of the dissipated and localised loss of stability together with formation of the plane state of deformation PSD under the plane stress state PSS, and the cracking criterion based on the technological index A_5 (five fold sample) were assumed as the criteria of instability. The majority of calculations have been carried out for a generalised model of strain and simplification of the 3rd type [1, 2]. It appears that they are two extreme cases (the generalised one providing the greatest strains and bending angles, and the strongest one providing the lowest strains and bending angles). For simplifications of types 1 and 2, values of permissible strains and bending angles are included between those extreme values. The calculations were realised for the top points of the elbow, where strains are the greatest, and the wall thickness is the smallest. The calculation results were presented as suitable graphs being useful nomograms, too. The paper continues the author's considerations presented in [1].

2. Critical strains and stresses

Let us consider a case of formation of plane stress state under plane state of deformation, see Eq. (3.16), Part I. Then

$$\varphi_{(i)e} = nz - \varphi_0, \tag{2.1}$$

where we can formally write that $\varphi_{(i)e} \equiv \varphi'''_{(i)e}$ and $\varphi_{(i)e}$ is the value of the substitute strain corresponding to the loss of stability, z is the subtangent including influence of the stress σ_{p} on the stability loss moment under PSS and in the moment of formation of the plane state of deformations PSD [1, 3–6], so that

$$z = \frac{1+r}{\sqrt{1+2r}}. \tag{2.2}$$

This case of stability loss refers to thin-walled tubes because of the assumed conditions resulting from the plane stress state.

According to the assumed considerations related to tube bending process we can assume that an allowable (admissible) value of the substitute strain $\varphi_{(i)\mathrm{all}}$, corresponding to an unlocalised stability loss for the case of tube bending at the bending machines, should be included into the following range:

$$\varphi_{(i)a} \le \varphi_{(i)\mathrm{all}} < \varphi_{(i)b1} \tag{2.3$_1$}$$

or

$$\varphi_{(i)a} \le \varphi_{(i)\mathrm{all}} < \varphi'''_{(i)b2} \tag{2.3$_2$}$$

and for the case of a localised stability loss:

$$\varphi'''_{(i)b2} \le \varphi_{(i)\mathrm{all}} \le \varphi_{(i)b1} \tag{2.3$_3$}$$

or

$$\varphi_{(i)e} \le \varphi_{(i)\mathrm{all}} \le \varphi_{(i)b1}. \tag{2.3$_4$}$$

Taking into account Eqs. (3.1), Part I, and (2.1), (2.2), $(2.3)_2$ for thin-walled tubes, we obtain

$$n - \varphi_0 \le \varphi_{(i)\mathrm{all}} \le \frac{1+r}{\sqrt{1+2r}} n - \varphi_0. \tag{2.4}$$

For isotropic materials we have $r = 1$, so

$$n - \varphi_0 \le \varphi_{(i)\mathrm{all}} \le \frac{2}{\sqrt{3}} n - \varphi_0. \tag{2.5}$$

Using equations obtained in works [1, 2, 7] for allowable values of bending angles α_b, depending on admissible values of deformations intensity in the stretched layers $\varphi_{(i)}$, we receive

$$\cos\left(k\frac{\alpha_{b\,\mathrm{all}}}{2}\right) = \frac{2R + d_1 - 2R\exp\sqrt{1.5\varphi^2_{(i)} - (\varphi_2^2 + \varphi_3^2)}}{d_1} = \frac{2R + d_1 - 2R\exp\varphi_1}{d_1}, \tag{2.6}$$

$$\cos\left(k\frac{\alpha'''_{b\,\mathrm{all}}}{2}\right) = \frac{2R + d'''_1 - 2R\exp\sqrt{1.5(\varphi'''_{(i)})^2 - (\varphi'''_3)^2}}{d_{\mathrm{ext}}} = \frac{2R + d'''_1 - 2R\exp\varphi'''_1}{d_{\mathrm{ext}}}, \tag{2.7}$$

where

$\alpha_{b\,\mathrm{all}}$ are allowable values of the bending angle counted appropriately from equations obtained for the general model of deformations cf. [1, 2, 7], when the value of intensity of deformation $\varphi_{(i)}$ reaches permissible value $\varphi_{(i)\mathrm{all}}$ (Eqs. (3.1) and (3.2), Part I),

$\alpha'''_{b\,\mathrm{all}}$ are allowable values of the bending angle counted from the deformation model of 3rd-type, when intensity value of the deformation $\varphi'''_{(i)}$ reaches the allowable value $\varphi'''_{(i)\mathrm{all}}$, obtained from Eqs. $(3.3)_2$ and (3.16), see Part I,

d_1 and d'''_1 are current diameters in points of top stretched layers of the elbow for appropriate generalised model of deformation and simplification of 3rd order [7, 8].

In the considered case of estimation of admissible strains, stresses, bending angles, or the wall thickness during cold bending of metallic tube at the bending machines (with no effect of displacement of the neutral axis of plastic bending, i.e., for $y_0 \approx 0$), inequalities $(2.3)_2$, (2.4), and (2.5) have been applied. The equations presented in papers [1, 7–9] resulting from the generalised model of deformations and simplifications of type 3 can be used for analysis of the strain quantity. Those relationships allow to describe initiation of the plane state of deformation in the plane stress state. It has been shown that the method resulting from the simplification of the 3rd type determines the most safe values for admissible strains, stresses, bending angles, or wall thickness. It means that for a given admissible value of the strain intensity $\varphi_{(i)\mathrm{all}}$ in the bending zone, admissible values of the bending angle are lower than those resulting from the generalised model of deformations and simplification of type 2 and 3. Thus, in considerations on the problem, a description resulting from simplification of type 3 will be used as the most safe and corresponding to the formation of PSD in PSS conditions.

Analysis of the problem for uniaxial tension of the outer layers of the bent tube is widely known in the literature and here it is limited to a solution of one simple example. Substituting (2.2) to (2.1), we obtain

$$\varphi'''_{(i)\mathrm{cr}} = \frac{1+r}{\sqrt{1+2r}} \cdot n - \varphi_0. \tag{2.8}$$

In the case of initiation of PSD during PSS, condition (2.8) is equal to condition $(3.3)_2$, see Part I.

From Eq. (2.8) we can calculate the value of $\varphi'''_{(i)\mathrm{cr}}$ and then from Eq. (2.7) we can determine the critical value of the bending angle $\alpha'''_{b\,\mathrm{cr}}$, which corresponds to the beginning of the stability loss (local initiation of PSD during PSS). It is widely known that the top (centre) of the layers subjected to tension is the most deformed in the bending zone ($\alpha = \beta = 0°$, see Fig. 2. Part I), so the beginning of the loss of stability should be expected there. Thus, the derived relationships applied to the top point of the layers subjected to tension for $\lambda_1 = 1$ can be applied in the following way: the values of $\varphi'''_{(i)\mathrm{cr}}$ are determined from the expression (2.8) for a given material and next for the given parameters of the bending process; the values of angle α'''_{cr} are determined from Eq. (2.7).

