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ISOCHRONOUS CREEP RUPTURE CURVES

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Isochronous creep rupture curves for damaged materials in the plane state of stress are investigated. The creep rupture criterion proposed by LITEWKA [6,8] in terms of damage evolution equation and yield criterion were used. The isochronous curves of Nimonic 80A show that even for the same material limit curves are varying and their shape depends on the load level.

1. INTRODUCTION

Metals and their alloys subjected to constant stress at elevated temperatures undergo time-dependent deformation and a rupture takes place in a finite time. The loci of constant rupture time for metals in multi axial states of stress constitutes the isochronous rupture curve. The problem of isochronous creep rupture curves was investigated by HAYHURST [1], TRĄMPCZYŃSKI et al. [2], CHRZANOWSKI and MADEJ [3,4,5]. HAYHURST suggested that the behaviour of metals and their alloys falls between two extremes. On the one hand the rupture behaviour of copper subjected to biaxial stresses approximately satisfies a maximum principal tensile stress criterion. On the other hand the rupture behaviour of an aluminium alloy is approximated by an octahedral shear stress criterion. In Fig.1 [2] the plane stress isochronous rupture loci for copper and aluminium alloys are shown. These curves are open and their shape varies with the material. CHRZANOWSKI and MADEJ [5] proposed the isochronous rupture curves shape depending on stress level. Figure 2 [4] illustrates the transformation of isochronous creep rupture curves in the state of plane stress.

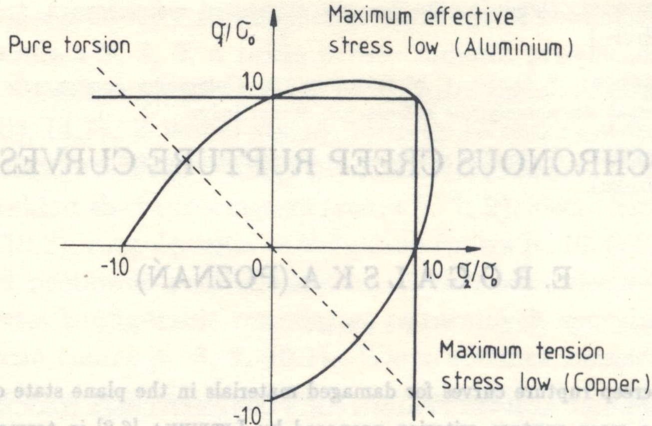

Fig. 1. Plane stress isochronous rupture loci for copper and aluminium alloys [2]

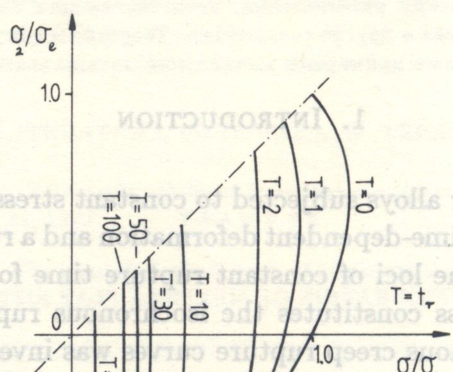

Fig. 2. Isochronous creep rupture curves in plane stress [4]

The aim of the paper is to present a method of constructing the isochronous curves by means of the creep rupture criterion proposed by LITEWKA [6,8] who described the damage using the symmetric second rank tensor. It was assumed in [6] that the creep rupture occurs at the critical value of the damage effect tensor **D** when the yield stress of the damaged material is equal to the stress applied to the material. In the following analysis, similar to that used in [6,8], the rupture criterion consists of two equations:

the yield criterion,

the damage evolution equation,

which describe the reduction of strength during the creep process and damage growth. The material is modelled as perfectly elastic-plastic and hence ultimate strength is equal to the uniaxial yield stress σ_o . The yield criterion for a critical configuration of the damage effect tensor $\mathbf{D}^{cr}$ is assumed as a collapse criterion for the deteriorated structure of the material [6].

