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Determination of the Initial Thickness of Tubes Subjected to Bending Part I. Derivation of the Basic Expressions and Relationships

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Part one of the paper presents the derived equations for calculations of the initial wall thickness g_0 of tubes to be bent for elbows, and a proper bending angle α_b. The expressions for calculating g_0 were presented for a suitable measure of the big actual radius R_i in the bending zone for an exact (general) solution and for three formal simplifications of the 1st, 2nd, and 3rd order. In any case, a value of g_0 depends on the radius and angle of bending, external diameter of the tube, and permissible elbow thickness according to the European-Polish Standards and recommendations of the European-Polish Technical Inspection Office, on the coordinates of the point where the thickness g_{1all} was determined, and the coefficient of the bending zone range k (defined during tests). In this paper, the external or internal diameter of the tube subjected to bending is applied as a parameter.

Key words: bending tubes, three simplifications, strains, initial wall thickness, discontinuous strain fields.

1. Introduction

During the tube bending process, variation of the wall thickness g_i can be observed in all zones of active bending and bent zones. The thickness is minimum at the top points of the elongated layers of the elbow and it is almost equal to the initial thickness g_0 at the beginning and end of the bending zone [1–13]. In paper [1] it was mentioned (in the range of elastic deformations) that wall thickness changing and ovality changing are of the same order. The wall thickness is usually maximum at the internal points of the compressed layers. The elbows must satisfy the conditions and criteria connected with the strength, designing, working safety and life, requirements of suitable standards and rules of technical inspection, so the initial thickness of a tube for bending should be chosen in a proper way. The elbow life should be comparable or even greater than the life of straight intervals of pipelines [14–16], because production costs for elbows are higher. Thus, the minimum thickness of the elbow at the top point should not be lower than the wall thickness of the pipeline straight interval.

If one wants to determine the required initial thickness of the pipe, one has to "invert" the expressions obtained in [5, 8–10] and calculate the initial thickness instead of the wall thickness in the bending zone. In this paper the author concentrated mainly on the analysis of elongated layers in the bending and bent zones because in those zones we observe reduction of the wall thickness, and damages, cracks, and other unfavourable processes usually take place there. Thus, the problem is the following: the minimum acceptable wall thickness, satisfying the strength criteria and requirements of technical inspection, as well as European-Polish standards, is given, and the initial thickness is the searched variable. According to the author's knowledge, empirical methods of selection of the tube initial thickness were used in the past for the given bending parameters, tube dimensions, and the material of the tube.

At present, two methods and approaches to the problem are known, namely:

- The calculated minimum wall thickness in the elbow $g_{1\,\mathrm{min}}$ is also the thickness for straight intervals and all the pipeline. It is not right from the economical point of view, because working conditions of the elbows are more difficult than those for the straight intervals, and damages of the pipelines will occur more often in the elbows [14, 15, 17–19] than in the straight intervals;
- The other method consists of separate calculations for straight elements and elbows of the pipeline. After calculations of the elbow strength and determination of the acceptable minimum thickness at the most dangerous points, the initial thickness of the tube is determined. Since the calculated thickness of the straight intervals is smaller than the calculated thickness of the elbow, it is also smaller than the initial thickness of the tube for the elbows. When external diameters of the elbows and the straight intervals are the same, at the joining point a change of the cross section will occur (external orifice). When external diameters are equal, there is no change of the cross section, on the contrary to the "external orifice" [8].

During a real bending process, after bending at the edges of the bending zone, the wall thickness is smaller in the elongated layers, and greater in the compressed layers as compared with the initial thickness of the tube [1, 2, 6, 8, 10, 11]. It is of a great importance when the straight intervals of elbows are too short. Then the joints of elbows with straight intervals are asymmetric (asymmetric contractions, asymmetric orifices form).

In this paper, the external or internal diameter of the tube subjected to bending is applied as a parameter because in Poland and other countries most tube manufacturers give the same basic dimensions: external diameter, wall thickness, and length ($d_{\text{ext}} \times g_0 \times l$). In practice, there are some specific cases of elbows made of the tubes where internal diameter, thickness, and length of the tube ($d_{\text{int}} \times g_0 \times l$) are dimensional parameters. This problem can occur during production of elbows of the same internal diameters, such as straight intervals, because it allows to make connections between them with no internal orifices. It also improves accuracy of mutual fitting of dimensions (internal diameter of the tube and external diameter of the mandrel of the bending machine) and reduces the number of mandrels or application of mandrels of variable geometry (variable diameter) that generate higher expenses.

