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Determination of the Initial Thickness of Tubes Subjected to Bending Part II. Discussion and Analysis of Obtained Results

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In this second part of the paper, applying epressions derived in Part I, the exemplary calculations of the initial thickness of a metallic tube subject to bending at bending machines are presented. The expressions for calculating of the initial thickness were presented for a suitable measure of the big actual radius R_i in the bending zone for an exact (general) solution and for three formal simplifications of the 1st, 2nd, and 3rd order. In the calculations the external or internal diameter of the tube subjected to bending is applied as a parameter. In this paper, the author shows that the calculated initial thickness of the tube (for the same parameters of bending) depending on the external diameter is lower than that calculated depending on the internal diameter. For example, the expression for calculation of deformations included in the UE Directive [1], contains dependence on d_{ext} not on d_{int}. The results of the calculations are presented in the graphs and tables enclosed.

Key words: bending tubes, three simplifications, strains, initial wall thickness, discontinuous strain fields.

1. Introduction

In part one of the paper, calculations of the initial thickness are done with the use of the expressions from a general scheme of description of the deformation state and three simplified methods. In the case of a generalised scheme of strain, the algebraic equation of the 2nd order is the basic equation applied for the calculations. In the case of the simplifications of the 1st, 2nd, and 3rd orders, the obtained expressions for g_0 are algebraic equations of the 1st degree, and they can be easily calculated with the use of a calculator. In all the cases the searched initial thickness of the tube g_0 depends on the bending radius R, the internal or external diameter of the tube, the bending angle, position of coordinates (α and β) of the point in the bending zone, for which the minimum allowable thickness g_{1all} was determined, and on the technological coefficient k, see [2–8]. When the given and calculated acceptable thickness of the elbow wall in the bending zone for the elongated layers ($\lambda_1 = 1$) [4, 6–8] is at the central point of the elbow and the top point of the bending zone, then $\alpha = \beta = 0°$. The introduced simplifications of the 1st, 2nd, and 3rd orders define greater values for the searched initial thickness of the wall as compared with the general solution, so such estimations of the real and searched initial thicknesses seem to be safer. This advantage, as well as simplicity of calculations, are the reason why application of simplifications of the 1st, 2nd, and 3rd orders seems to be interesting and promising. An empirical method of calculations of the acceptable wall thickness for bend tubes for different bending radii included in the interval (1–5) $\times\, d_{\text{ext}}$ is presented in the European Standard EN [1].

In this paper, the external or internal diameter of the tube subjected to bending is applied as a parameter, because in Poland and other countries most tube manufacturers give the same basic dimensions: external diameter, wall thickness, and length ($d_{\text{ext}} \times g_0 \times l$). In practice, there are some specific cases of elbows made of tubes where internal diameter, thickness, and length of the tube ($d_{\text{int}} \times g_0 \times l$) are dimensional parameters. This problem can occur during production of elbows of the same internal diameters, such as straight intervals, because it allows to make connections between them with no internal orifices. It also improves accuracy of mutual fitting of dimensions (internal diameter of the tube and external diameter of the mandrel of the bending machine) and reduces the number of mandrels or application of mandrels of variable geometry (variable diameter) generating higher expenses.

In this paper, the author shows that the calculated initial thicknesses of the tube (for the same parameters of bending) depending on the external diameter are lower than those calculated depending on the internal diameter. Thus, the "dimensional reference base" is different in the case of the internal diameter – in such a case the wall thickness increases outside the tube and increases its external dimensions applied to the calculations of strains. When the external diameter is the base, the wall thickness grows to the external part of the tube and the external dimensions used for the calculations of strains do not increase. Dimensional, constructional, technological, and operating problems connected with welded joints calculated in such a way and then produced elbows (for a given external or internal diameter) are discussed in [5]. For example, the expression for calculation of deformations included in the UE Directive [1], contains dependence on d_{ext}, not on d_{int}.