2. CREEP PROCESS OF NIMONIC 80A

The theoretical description of the creep process in the plane state of stress of Nimonic 80A at 1023°K is presented below. Further considerations are based on the collapse criterion formulated in papers [9,10] and on the damage evolution equation proposed by LITEWKA [6,8]. The collapse criterion is expressed as an isotropic scalar function of the stress tensor $\boldsymbol{\sigma}$ and the damage effect tensor $\mathbf{D}^{cr}$ as follows:

$$C_1 \operatorname{tr}^2\boldsymbol{\sigma} + C_2 \operatorname{tr}\mathbf{S}^2 + C_3 \operatorname{tr}\mathbf{D}^{cr}\boldsymbol{\sigma}^2 - \sigma_0^2 = 0, \tag{2.1}$$

where $\boldsymbol{\sigma}$ describes the state of stress in the creep process, σ_o is the yield stress of virgin material at the test temperature and **S** is the stress deviator. The constants C_1, C_2, C_3 can be calculated from the following set of equations [6]:

$$\begin{aligned} C_1 + \frac{2}{3}C_2 + D_1^{cr}C_3 &= (\sigma_0/\sigma_{10})^2, \\ C_1 + \frac{2}{3}C_2 + D_2^{cr}C_3 &= (\sigma_0/\sigma_{20})^2, \\ 4C_1 + \frac{2}{3}C_2 + (D_1^{cr} + D_2^{cr})C_3 &= (\sigma_0/T_0)^2, \end{aligned} \tag{2.2}$$

where σ_{10}, σ_{20} and T_o are yield stresses for damaged material under uniaxial and biaxial loads respectively. Based on the linear dependence between the stress $\boldsymbol{\sigma}$ and damage $\boldsymbol{\Omega}$ tensors, the yield stresses $\sigma_{10}, \sigma_{20}, T_o$ can be derived from the relations [6,7]:

$$\begin{aligned} \sigma_{10} &= T_0 = \sigma_0(1-\Omega_1^{\rm cr}), \\ \sigma_{20} &= \sigma_0(1-\Omega_2^{\rm cr}). \end{aligned} \tag{2.3}$$

The values Ω_1^{cr} and Ω_2^{cr} are the principal values of the damage tensor $\boldsymbol{\Omega}$ which describes the evolution of damage. The principal values of the damage effect tensor $\mathbf{D}$ at the damage tensor $\boldsymbol{\Omega}$ are related by the formula

$$D_i = \frac{\Omega_i}{1-\Omega_i}. \tag{2.4}$$

It was assumed that only the tensile stress can cause the damage and that is why the modified stress tensor $\boldsymbol{\sigma}^*$ was introduced in [6]. This tensor is expressed in terms of positive principal values of the stress tensor $\boldsymbol{\sigma}$, in the form

$$\begin{aligned} \sigma_i^* &= H(\sigma_i)\sigma_i, \quad i=1,2,3, \\ H(\sigma_i) &= \begin{cases} 1 \text{ when } \sigma_i > 0, \\ 0 \text{ when } \sigma_i \leq 0, \end{cases} \end{aligned} \tag{2.5}$$

where H(σ_i) is a Heaviside function. This means that in the case of negative principal values of the stress tensor $\boldsymbol{\sigma}$, the appropriate principal components of the modified stress tensor $\boldsymbol{\sigma}^*$ are equal to zero.

The damage evolution equation proposed by LITEWKA [6] has the following form of the tensor function:

$$\dot{\boldsymbol{\Omega}} = \Phi_e^2 B \boldsymbol{\sigma}^*, \tag{2.6}$$

where $\dot{\boldsymbol{\Omega}}$ and $\boldsymbol{\sigma}^*$ are the rate-damage tensor and the modified stress tensor, respectively. The scalar multiplier Φ_e is the strain energy stored under the given state of stress $\boldsymbol{\sigma}$, and B is a temperature-dependent material constant. Further considerations are based on the following form of the damage evolution equation [6]:

$$\dot{\boldsymbol{\Omega}} = \left[\frac{1}{3}(1-2\nu)\,\mathrm{tr}^2\boldsymbol{\sigma} + (1+\nu)\,\mathrm{tr}\mathbf{S}^2 + \frac{D_1}{(1+D_1)}\,\mathrm{tr}\boldsymbol{\sigma}^2\mathbf{D}\right]^2 k\boldsymbol{\sigma}^*, \tag{2.7}$$