Then Eqs. (2.1), see Part I, are transformed for simplification of the 3rd order [1, 2, 7, 8], the condition of plastic incompressibility and the expression for the strain intensity are used for derivation of suitable logarithmic components of plastic strains φ'''_1, φ'''_3, and thickness g'''_1. The value of the equivalent (reduced) critical plastic stress σ_{ps}, corresponding to the strain (2.8), is determined from the constitutive Eq. (2.3), see Part I, after substituting the strain value $\varphi_{(i)} = \varphi'''_{(i)\mathrm{cr}}$. Suitable stress components can be determined (see Fig. 1) for the point s on the H-M-H ellipse of plasticity. According to theory of the associated laws of plastic flow, that point determines initiation of PSD in PSS condition. The basic set of quasi-linear partial differential equations of statics for the characteristics under H-M-H yield condition of plasticity is parabolic type in that point [1, 7, 10–13]. Thus,

$$\sigma'''_{1s} = \frac{1+r}{\sqrt{1+2r}}\sigma'''_{\mathrm{p}}, \qquad \sigma'''_{2s} = \frac{r}{\sqrt{1+2r}}\sigma'''_{\mathrm{p}}. \tag{2.9}$$

From considerations concerning the stress state during the process of tube bending it appears that in the top points of the cross section of the layers subjected to tension (the points of intense contact of the bent tube with the mandrel in the bending zone) there is a stress state included between the axis σ_1 and the point s on the ellipse of plasticity (see Fig. 1). For this range of the stress

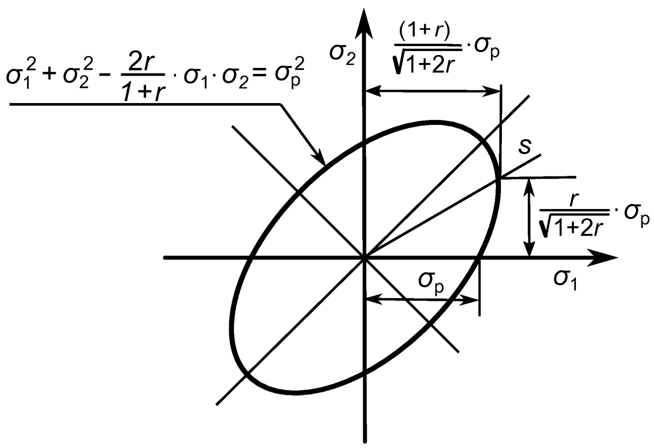

FIG. 1. Stress condition arising at the H-M-H yield ellipsis at the onset of instability.

states, the set of differential partial equations of statics for the stress field characteristics is of the hyperbolic type [1, 7, 10–13]. Occurrence of the stability loss in dispersed and localised forms (especially as localised strains in the form of slip lines) during tube bending for pipeline elbows causes a high acceleration of degradation processes of creep, and next, failure during their operation. It results from the fact that in the points of the elbow where the localised strain states occur during the manufacturing stage, the strain localisation processes are intensified during operation. Faster continuation and development of strain localisation cause crack of the elbow and failure.

When the stress states are above the point s (biaxial tension states, see Fig. 1), then increments of the circumferential strain in the elongated layers will be positive ($d\varphi_2 > 0$ or $d\varepsilon_2 > 0$), see [4], and the states of the localised stability loss will be to the right from the point E, on the line EUC (see Fig. 3, Part I). Such stress states and strain states occur during plastic expanding of tubes.

In next sections (Secs. 3 and 4) the calculations and analysis obtained results were performed for a generalised model of strain and for simplification of the 3rd order [1, 7, 8]. They are two extreme cases (the most gentle one, where the greatest strains and bending angles are admissible, and the most "sharp" one, where the lowest strains and bending angles are possible, respectively), see expressions $(2.3)_3$, (2.6), and (2.7).

3. Results and discussion

In Figs. 2 and 3 the graphs representing computational values of substitute deformations $\varphi_{(i)c}$, see Eq. (3.16), Part I, depending on the rate of the hardening

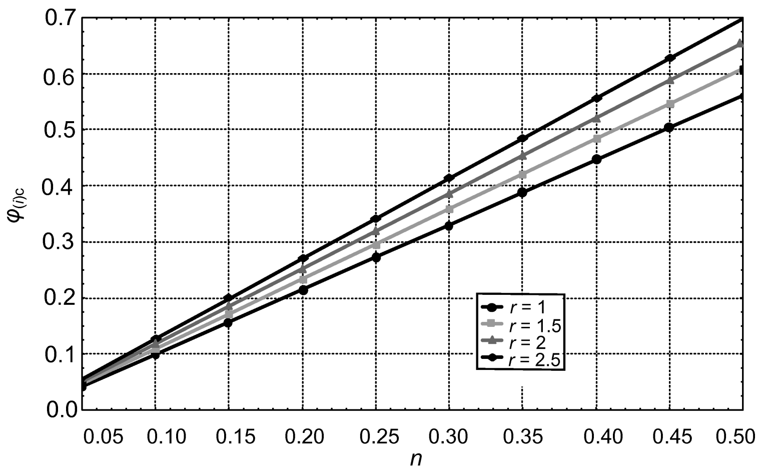

Fig. 2. Admissible equivalent strains as a function of the normal anisotropy index for selected values of the hardening exponent [1].

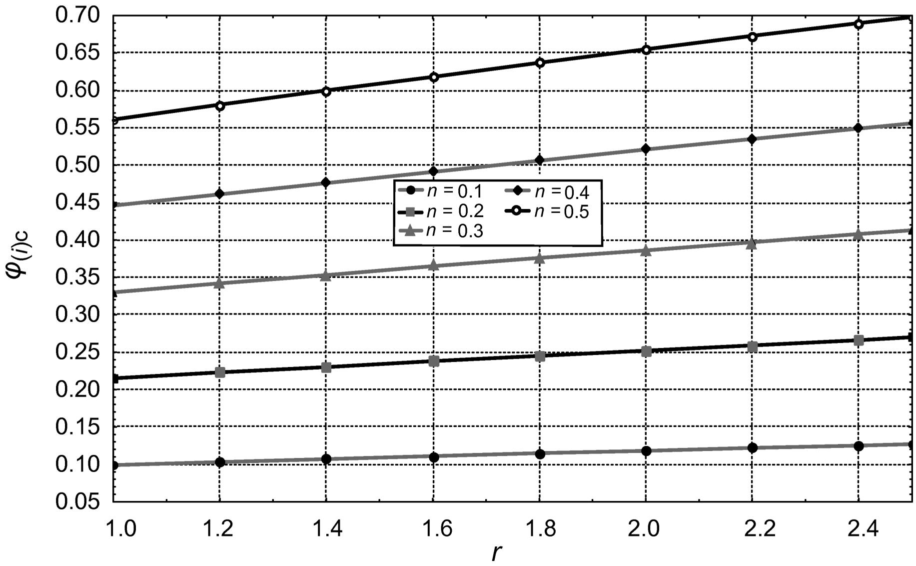

FIG. 3. Admissible equivalent strain as a function of the normal anisotropy index r for selected values of the hardening exponent [1].