2. Analysis and discusion of the obtained results

2.1. Initial thickness g_0 depending on $d_{\rm ext}$

Based on the experimental data [5, 7, 9–12] and for simplicity of calculations we assumed that $y_0 \cong 0$, and hence $\beta_0 \cong 0$. This means that in the calculations of strains and thickness the moving of neutral axis of the plastic bending was omitted.

Figure 1 presents the calculation results of variation of the initial wall thickness (g_0, g_0', g_0'', and g_0''') of the tube for bending in order to obtain an elbow of the minimum thickness $g_1 = 4.5$ mm measured at the top point ($\alpha = \beta = 0°$) of the elbow in the elongated layers depending on the bending angle $k\alpha_b$, $k\alpha_b \in \langle 0°;\ 180° \rangle$. The appropriate calculations' results presented in Fig. 1 are based on the obtained expressions [(3.1)–(3.3), (3.6), (3.8), and (3.10), Part I]. Calculations were carried out for the tube $d_{\rm ext} = 44.5$ mm, bent by the radius $R = 80$ mm ($R = 1.8 \times d_{\rm ext}$) according to [2] for a general strain scheme and three simplified methods. For the data from [2] we can assume that $k \approx 3$ for bending with a put forward mandrel, and $k \approx 2.5$ for bending with a mandrel not put forward or with no mandrel. For the data from [11], we can assume that $k \approx 1$ for mild bendings with no mandrel and $k \approx 2$ for mild bendings with a mandrel.

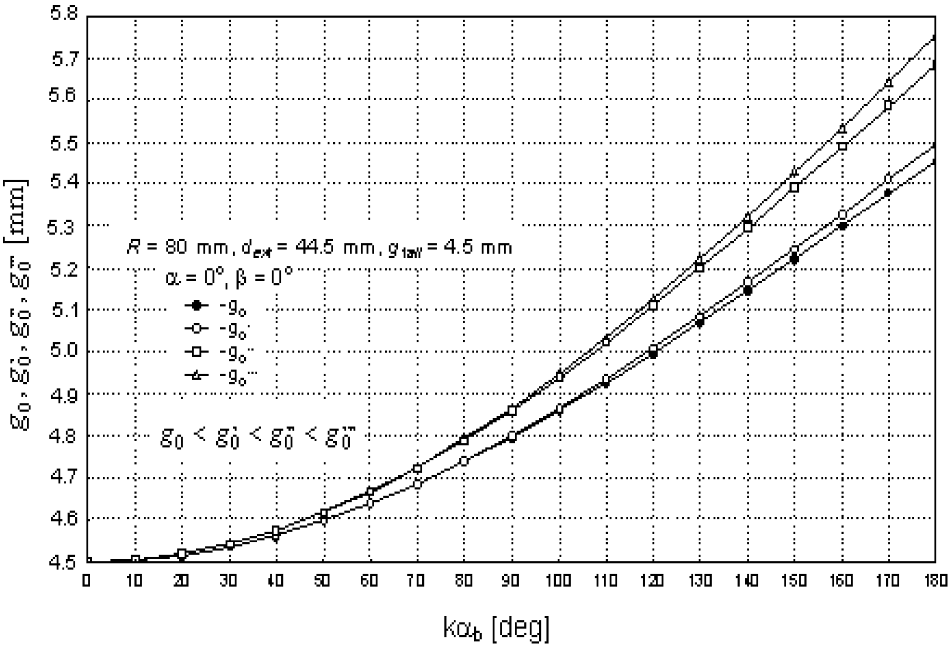

Fig. 1. Variation of the initial thickness versus $k\alpha_g$ for four methods of calculations.

From the figures it appears that the method of the general strain scheme defines the minimum initial thickness of the wall g_0, and the simplified methods define higher initial thicknesses, respectively, so that $(g_0 < g_0' < g_0'' < g_0''')$. It means that the simplified methods define a more safe initial thickness of the wall. Thus, the methods seem to be simple and interesting from the point of view of their usability in future calculations. The suitable initial thickness of the tube wall increases as the bending angle $k\alpha_b$ does.