where ν, E is the Poisson's ratio and Young's modulus for the virgin material. Equation (2.7) contains only one unknown constant $k = B/4E^2$. The damage evolution equation expressed in terms of the principal values of the damage-rate $\dot{\mathbf{\Omega}}$ and stress $\boldsymbol{\sigma}$ tensors can be written in the form

$$
\begin{aligned}
\dot{\Omega}_1 &= \left[M + \Omega_1^2\left(\frac{1}{1-\Omega_1} + \frac{nm^2}{1-n\Omega_1}\right)\right]^2 k\sigma_1^5, \\
\dot{\Omega}_2 &= n\dot{\Omega}_1,
\end{aligned}
\tag{2.8}
$$

where

$$
\begin{aligned}
M &= 1 + m^2 - 2m\nu, \\
m &= \sigma_2/\sigma_1 \text{ and } n = \Omega_2/\Omega_1, \\
n &= \begin{cases} m & \text{for } 0 < m < 1, \\ 0 & \text{for } m \le 0. \end{cases}
\end{aligned}
$$

Neglecting the term with Ω_1^4 in Eqs.(2.8) one can obtain the following form of the damage evolution equation:

$$
\begin{aligned}
\dot{\Omega}_1 &= \left[M^2 + 2M\Omega_1^2\left(\frac{1}{1-\Omega_1} + \frac{nm^2}{1-n\Omega_1}\right)\right] k\sigma_1^5, \\
\dot{\Omega}_2 &= n\dot{\Omega}_1.
\end{aligned}
\tag{2.9}
$$

The constant k can be calculated from the result of any creep rupture test. Using the experimental results for the uniaxial tension creep rupture test obtained in [11] the value of constant $k = 3{,}8 \cdot 10^{-15}\text{MPa}^{-5}\text{h}^{-1}$ was calculated from Eq.(2.9) specified for the uniaxial tension

$$
k = (\sigma_1^5 t_r)^{-1} \int \frac{1-\Omega_1}{2\Omega_1^2 - \Omega_1 + 1} d\Omega_1. \tag{2.10}
$$

The critical value of the damage tensor Ω_1^{cr} can be calculated from Eqs. (2.1) - (2.3) for a given state of stress. The value σ_o should be determined experimentally, or can be calculated together with k from the results of two creep tests. These two constants k and σ_o together with Eqs. (2.1) and (2.9) are used to determine the theoretical curve of the rupture time t_r against effective stress σ_e determined according to the Huber-Mises yield criterion. For example, the integrated damage

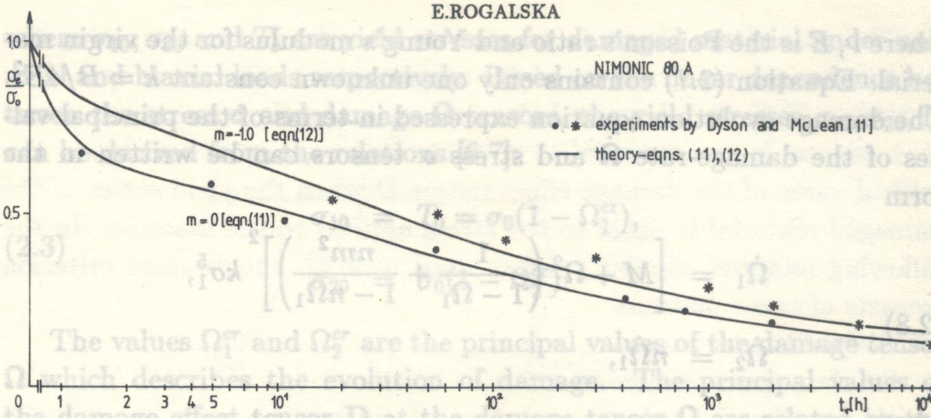

Fig. 3. Variation of the applied stress with the time to rupture

evolution equation for the uniaxial tension, where $\sigma_1^* = \sigma_1$ and $\sigma_2^* = 0$, has the form

$$(2.11)\qquad -0{,}25 \ln |(\Omega_1^{cr})^2 - 0{,}5\Omega_1^{cr} + 0{,}5| + 0{,}5669 \operatorname{arctg}(1{,}5118\Omega_1^{cr} - 1) + 0{,}03159 = k\sigma_1^5 t_r.$$