coefficient n and depending on the rate of the normal anisotropy coefficient r are presented. The calculations were performed for the case of formation of the plane state of deformation in the conditions of the plane state of stress. Conditions of the formation of PSD in PSS results from initiating of located loss of the stability (e.g., in the form of the appearance of the local groove). Material of the pipe was assumed as rigid-plastic with the isotropic hardening.

As it can be seen, the $\varphi_{(i)c}$ value significantly increases with the increase of the hardening coefficient n and insignificantly increases with the increase of the normal anisotropy coefficient r. From the technological point of view it means that the increase of the hardening coefficient n has a positive effect on the process of cold pipes bending when the value of admissible stresses is increased. For a given process of pipe bending an allowable value of the angle of the bending $\alpha'''_{b\,\mathrm{all}}$ is also increased.

The increase of r causes that thinning of the wall of the bent pipe is smaller, which means resistance to thinning; so materials with higher values of r coefficient exhibit slightly better properties for bending.

The increase of the hardening coefficient n improves significantly bending properties of material, since this exponent is numerically equal to the value of uniform elongation in a test of uniaxial stretching and is partly related to the texture and grain size of steel of a given sort [1, 3–5, 14, 15]. It follows that the increase of a uniform elongation of material improves the conditions of cold bending of pipes, so it promotes the increase of acceptable strains and bending angles.

Figure 4 presents a change of the wall thickness g_1''' of the elbow (knee) of the pipeline in its top (central) point of the elongated layers of the bending zones depending on the value of the bending angle $k\alpha_b$, for the assumed technological-material coefficient of correction of the strain distribution $\lambda_1 = 1$, Part I, and [1, 2, 7–9, 16]. The graphs were obtained for different bending radii R, included into the interval $R \in \langle (1 \div 5) \times d_{\text{ext}} \rangle$ and without including displacement of the neutral axis of plastic bending ($y_0 = 0$). For calculations, a standard tube $\varnothing 44.5 \times 4.5$ mm, used for many calculations and tests was earlier applied [1, 2, 7–9, 16–19]. From the obtained graphs it appears that wall thickness decreases as the bending angle $k\alpha_b$ increases and the bending radius R decreases.

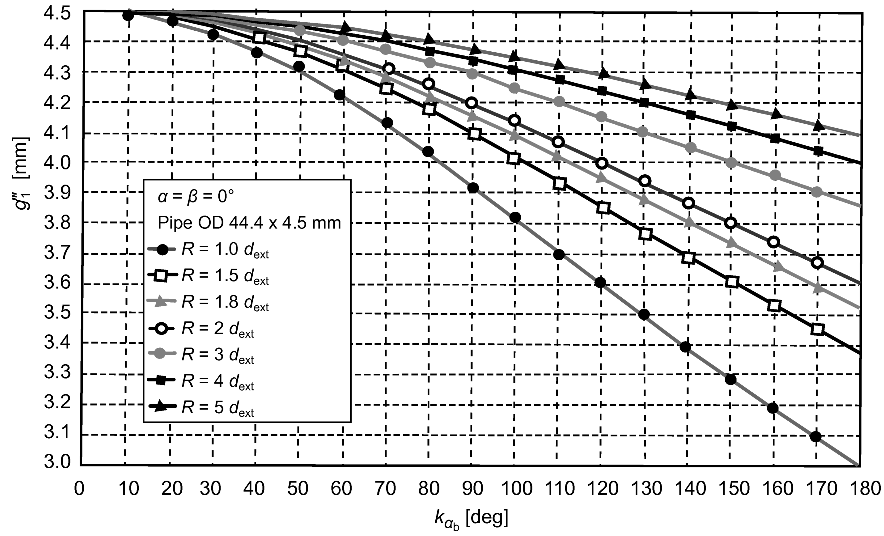

FIG. 4. Wall thickness at the apex point as a function of bending angle for selected values of the bending radius [1].

Figure 5 shows a change of the plastic strain intensity $\varphi_{(i)}'''$ calculated in the central (top) point of elongated layers of the bending zone, depending on the bending angle $k\alpha_b$. The calculations were done for different bending radii R included, as previously, in the interval $R \in \langle (1 \div 5) \times d_{\text{ext}} \rangle$ for a tube $\varnothing 44.5 \times 4.5$ mm, see [1, 2, 7–9, 16–19]. From Fig. 5 it appears that a value $\varphi_{(i)}'''$ increases together with an increase of the bending angle $k\alpha_b$ and decreases as the bending radius R decreases. Thus, our previous expectations seem to be right. When the bending radius R tends to infinity (straight tube), then $\varphi_{(i)}'''$ tends to zero and as a consequence g_1 tends to g_0, so there is no physical bending. Then $\alpha_b = 0°$, because $k > 0$.

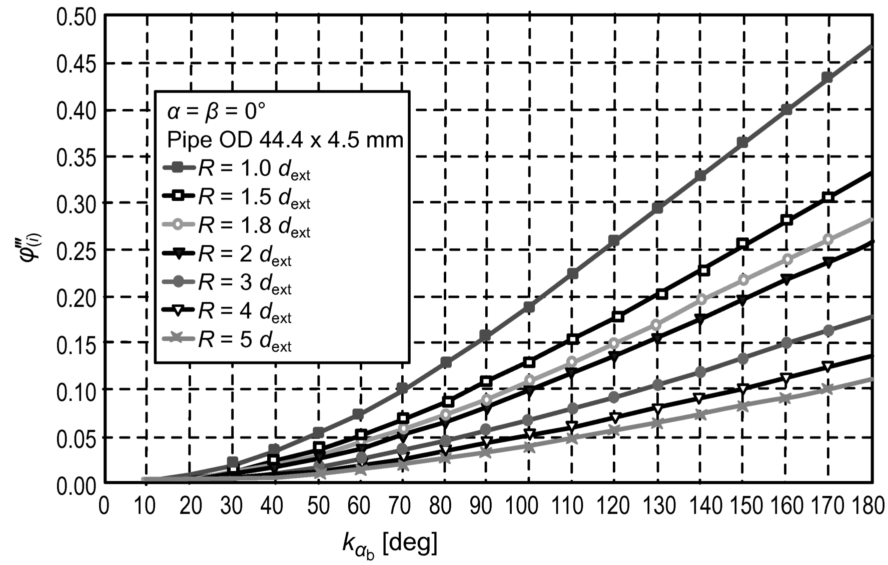

FIG. 5. Equivalent strain value at the bend apex point as a function of bending angle for selected values of the bending radius [1].