Figure 2 shows the results of calculations for determination of the initial wall thickness of the tube for bending depending on the bending radius R, when $(0.5 \times d_{\text{ext}}) \le R \le (5 \times d_{\text{ext}})$. The calculations' results presented in Fig. 2 are based on the obtained expressions [(3.5), (3.7), (3.9), and (3.11), Part I]. The calculations were also made for a tube of the external diameter $d_{\text{ext}} = 44.5$ mm and the required minimum wall thickness $g_1 = 4.5$ mm at the elbow top point $(\alpha = \beta = 0°)$ for elongated layers $(\lambda_1 = 1)$, when $k\alpha_b = 180°$. From the figure it appears that $(g_0 < g_0' < g_0'' < g_0''')$, and it means that the simplified methods determine a higher initial thickness of the tube for bending, so they give safe estimations. From the figure it also appears that the initial thickness of the wall calculated according to the three methods decreases as the bending radius R increases. Such thickness is equal to g_1, when the bending radius R tends to an infinitely high value, and it means a lack of bending.

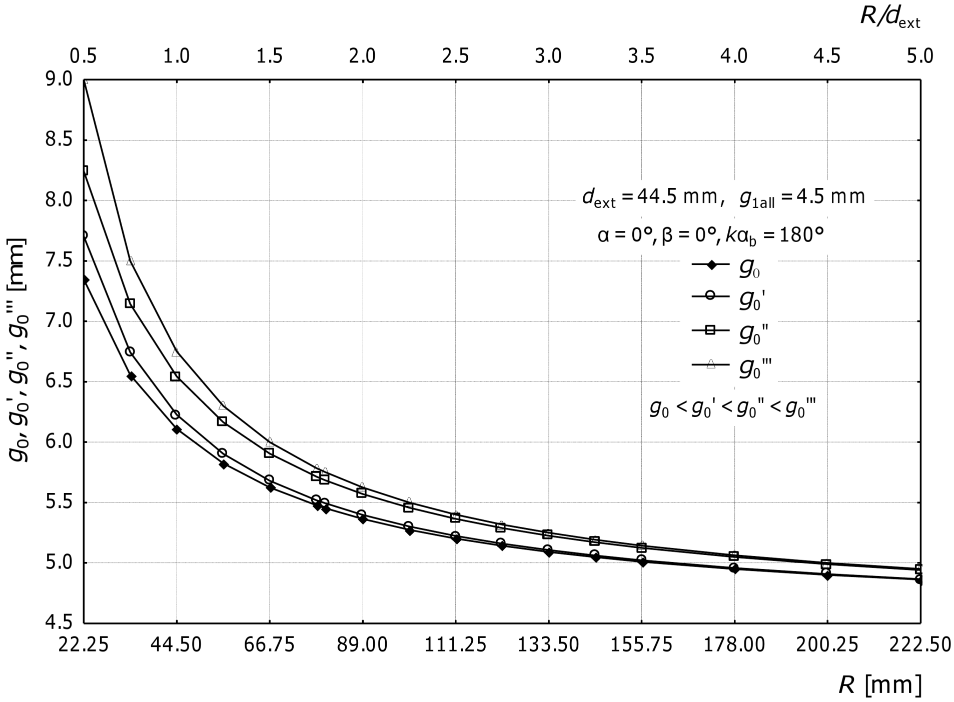

Fig. 2. Variation of the initial thickness versus R for four methods of calculations.