For the pure torsion, where $\sigma_1 = \tau, \sigma_2 = -\tau$ and $\sigma_1^* = \tau, \sigma_2^* = 0$, the damage evolution equation now has the following form:

$$(2.12)\qquad -0{,}0961 \ln |(\Omega_1^{cr})^2 - 0{,}33\Omega_1^{cr} + 0{,}33| + 0{,}0719 \operatorname{arctg}(1{,}0680\Omega_1^{cr} - 0{,}5338) + 0{,}06882 = k\tau^5 t_r.$$

The theoretical curve of the rupture time against the effective stress σ_e, together with the experimental results of Dyson and McLean [11] is shown in Fig.3.

3. Isochronous creep rupture curves

The locus of points in the σ_1, σ_2 coordinates having the same rupture times was constructed by employing the creep rupture criterion in the form of Eqs. (2.1) and (2.9). For the states of positive stresses σ_1, σ_2 where $(m = n)$, the following set of equations was derived:

$$(3.1)\qquad t_r = (k\sigma_1^5)^{-1} \int \left[M^2 + 2M\Omega^2 \left(\frac{1}{1-\Omega_1} + \frac{m^3}{1-m\Omega_1} \right) \right]^{-1} d\Omega_1,$$

where

$$\sigma_1 = \sigma_0 \left[C_1(m+1)^2 + \frac{2}{3}C_2(1-m+m^2) + C_3\Omega_1^{cr}\left(\frac{1}{1-\Omega_1^{cr}} + \frac{m^3}{1-m\Omega_1^{cr}}\right)\right]^{-0,5},$$

$$C_1 + \frac{2}{3}C_2 + \frac{\Omega_1^{cr}}{1-\Omega_1^{cr}}C_3 = \frac{1}{(1-\Omega_1^{cr})^2},$$

$$C_1 + \frac{2}{3}C_2 + \frac{m\Omega_1^{cr}}{1-\Omega_1^{cr}}C_3 = \frac{1}{(1-m\Omega_1^{cr})^2},$$

$$4C_1 + \frac{2}{3}C_2 + \left(\frac{\Omega_1^{cr}}{1-\Omega_1^{cr}} + \frac{m\Omega_1^{cr}}{1-\Omega_1^{cr}}\right)C_3 = \frac{1}{(1-\Omega_1^{cr})^2}.$$

The integrated damage evolution equation for the equal biaxial tension ($m = 1$) has the following form:

$$t_r = (k\sigma_1^5)^{-1}\left[-0{,}08929 \ln\left|(\Omega_1^{cr})^2 - 0{,}35\Omega_1^{cr} + 0{,}35\right| + \right.$$
$$\left. + 0{,}2607 \operatorname{arctg}(1{,}7696\Omega_1^{cr} - 0{,}3097) + 0{,}17185\right]. \tag{3.2}$$

For the states of stress where one of the stresses is negative $m \leq 0$ and $\sigma_1 > 0$, the following set of equations was used:

$$t_r = (k\sigma_1^5)^{-1}\int\left[M^2 + 2M\Omega^2\frac{1}{1-\Omega_1}\right]^{-1} d\Omega_1, \tag{3.3}$$

where

$$\sigma_1 = \sigma_0\left[C_1(m+1)^2 + \frac{2}{3}C_2(1-m+m^2) + C_3\frac{\Omega_1^{cr}}{1-\Omega_1^{cr}}\right]^{-0,5},$$

$$C_1 + \frac{2}{3}C_2 + \frac{\Omega_1^{cr}}{1-\Omega_1^{cr}}C_3 = \frac{1}{(1-\Omega_1^{cr})^2},$$

$$C_1 + \frac{2}{3}C_2 = 1,$$

$$4C_1 + \frac{2}{3}C_2 + \frac{\Omega_1^{cr}}{1-\Omega_1^{cr}}C_3 = \frac{1}{(1-\Omega_1^{cr})^2}.$$