Figure 6 presents the results of calculations of logarithmic strain components φ_1, φ_2, φ_3 and the strain intensity $\varphi_i \equiv \varphi_{(i)}$, depending on the bending angle

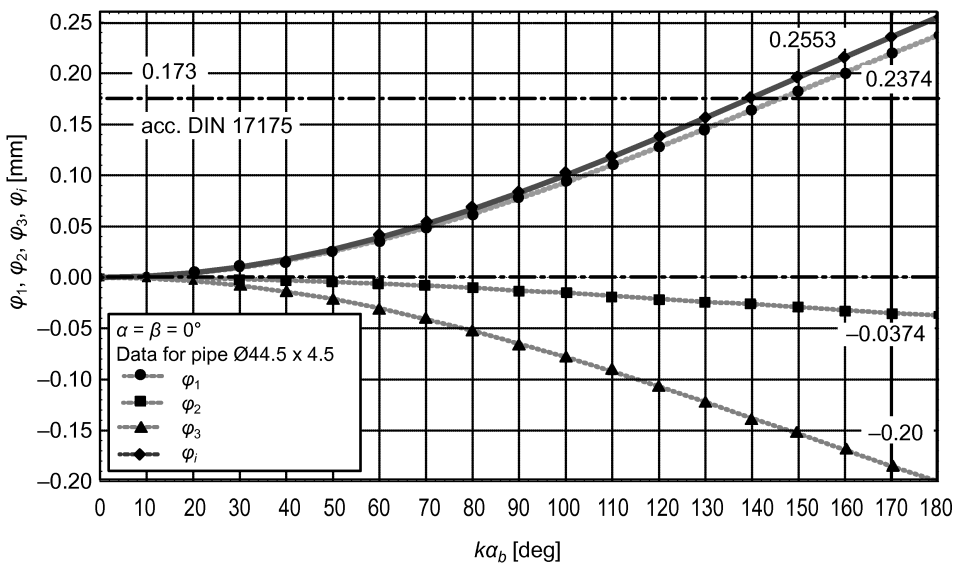

FIG. 6. Strain and strain intensity components as functions of the bending angle, where $\alpha_g \equiv \alpha_b$.

($k\alpha_b$) for the top point of the elbow ($\alpha = \beta = 0°$). Let us put the experimental strains (for example, determined under uniaxial tension) on the Y-axis. Then we are able to determine the value of the admissible bending angle after exceeding of which technological values of permissible strains, for example A_u or A_5, or the strains corresponding to the localised form of loss stability were exceeded. Here A_u and A_5 mean the following: A_u is the value of the uniform strain and A_5 is the strain at the failure of the specimen (five-fold sample) during a test of uniaxial tension.

From the experimental data presented in [17] we have $A_u \approx 0.173$ for steel St.35.8 according to DIN 17175. According to the graphs shown in Figs. 5 and 6, it corresponds to the bending angle $k\alpha_b \approx 145°$. It means that after exceeding the bending angle 145°, for example for $k\alpha_b = 180°$ in the external elongated layers limited by the angle $\beta(0° \leq \beta \leq 45°)$ (see Fig. 2, Part I, and [7, 17, 19, 25]) there are the strains exceeding the value of the uniform strain A_u. After exceeding the value of A_u, a phenomenon of the dispersed stability loss can occur and next localisation of plastic strains can be observed, like in tests of uniaxial and biaxial tension [3, 4, 6, 10, 12, 14, 15, 18, 20].

The wall thickness g_1 in the top central points of the elbow in the elongated layers, corresponding to the bending angle $k\alpha_b \approx 145°$ can be graphically determined from Fig. 7 or analytically defined from Eqs. (1), see Part I, using the condition of plastic incompressibility of the material ($\varphi_1 + \varphi_2 + \varphi_3 = 0$). The calculated and read out from Fig. 7 or Fig. 8 wall thickness is $g_1 \cong 3.88$ mm.

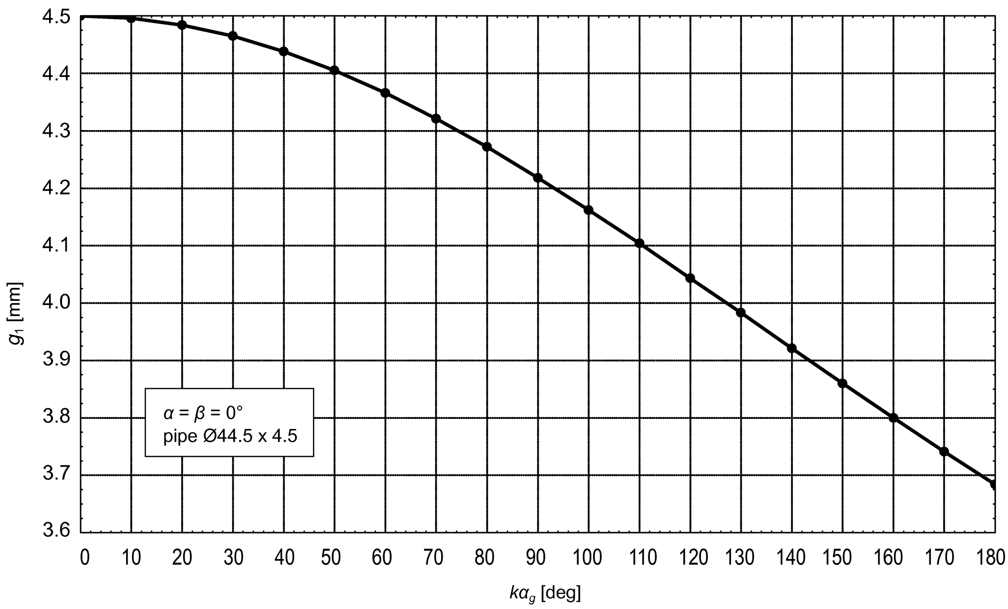

FIG. 7. Variation of the wall thickness at the apex point of the elongated area, where $\alpha_g \equiv \alpha_b$.