Figure 3 shows the calculation results for determination of the initial thickness of the tube wall depending on the required acceptable wall thickness g_1 from the interval ($1 \le g_1 \le 10$ mm). The calculations were done for a tube of the external diameter $d_{\rm ext} = 44.5$ mm, subjected to bending at the radius $R = 80$ mm according to Franz [9]. Like in the previous cases, now the simplified methods determine a greater initial wall thickness g_0 of the tube for bending as compared to that obtained according to the general scheme of strain. The differences are not big (one to some per cent), and the methods of calculations are simpler. In all the cases, calculations can be done with the use of an electronic calculator.

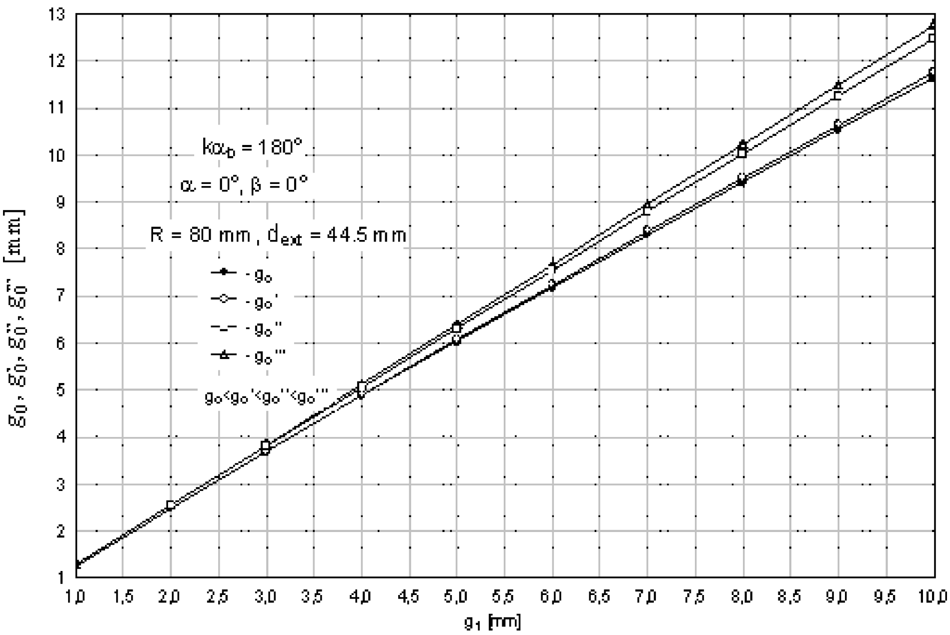

FIG. 3. Variation of the initial thickness of the tube wall depending on the calculated wall thickness g_1 satisfying the conditions of the EU Standards [1] and UDT, see [12, 13].

The inequalities (2.1) are resulting from Example 1, expressions [(3.1)–(3.11), Part I] for the same bending parameters and the tube external diameter $d_{\rm ext}$ and from Figs. 1–3:

$$
\begin{aligned}
&g_0(d_{\rm ext}) < g_0'(d_{\rm ext}) < g_0''(d_{\rm ext}) < g_0'''(d_{\rm ext})\\
&\text{or}\quad g_{0\rm ext} < g_{0\rm ext}' < g_{0\rm ext}'' < g_{0\rm ext}'''.
\end{aligned}
\tag{2.1}
$$

2.2. *Initial thickness g_0 depenging on d_{int}*

As in Subsec. 2.1, based on the experimental data [5, 7, 9–11] we admitted to the calculations y_0 ($y_0 \cong 0$), and hence $\beta_0 \cong 0$. From expressions [(4.5), (4.7), (4.9), (4.11) and (4.4)–(4.6), (4.10)–(4.12), (4.14), and (4.16), Part I] after the calculations we obtained the following results, see Table 1.

Table 1. Maximum values of the initial wall thickness calculated from the generalised theory and the three simplified methods, expressed as a true logarithmic strain depending on the bending radius R for the minimum wall thickness value set to g_1 = 4.5 mm. The other parameters assumed in the calculation are d_{ext} = 44.5 mm and d_{int} = 35.5 mm.