For the states of negative stresses ($\sigma_1, \sigma_2 < 0$) we examine the following cases:

a) for $-2{,}363 < m \leq 0$

$$(3.4)\qquad t_r=(k\sigma_1^5)^{-1}\Big[-0{,}25M^{-1}\ln|(\Omega_1^{cr})^2-0{,}5M\Omega_1^{cr}+0{,}5M|+ +\frac{4-M}{2M\sqrt{M(8-M)}}\operatorname{arctg}\frac{4\Omega_1^{cr}-M}{\sqrt{M(8-M)}}+C\Big]$$

the integral constant C is equal to:

$$(3.5)\qquad C=0{,}25M^{-1}\ln|0{,}5M|-\frac{4-M}{2M\sqrt{M(8-M)}}\operatorname{arctg}\frac{-M}{\sqrt{M(8-M)}},$$

b) $m=-2{,}363$

$$t_r=(k\sigma_1^5)^{-1}\left[0{,}03125\ln|(\Omega_1^{cr})^2-4\Omega_1^{cr}+4|+0{,}0625(\Omega_1^{cr}-2)^{-1}+C\right],$$

the integral constant C is equal to:

$$C=0{,}07457$$

c) $m<-2{,}363$

$$(3.6)\qquad t_r=(k\sigma_1^5)^{-1}\Big[-0{,}25M^{-1}\ln|(\Omega_1^{cr})^2-0{,}5M\Omega_1^{cr}+0{,}5M|+ +\frac{4-M}{4M(M^2-8M)^{0,5}}\ln\frac{4\Omega_1^{cr}-M-(M^2-8M)^{0,5}}{4\Omega_1^{cr}-M+(M^2-8M)^{0,5}}+C\Big],$$

the integral constant C is equal to:

$$C=0{,}25M^{-1}\ln|0{,}5M|-\frac{4-M}{4M(M^2-8M)^{0,5}}\ln\frac{M+(M^2-8M)^{0,5}}{M-(M^2-8M)^{0,5}}.$$

Figure 4 shows the isochronous curves of Nimonic 80A calculated from Eqs. (3.1) - (3.6). It is seen that the shape of the curve depends on the load level. For a high load level ($\sigma_e \to \sigma_o$), the time-to-rupture is relatively short ($t \to 0$) and the damage approaches zero. For low load level, the effective stress is much smaller than σ_o , the time-to-rupture is increasing to infinity and the damage approaches unity. The plot of variation of the damage at failure against the time-to-rupture is shown in Fig.5. It can be observed that for a lower level of stresses the rupture behaviour has a brittle character because the damage approaches unity. On the contrary for high stresses the rupture behaviour has a ductile character, as it is for metals at normal temperatures. The isochronous curves are closed everywhere. For the states of negative stresses failure appears when the value of the effective stress σ_e is equal to σ_o of the virgin material.