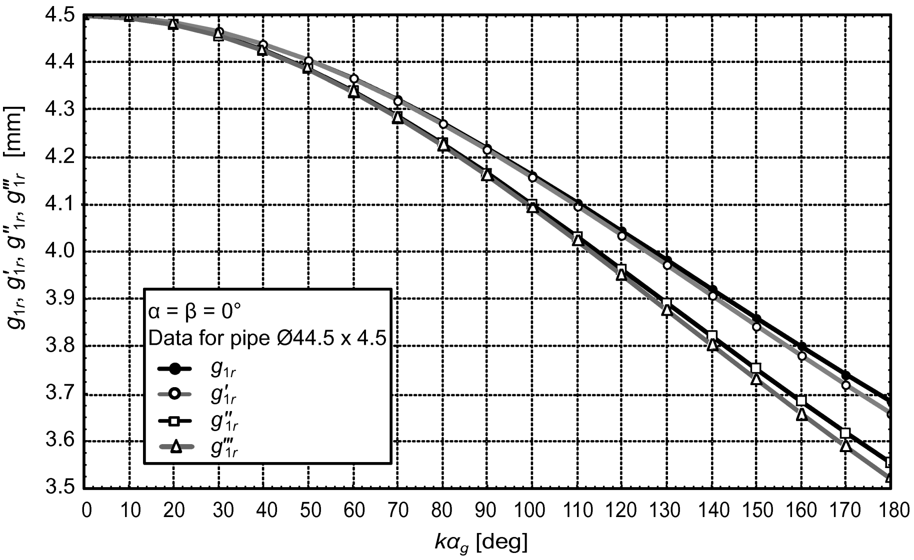

FIG. 8. Variation of the wall thickness value at the bend apex versus bending angle according to four computing methods, where $\alpha_g \equiv \alpha_b$.

Figure 8 presents the calculation results for changes of the wall thickness (g_{1r}, g'_{1r}, g''_{1r}, g'''_{1r}) depending on the value of the bending angle $k\alpha_b$. The subscript r means calculations in measures of real strains (logarithmic strains). The thickness is calculated in the top point of the elbow ($\alpha = \beta = 0°$) of the elongated layers ($\lambda_1 = 1$) for the bent tube $\varnothing 44.5 \times 4.5$ mm and the bending radius $R = 80$ mm, such that $R \cong 1.8 \cdot d_{\text{ext}}$. The tube was made of steel St 35.8 according to DIN 17175 [17].

Figure 9 shows the obtained results of calculations of logarithmic measures of strain components and strains intensity ($\varphi_{(i)}$, $\varphi'_{(i)}$, $\varphi'''_{(i)}$) depending on the bending angle ($k\alpha_b$) calculated according to the equations for the general model and the simplified models of the 1st and 3rd order, described in [2, 7, 8]. The calculations were realised for the top point of the bent elbow in the bending zone, being also the central point of the elbow ($\alpha = \beta = 0°$) in the elongated layers. Only two extreme simplifications (1st and 3rd order) are considered, because additional graphs for simplification of the 2nd order could make the figures low-readable. Let us put admissible values of the experimental substitute strain on the Y-axis, like in Fig. 5. Now we are able to determine an approximate value of the admissible critical bending angle $\alpha_{b\text{cr}}$ – at first, the coefficient k should be determined. Exceeding that coefficient causes exceeding the admissible strains A_5 or A_u under uniaxial tension for tube steels. Figure 6 shows $\varphi_{(i)}$ equal to 0.173; that value was determined for steel St 35.8 [17] during a test of simple tension (according to the standard DIN 17175). Values of the bending angle read

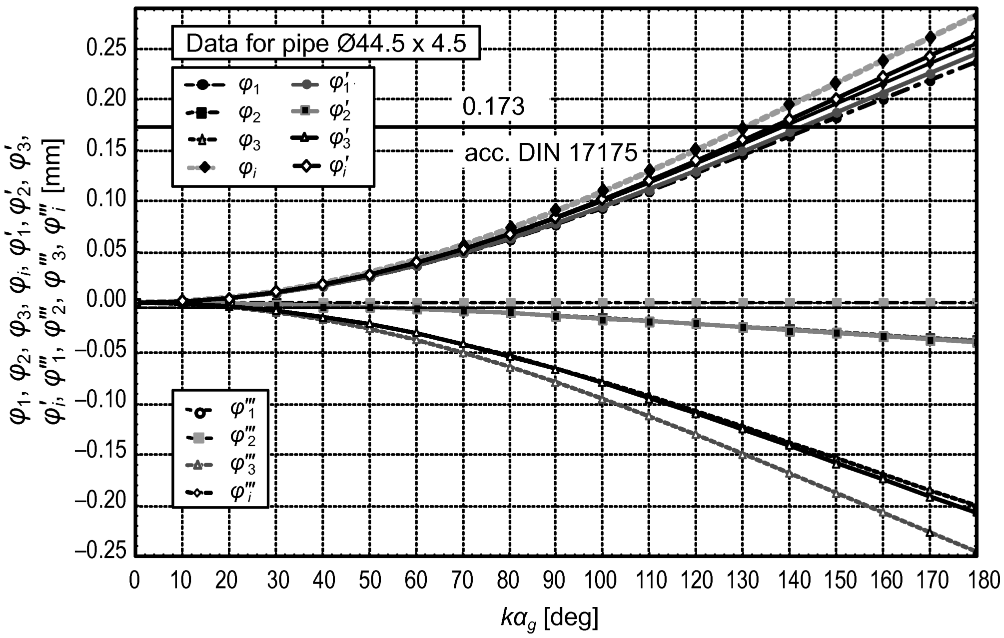

FIG. 9. Strain components and strain intensity versus the bending angle according to three computing methods ($\alpha_g \equiv \alpha_b$).

out on the X-axis for suitable strain intensities ($\varphi_i \equiv \varphi_{(i)}$, $\varphi'_i \equiv \varphi'_{(i)}$, and $\varphi'''_i \equiv \varphi'''_{(i)}$), see Fig. 9, determined for three calculation methods (the exact method and two simplified methods 1st and 3rd type) oscillate around the following values of the bending angles $\alpha_{b\,\mathrm{cr}} \approx 140^\circ$ for $k = 1$, $\alpha_{b\,\mathrm{cr}} \approx 57^\circ$ for $k = 2.5$, and $\alpha_{b\,\mathrm{cr}} \approx 47^\circ$ for $k = 3$, see [1, 2, 7].

From Fig. 8 it also results that application of simplified measures of the 1st, 2nd, and 3rd order, respectively, causes determination of a greater decrease of the material thickness in the bending zone and greater strain and strain intensity components as compared to the results obtained for the generalised model of deformation. Thus, the simplified descriptions will determine lower (more safe) values and safer limitations for the allowable bending angle $\alpha_{b\,\mathrm{all}}$. When the angle $\alpha_{b\,\mathrm{all}}$ is exceeded, we can observe effects connected with localisation of plastic strains or another form of stability loss and cracking. The simplified measures of deformations derived in [2, 8] can be also used, e.g., for practical reasons, when the bending process causes a greater decrease of the wall thickness in the elongated layers, greater components of the strains, and substitute strain in the bending zone. Because of a simplified form of the expressions they can be calculated with the use of calculator under real conditions (production, repair in situ, at the object and other).