R [mm]	R [$\widetilde{r} \times d_{\text{ext}}$]	$g_{0\text{ext}}$ [mm]	$g_{0\text{int}}$ [mm]	$g'_{0\text{ext}}$ [mm]	$g'_{0\text{int}}$ [mm]	$g''_{0\text{ext}}$ [mm]	$g''_{0\text{int}}$ [mm]	$g'''_{0\text{ext}}$ [mm]	$g'''_{0\text{int}}$ [mm]
22.250	$R = 0.5 \times d_{\text{ext}}$	7.347	7.829	7.704	8.338	8.243	9.000	9.000	10.141
33.375	$R = 0.75 \times d_{\text{ext}}$	6.556	6.798	6.744	7.040	7.144	7.500	7.500	7.968
44.500	$R = 1 \times d_{\text{ext}}$	6.110	6.256	6.226	6.400	6.543	6.750	6.750	7.003
55.625	$R = 1.25 \times d_{ext}$	5.824	5.922	5.903	6.014	6.165	6.300	6.300	6.458
66.750	$R = 1.5 \times d_{\text{ext}}$	5.625	5.694	5.680	5.760	5.905	6.000	6.000	6.108
77.875	$R = 1.75 \times d_{\text{ext}}$	5.478	5.530	5.520	5.578	5.715	5.786	5.786	5.865
80.000	$R \cong 1.8 \times d_{\text{ext}}$	5.454	5.503	5.495	5.549	5.685	5.752	5.752	5.826
89.000	$R = 2 \times d_{\text{ext}}$	5.364	5.405	5.398	5.542	5.571	5.625	5.625	5.685
100.125	$R = 2.25 \times d_{\text{ext}}$	5.275	5.307	5.302	5.337	5.457	5.500	5.500	5.547
111.250	$R = 2.5 \times d_{\text{ext}}$	5.202	5.229	5.224	5.253	5.365	5.400	5.400	5.438
122.375	$R = 2.75 \times d_{\text{ext}}$	5.142	5.164	5.160	5.184	5.289	5.318	5.318	5.349
133.500	$R = 3 \times d_{\text{ext}}$	5.091	5.110	5.107	5.127	5.226	5.250	5.250	5.276
144.625	$R = 3.25 \times d_{\text{ext}}$	5.048	5.064	5.061	5.080	5.171	5.192	5.192	5.215
155.750	$R = 3.5 \times d_{\text{ext}}$	5.010	5.024	5.022	5.034	5.125	5.143	5.143	5.162
178.000	$R = 4 \times d_{\text{ext}}$	4.949	4.960	4.958	4.970	5.049	5.063	5.063	5.077
200.250	$R = 4.5 \times d_{\text{ext}}$	4.901	4.910	4.908	4.920	4.989	5.000	5.000	5.011
222.500	$R = 5 \times d_{\text{ext}}$	4.862	4.869	4.866	4.875	4.941	4.950	4.950	4.959

From the calculations presented in Table 1, the following inequalities are obtained:

$$
\begin{aligned}
&g_0(d_{\text{ext}}) < g_0(d_{\text{int}}), && g'_0(d_{\text{ext}}) < g'_0(d_{\text{int}}),\\
&g''_0(d_{\text{ext}}) < g''_0(d_{\text{int}}), && g'''_0(d_{\text{ext}}) < g'''_0(d_{\text{int}}),
\end{aligned} \tag{2.2}
$$

or

$$
g_{0\text{ext}} < g_{0\text{int}}, \qquad g'_{0\text{ext}} < g'_{0\text{int}}, \qquad g''_{0\text{ext}} < g''_{0\text{int}}, \qquad g'''_{0\text{ext}} < g'''_{0\text{int}}. \tag{2.3}
$$

The above inequalities are valid for suitable parameters of the tube bending process, for any bending radius R and geometrical dimensions of the bend tube, i.e., for the given external and internal diameters of the tube, and the required thickness of the elbow in the vertex points of the elongated layers. From Table 1 and Fig. 4 it appears that when the bending radius R decreases, then differences between the calculated initial wall thicknesses increase. When the bending radius R increases, then the differences in the calculated thicknesses decrease. When the bending radius R increases to infinity, differences in the calculated thicknesses tend to zero, and the values of the calculated initial thickness tend to the required thickness g_1, and that means no bending.