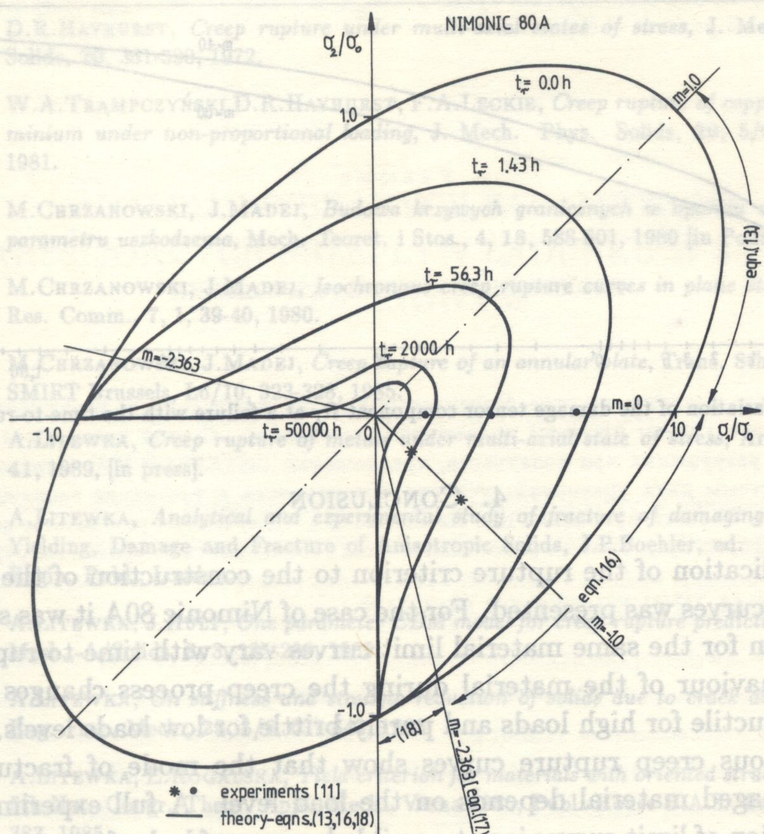

Fig. 4. Isochronous creep rupture curves in plane stress for Nimonic 80A

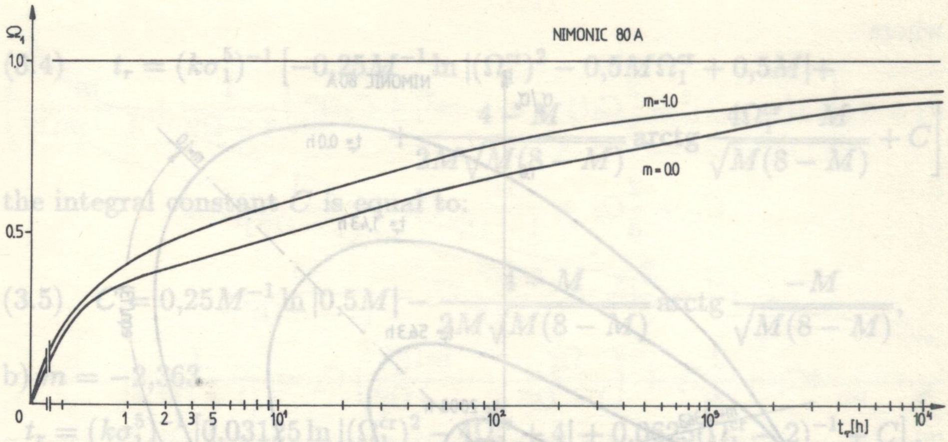

Fig. 5. Variation of the damage tensor component Ω_1 at a failure with the time-to-rupture

4. Conclusion

Application of the rupture criterion to the construction of the limit rupture curves was presented. For the case of Nimonic 80A it was shown that even for the same material limit curves vary with time-to-rupture. The behaviour of the material during the creep process changes from purely ductile for high loads and purely brittle for low loads levels. The isochronous creep rupture curves show that the mode of fracture of the damaged material depends on the load level. A full experimental verification of limit curves is not possible because of lack of experimental data on the creep process in plane state of stress.

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Streszczenie

IZOCHRONICZNE KRZYWE ZNISZCZENIA

W pracy przedstawiono sposób wyznaczenia izochronicznych krzywych zniszczenia przy pełzaniu z uwzględnieniem procesu uszkodzenia materiału. Wykorzystano w tym celu kryterium zniszczenia przy pełzaniu, zaproponowane przez Litewkę [6,8], wyrażone w formie układu dwóch równań: warunek plastyczności i równania ewolucji uszkodzenia. Obliczenia przeprowadzono dla Nimoniku 80A. Uzyskane wyniki pokazują, że kształt krzywych granicznych zmienia się zależnie od poziomu obciążenia.

Резюме

ИЗОХРОННЫЕ КРИВЫЕ РАЗРУШЕНИЯ

В работе представлен способ определения изохронных кривых разрушения при ползучести с учетом процесса повреждения материала. С этой целью использован критерий разрушения при ползучести, предложений ЛИТЕВКОЙ [6,8], выраженнй в форме системы двух уравнений: условия пластичности и уравнения эволюции повреждения. Расчети проведены для Нимоника 80А. Полученные результаты показывают, что форма предельных кривых изменяется в зависимости от уровня нагружения.

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