4. Simple examples of calculations of critical states

Let us assume that the material of the bent tube has the dimensions: ∅44.5 × 4.5 mm, bending radius $R = 80$ mm ($R \cong 1.8 \cdot d_{\text{ext}}$), and is described by the following material parameters: $n \approx 0.2$; $r \approx 1.5$; $\varphi_0 \approx 0.016$; $D \approx 550$ MPa, and $R_m \approx 420$ MPa. These values can be related to the boiler steel K10 or St 35.8 according to DIN 17175.

All the examples are calculated according to the scheme shown in Fig. 10.

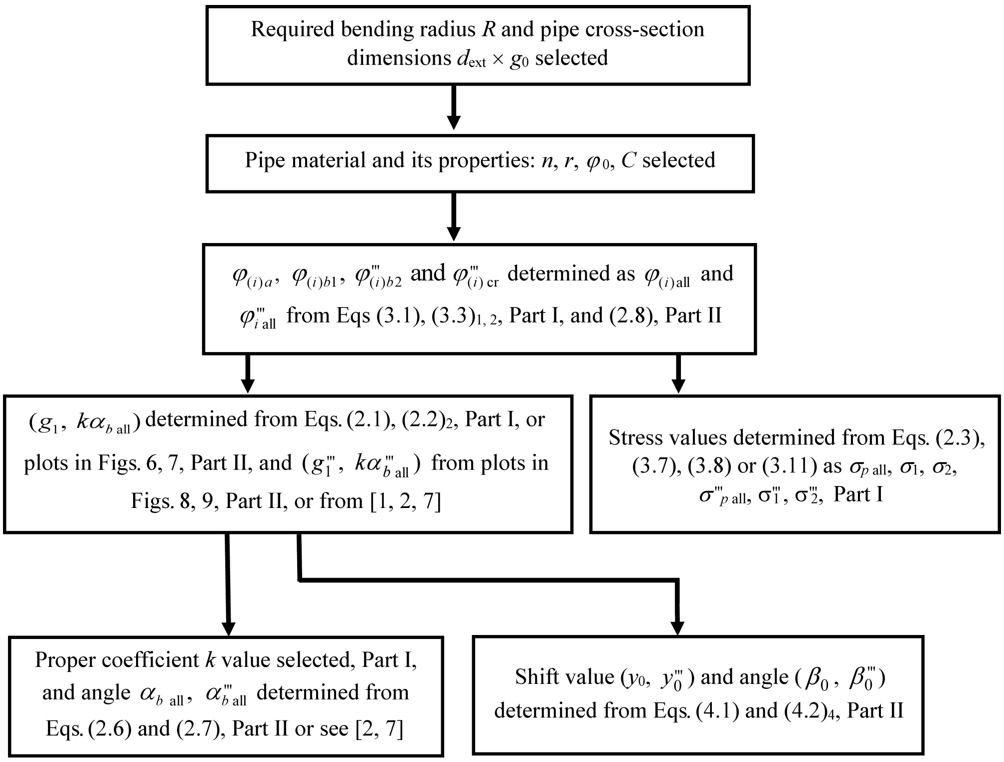

Fig. 10. Flow chart showing the computation stages.

Example 1. When the strain state expressed by Eq. (3.1), see Part I is obtained, then we obtain that $\varphi_{(i)a} = 0.184$. The allowable value of the bending angle corresponding to the above value, read out from Fig. 6 is $k\alpha_b \cong 145°$. Thus, $\alpha_{b\,\text{all}} \cong 145°$ for ($k = 1$), $\alpha_{b\,\text{all}} \cong 72.5°$ for ($k = 2$), $\alpha_{b\,\text{all}} \cong 58°$ for ($k = 2.5$), and $\alpha_{b\,\text{all}} \cong 48.3°$ for ($k = 3$). The calculated values of wall thickness g_1 and g_2 for ($\lambda_1 = \lambda_2 = 1$) are ($g_1 \cong 3.88$ mm, see Fig. 7) and $g_2 \cong 5.66$ mm [1, 7]. During a test of simple tension, Franz [17] obtained the strain value equal to ($A_u = 0.180$). The value $\varphi_{1\,\text{all}}$ calculated from [Eq. (2.1), Part I] equals $\varphi_{1\,\text{all}} \cong 0.173$

and it is the same as that given in [17] for the case of uniform strains. From the calculations and Figs. 5, 9 it appears that $k\alpha_b''' \approx 135°$. Thus for $(k = 1)$, $k\alpha_b' \approx 142°$, $k\alpha_b'' \approx 138°$ and $k\alpha_b''' \approx 135°$, therefore $(k\alpha_b > k\alpha_b' > k\alpha_b'' > k\alpha_b''')$. A value of the yield stress calculated according to Eq. (2.3), Part I, reaches the value $\sigma_{\mathrm{p\,all}} \cong 398.6$ MPa.

An approximate position of the neutral layer of plastic bending (see Fig. 11) corresponding to the considered case can be determined from Eq. (4.1), see [2, 7, 9]. Expression (4.1) is a generalisation of the expression presented in [21] for zones of active bending. Thus

$$(4.1)\quad y_0 \cong \lambda_0 \frac{0.42}{\widetilde{r}}\left(r_{\mathrm{int}} + \frac{g_0}{2}\right)\left[\cos(k\alpha) - \cos(k\frac{\alpha_b}{2})\right] \quad \text{and} \quad y_{0\,\max} = \lambda_0 \frac{0.42}{\widetilde{r}} r_m,$$

where λ_0 is the technological-material correction coefficient of displacement of the neutral layer of plastic bending [2, 7, 9, 16, 21], $y_0 = y_{0\,\max}$ when $\alpha = 0°$, and $k\alpha_b = 180°$.

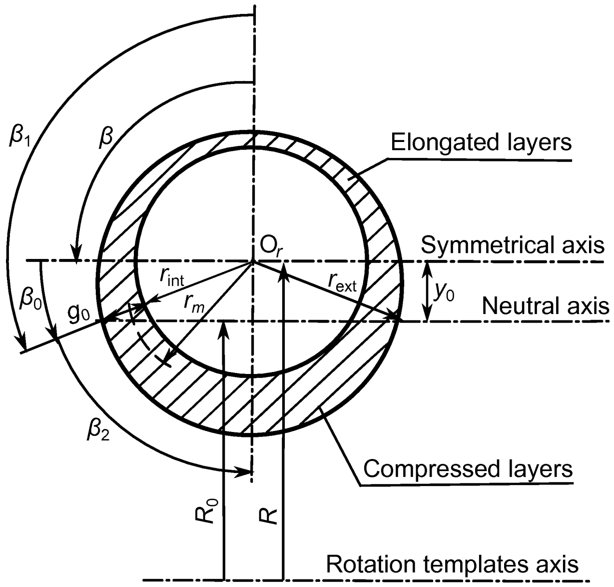

Fig. 11. Schematic picture of cross-section of the elbow and its characteristic parameters.