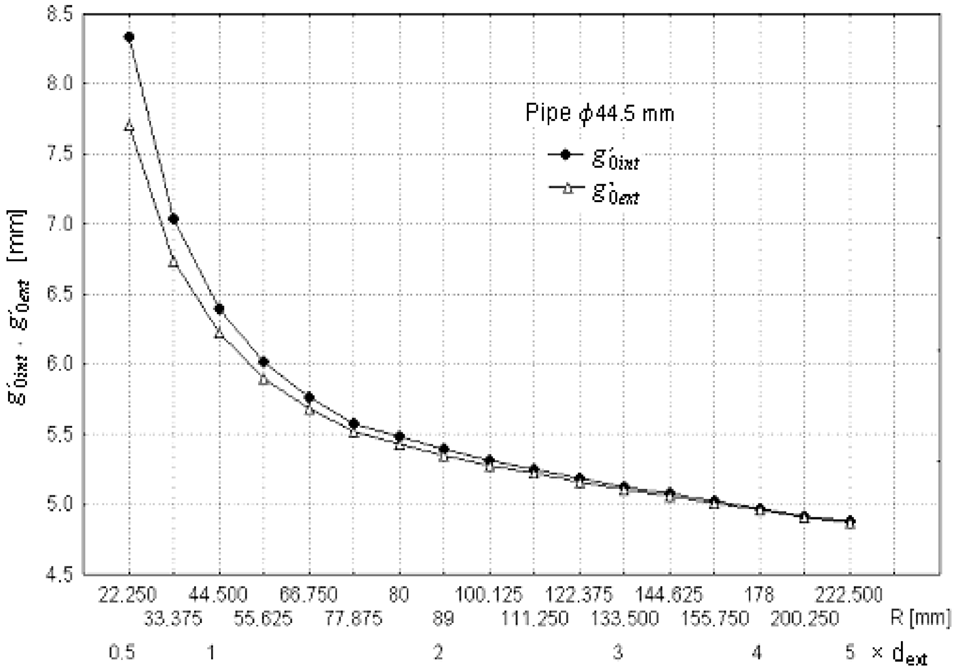

FIG. 4. Variation of the initial thickness ($g'_{0\text{ext}}$, $g'_{0\text{int}}$) of the wall of the tube for bending, calculated according to the simplification of the 1st order and in measures of logarithmic strains versus the bending radius R, when the required minimum wall thickness of the bent elbow is $g_1 = 4.5$ mm.

From Table 1, inequalities (2.2), (2.3) and Fig. 4, it is seen that the initial thicknesses of the walls of tubes subjected to bending calculated depending on the external and internal diameters in the measures of logarithmic (real) strains are not equal. It is an effect of assumption of different dimensional reference geometrical bases for strains' calculations. When the internal diameter is assumed as the geometrical dimensional reference base, the calculated thicknesses of the tube for bending are higher than in the case of the external diameter assumed as the geometrical dimensional reference base [5].