According to tests, it is possible to assume that $\lambda_0 \in \langle 0; 1\rangle$, $\widetilde{r}$ is the relative bending radius $(R = \widetilde{r} \times d_{\mathrm{ext}})$. Thus

$$(4.2)\qquad \widetilde{r} = \frac{R}{d_{\mathrm{ext}}}, \quad r_m = r_{\mathrm{int}} + \frac{g_0}{2}, \quad R_0 = R - y_0 \quad \text{and} \quad \frac{y_0}{r_{\mathrm{ext}}} \cong \sin\beta_0.$$

The maximum displacement of the neutral axis for free bending can be determined with use of Eq. (4.1) for $\alpha = 0°$ and $k\alpha_b = 180°$ and $\lambda_0 = 1$. Assuming that in the considered case of not-free bending $\lambda_0 \approx 0.5$, after calculations we obtain $y_0 \approx 1.63$ mm and $y_{0\,\max} \approx 2.33$ mm.

The radius R_0 defines a new (instantaneous for a given bending angle) position of the neutral layer of plastic bending for $\alpha = 0°$, then $R_0 = R_0(\alpha,\, \alpha_b)$. Thus, $R_0 = R - y_0 \cong 78.37$ mm.

When $\alpha = 0°$, $k\alpha_b = 180°$, and $\lambda_0 = 0.5$ in the case of not free bending, then $y_0 = y_{0\,\max}$. Thus

$$\frac{y_0}{d_{\text{ext}}} \cong 0.037 \qquad \text{or} \qquad \frac{y_0}{d_{\text{ext}}} \cong 3.7\%.$$

When $y_0 = y_{0\,\max}$, then

$$\frac{y_{0\,\max}}{d_{\text{ext}}} \cong 0.052 \qquad \text{or} \qquad \frac{y_{0\,\max}}{d_{\text{ext}}} \cong 5.2\%.$$

Such a low value of y_0 does not strongly influence the plastic strain distribution. Moreover, the value of y_0, assumed in this paper ($\lambda_0 \approx 0.5$) for not-free bending and removed clearances between devices of the bending machine and bent tube, especially in the layers subjected to compression, can be even lower because of kinematically permissible displacement of the material particles upward along the perimeter [2, 17, 21]. In the considered case of bending, it is displacement downward in direction of the centre of the template rotation.

Example 2. When the strain state expressed by Eqs. $(3.3)_1$ and $(3.3)_2$, Part I, is obtained, then we have $\varphi_{(i)b1} \approx 0.257$ and $\varphi'''_{(i)b2} \cong 0.234$.

When the calculated allowable bending angle corresponding to a defined value of $\varphi_{(i)b1}$ is equal to $k\alpha_{b\text{all}} \approx 180°$, see Fig. 6, it means that for the generalised scheme of strain that form of stability loss can occur at the end of bending. Thus, from numerical calculations according to Eqs. (2.1) and (2.2), Part I, for $k\alpha_b = 180°$ or from Figs. 7 and 8 we obtain $g_{1\,\min} \cong 3.68$ mm.

The numerically calculated $g_{2\,\max}$ for the layers subjected to compression for $\lambda_2 = 1$ is $g_{2\,\max} \cong 6.0$ mm, and for $\lambda_2 = 0.5$, $g_2 \cong 5.0$ mm. From Fig. 8 we obtain $k\alpha'''_b \approx 158°$. Thus, it appears that $k\alpha_b > k\alpha'''_b$. It means that for the strain scheme of the 3rd type such a stability loss occurs, and this estimation in this sense is safer in comparison with the generalised model of strain. The calculated value $\varphi'''_{1\,\text{all}} \approx 0.203$ was obtained, see Fig. 9.

The following values of the plasticising stress and the principal stress components were obtained from Eqs. (2.3) and (3.11), Part I: $\sigma_p \cong 424.2$ MPa and $\sigma'''_{p\,s} \cong 417$ MPa, and $\sigma_1 \approx 524.4$ MPa, $\sigma_2 \approx 251.8$ MPa, and $\sigma'''_1 \cong 521$ MPa, $\sigma'''_2 \cong 313$ MPa.

When the bending state for the strain value equal to $\varphi_{(i)b1} \approx 0.257$, resulting from a general scheme of strain is reached, then $y_0 = y_{0\,\max} \approx 2.33$ mm and $R_0 = R_{0\,\min}$. Like in Example 1, the following values were obtained: $y_{0\,\max}/d_{\text{ext}} \approx 0.052$ and $y_{0\,\max}/d_{\text{ext}} \approx 5.2\%$ and $R_{0\,\min} \approx 77.67$ mm.

EXAMPLE 3. When the condition (2.8) or $(3.3)_2$, Part I, is reached, then in the given bending process a state connected with occurrence of PSD during PSS can appear in the top points of the elongated layers of the bending zone. The quantities characterising that process reach their allowable values after exceeding of which localisation of plastic deformation can occur in PSS condition and, as a consequence, a local furrow can be observed.

When

$$\varphi'''_{(i)b} = \varphi'''_{(i)\text{all}}, \qquad \text{then} \qquad \alpha'''_b = \alpha'''_{b\text{all}}, \tag{4.3}$$

then, according to Eq. (2.8) or Eqs. $(3.3)_2$ or (3.16) and (3.17), Part I, we obtain $\varphi'''_{(i)\text{all}} = 0.234$. Thus, it appears that the condition (2.8) is the same as the condition $(3.3)_2$ in Part I. The values g'''_1 and α'''_b obtained from Figs. 8 and 9 reach the following values: $\alpha'''_{b\,\text{all}} \cong 158°$ for $k = 1$, $\alpha'''_{b\,\text{all}} \cong 79°$, for $k = 2$, $\alpha'''_{b\,\text{all}} \cong 63°$, for $k = 2.5$, and $\alpha'''_{b\,\text{all}} \cong 53°$, for $k = 3$, and $g'''_{1\,\text{all}} \cong 3.66$ mm. Also from Fig. 8 we obtain the value of $\varphi'''_{1\,\text{all}} \cong 0.203$.

The plasticising stress calculated according to Eq. (2.3) reaches the value $\sigma'''_{ps} \cong 417$ MPa and the components of the plane stress state calculated according to Eqs. (2.9) are $\sigma'''_{1s} \cong 521$ MPa and $\sigma'''_{2s} \cong 313$ MPa. For a body of normal anisotropy ($r \neq 1$ we have, that

$$\sigma'''_{1s} \neq 2\sigma'''_{2s} \qquad \text{when} \qquad r \neq 1. \tag{4.4}$$

Now an approximate position of the neutral layer for the considered state determined from Eq. (4.1) is $y'''_0 \approx 1.89$ mm. Since $y'''_0 \cong R - R'''_0$, so the radius R'''_0 defining a new position of the neutral layer for the angle $\alpha = 0°$ is determined from the following expression:

$$R'''_0 = R - y'''_0. \tag{4.5}$$

Thus, $R'''_0 \cong 78.11$ mm and

$$\frac{y'''_0}{d_{\text{ext}}} \cong 0.042 \qquad \text{or} \qquad \frac{y'''_0}{d_{\text{ext}}} \cong 4.2\%.$$

When $k\alpha_b = 180°$, then $y'''_0 = y'''_{0\,\max}$ and then

$$\frac{y'''_{0\,\max}}{d_{\text{ext}}} \cong 0.052 \qquad \text{or} \qquad \frac{y'''_{0\,\max}}{d_{\text{ext}}} \cong 5.2\%.$$

EXAMPLE 4. The technological coefficient A_5 from the tables for tube steel K10 is $A_5 \cong 0.250$. The logarithmic longitudinal strain corresponding to this critical value and calculated from the relation $\varphi_{1\,\text{cr}} = \varphi'''_{1\,\text{cr}} = \ln(1 + A_5)$ is $\varphi_{1\,\text{cr}} = \varphi'''_{1\,\text{cr}} \approx 0.223$.

The critical bending angles corresponding to that state determined from Figs. 6 and 9 reach the values $k\alpha_b \approx 176^\circ$ and $k\alpha'''_{b\,\mathrm{cr}} \approx 168^\circ$. It means that the cracking (fracture) moment calculated according to that criterion (for strain scheme of 3rd type) will occur later than the moments of occurrence of the dissipated and localised stability loss calculated according to relationships (3.1) and $(3.3)_2$, Part I, or (2.8) in Part II, for boiler steel K10 [1, 2, 7] or St 35.8 according to DIN 17175 [17] and normal anisotropy coefficient $r = 1.5$. This is due to the quantities ($\varphi_{1\,\mathrm{all}}$ from Example 1, have the value almost equal to $\varphi_{1\,\mathrm{cr}}$) and ($\varphi'''_{1\,\mathrm{all}}$ from Examples 2 and 3, have lower values than $\varphi'''_{1\,\mathrm{cr}}$). Thus, when ($\varphi_{1\,\mathrm{all}} \approx \varphi_{1\,\mathrm{cr}}$) and ($\varphi'''_{1\,\mathrm{all}} < \varphi'''_{1\,\mathrm{cr}}$), then ($\alpha_{b\,\mathrm{all}} \approx \alpha_{b\,\mathrm{cr}}$) and ($\alpha'''_{b\,\mathrm{all}} < \alpha'''_{b\,\mathrm{cr}}$). The critical value of the strain intensity from Fig. 8, corresponding to the value A_5 is $\varphi'''_{(i)\mathrm{cr}} \approx 0.258$. The critical value of the strain intensity determined from Figs. 6 or 9 for that state is $\varphi_{(i)\mathrm{cr}} \approx 0.252$.

5. Final remarks and conclusions

1. The condition of possible initiation of the stability loss in the localised form (initiation of the biaxial (plane) strain state) in the plane stress state condition, determines greater admissible strain intensities than those for the case of stability loss in the dispersed form (maximum of the drawing force) and lower ones for the localised stability loss $d(\sigma_p \cdot g) = 0$, during biaxial tension Eqs. $(3.2)_1$ and $(3.3)_1$, Part I. In the case of dispersed stability loss during uniaxial uniform tension (see [2–5, 14, 18, 20]), an admissible value of strain intensity is comparable with the value of the coefficient of metal plastic hardening.

   A new element of this paper is the extension of the criterion of strain localisation (formulated by Marciniak [4]) for sheets, for the case of tube bending. In the case of a generalised scheme of strain and simplification of the 1st order, such an extended criterion (with and without including the effect of influence of the neutral axis displacement of plastic bending y_0) depends on the geometrical dimensions of the bent tube (approximately on its wall thickness s^*).
2. In the case of the strain model resulting from the simplifications of the 2nd and 3rd order we can state that in the external top point of the bent elbow, where there is the plane stress state, the condition $d(\sigma_p \cdot g) = 0$ corresponds to the parabolic point s at the H-M-H ellipse of plasticity, where ($d\varphi_2 = 0$). Physically it means a local initiation of PSD.
3. The tube bending process described by a generalised strain scheme (including influence of displacement of the neutral axis y_0 of plastic bending on the plastic strain state) presented in [2, 7, 9, 16] and suitable simplified

methods considered in this papers, means that displacement of the neutral layer of the plastic bending proceeds downwards (in the direction of the layers subjected to compression, see Fig. 11) and it increases as the bending angle increases. In the considered examples, the value of this displacement for the top points of the elongated layers (see Examples 1, 3) is $\sim(1\div 2$ mm). This value is proportionally comparable (near twice as big) to the value of the decrease of the wall thickness for these points of elongated layers, see Figs. 7 and 8.

4. We can formulate practical recommendations resulting from the calculations that the elbows of the pipelines in pressure devices working at elevated and high temperatures should be made for bending angles which do not exceed the obtained values defined in Examples 1–4. Exceeding the allowable values of the bending angles (also admissible strains) could cause reduction of the time of their operation (life time), especially for elbows working at elevated and high temperatures. Occurrence of localised strains during tube bending for elbows of the pipelines causes accelerated degradation processes of creep (because in such places those processes could concentrate) and lead to occurrence of cracks and dangerous failures, see [1, 2, 7, 13, 22–28]. Let us note that the analytical expressions for principal components of the strain state derived in [2, 7–9, 16, 19] can help in a future analysis and evaluation of tube usability for bending with the use of the methods of defining the curves of limit strains, like in the case of evaluation of sheet drawability [3–6, 14, 18, 20, 29–31].
5. During hot tube bending or with preheating, it is necessary to consider another form of the constitutive equation (not Eq. (2.3), Part I), in which material parameters will be dependent on temperature [29]. For example for the majority of metallic materials the coefficient of metal plastic hardening n decreases as the temperature increases [4, 5, 12, 29]. The terms of loss of stability should be also considered. On the other hand, softer and more plastic material could be characterised by greater values of permissible and critical strains. At suitably high temperatures it is assumed that the strength limit is almost the same as the yield point (no hardening effect) [12, 19].

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