3. Conclusions

1. From Fig. 1 it appears that the method of the general strain scheme defines the minimum initial thickness of the wall g_0, and the simplified methods define a higher initial thickness, respectively, so that ($g_0 < g_0' < g_0'' < g_0'''$). It means that the simplified methods define a more safe initial thickness of the wall. Thus, the methods seem to be simple and interesting from the point of view of their usability in future calculations. The suitable initial thickness of the tube wall increases as the bending angle $k\alpha_b$ does. Calculations were also carried out for a tube of the external diameter $d_{\text{ext}} = 44.5$ mm and the required minimum wall thickness $g_1 = 4.5$ mm at the elbow top point ($\alpha = \beta = 0°$) for the elongated layers ($\lambda_1 = 1$), when $0° \leq k\alpha_b \leq 180°$.
2. From Fig. 2 it appears also that $g_0 < g_0' < g_0'' < g_0'''$, and it means that the simplified methods determine a higher initial thickness of the tube for bending, so they give safe estimations. From the figure it also appears that the initial thickness of the wall calculated according to three methods decreases as the bending radius R increases. Such a thickness is equal to g_1, when the bending radius R tends to an infinitely high value, and it means a lack of bending. Calculations were also made for a tube of the external diameter $d_{\text{ext}} = 44.5$ mm and the required minimum wall thickness $g_1 = 4.5$ mm at the elbow top point ($\alpha = \beta = 0°$) for the elongated layers ($\lambda_1 = 1$), when $k\alpha_b = 180°$.
3. Figure 3 shows the calculation results for determination of the initial thickness of the tube wall depending on the required acceptable wall thickness g_1 from the interval ($1 \leq g_1 \leq 10$ mm). Like in the previous cases, now the simplified methods determine a greater initial wall thickness g_0 of the tube for bending as compared with that obtained according to the general scheme of strain. The differences are not big (one to some per cent).
4. From Table 1, inequalities (2.2), (2.3) and Fig. 4, it is seen that the initial thicknesses of the walls of tubes subjected to bending calculated depending on the external and internal diameters in the measures of the logarithmic (real) strains are not equal. It is an effect of the assumption of different dimensional reference geometrical bases for strains' calculations. When the internal diameter is assumed as the geometrical dimensional reference base, the calculated thicknesses of the tube for bending are higher than in the case of the external diameter assumed as the geometrical dimensional reference base [5].
5. After the calculations of g_0, g_0', g_0'', and g_0''' we must take for bending the first next tube of a greater thickness from a catalogue of the manufac-

turer, according to the European and Polish standards and rules (European EN [1] and of other countries' Standards).

6. In the future, it is possible to formulate useful nomograms for determination of the initial thickness of the tube for bending for different parameters of the bending process, for example for different external diameters and different acceptable and calculated wall thicknesses g_1, according to the European and of other countries' standards, technological requirements [14] (or any other existing and acceptable criteria and conditions). When the initial thicknesses for generalised and simplified models are calculated depending on d_{ext} or d_{int}, the results are different, see Table 1, Fig. 4, and inequalities (2.2) and (2.3). The UE Directive [1] contains the dependence on d_{ext} not on d_{int}.
7. One should remember that tube bending is not a free process but depends on the bender, its stiffness, shape of working tools, type of applied mandrels, bending parameters such as R_m, tube dimensions ($d_{\text{ext}} \times g_0$), tube material, and others [1–16]. In the production practice one should increase the bending angle by a few steps in order to eliminate the phenomenon of springing. Springing has no influence on the calculated initial thickness because the active bending angle α_b means the bending angle of plastic deformations.
8. In the future, the methods of calculations of elbows should be improved, taking into account the criteria and conditions of stability loss derived in [4, 5, 17–22] or other papers and using FEM. It will provide a more precise determination of the required and acceptable wall thickness satisfying the criteria of strength, life, and working safety comparable with those for straight intervals and other elements of pipelines.
9. The solution of the problem of pipe bending on benders in the framework of the nonlinear solid mechanics [23–27] using the Finite Elements Method (FEM) is open to further studies.
10. This work can be treated as the first step and the next steps could be a development of nomograms and tables for bending tubes of various dimensions ($d_{\text{ext}} \times g_0$) and ($d_{\text{int}} \times g_0$) for various R. As it has been shown in the paper, when the initial thicknesses g_{0l} or g_{0r} (for the generalised and simplificated methods) are calculated depending on d_{ext} or d_{int}, the results are different.

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