For example, the expression for calculation of deformations included in UE Directive [13], contains dependence on d_{ext} not on d_{int}.

2. Basic geometrical descriptions and relationships

The notations applied in Fig. 1 and in the equations are:

d_j – local "actual diameter" of the bent elbow, $d_j = 2r_j$ ($j = 1$ for the elongated layers, $j = 2$ for the compressed layers),

d_{ext} and d_{int} – external and internal diameters of the tube subjected to bending, respectively, such that $d_{\text{ext}} > d_{\text{int}}$ and also $d_{\text{ext}} = 2r_{\text{ext}}$, and $r_{\text{ext}} = r_{\text{int}} + g_0$, and we assume that $d_{\text{int}} = \text{const}$,

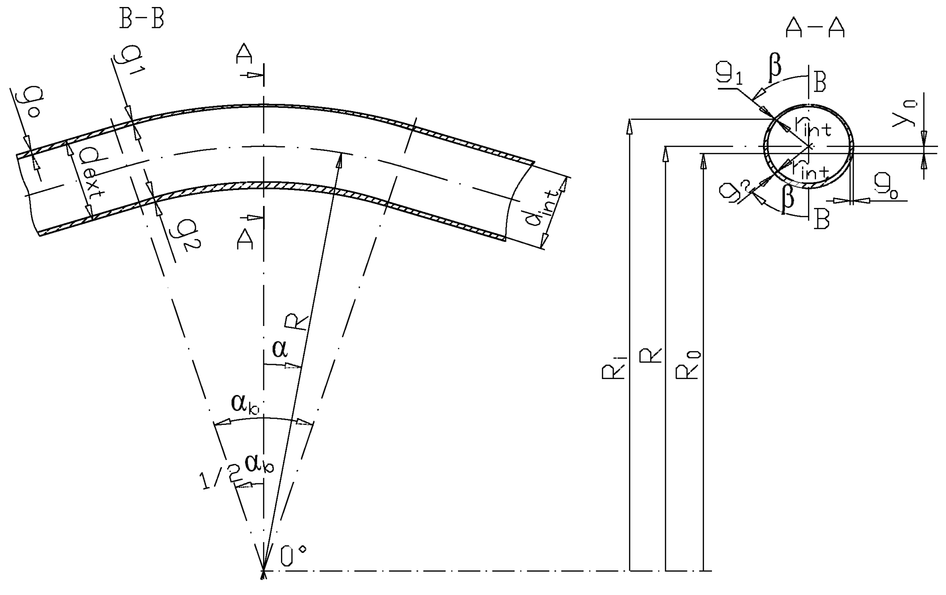

Fig. 1. Geometric and dimensional quantities occurring during tube bending.

g_0 – initial thickness of the tube subjected to bending,

g_j – local actual thickness of the elbow in the bending and bent zones where ($j = 1$ or $j = 2$),

$g_{1\,\mathrm{min}}$ – minimum wall thickness in the elongated layer,

$g_{2\,\mathrm{max}}$ – maximum wall thickness in the compressed layer,

k – technological-material coefficient determined from tests which defines the range of the bending zone so that $k\alpha_b = 180°$. Theoretically, $k \in \langle 1;\ \infty\rangle$. For practical purposes we can assume that $k \in \langle 1;\ 6\rangle$, see Śloderbach [6–9],

r_j and r^* – local small and locally small active actual radii of the bent elbow,

r_{ext} and r_{int} – external and internal radii of the tube subjected to bending,

R and $\widetilde{r}$ – bending radius (initial position of the neutral layer) and relative bending radius, where $\widetilde{r} = R/d_{\mathrm{ext}}$,

R_j – local "big active actual radius" of the elbow connected with the longitudinal strain,

R_0 – radius of the locally actual position of the neutral layer in the tube bending process,

y_0 – local and instantaneous displacement of the neutral axis in relation to the initial state.

<u>*Greek letters*</u>

α – actual angle of the bending zone determined in the main bending plane and in the parallel planes, $\alpha \in \left\langle 0°,\ \frac{\alpha_b}{2}\right\rangle$; when $\alpha_b = 0$ (no bending), then $\alpha = 0$,

α_b – bending angle of plastic deformations measured in the bending zone, $\alpha_b \in \langle 0°,\ 180°\rangle$, α_{pl} – angle of the plateau zone,

α_0 – angle of the elbow bend (angle of rotation of the template, drawing die of the bending machine), and in practice $\alpha_0 \in \langle 0°,\ 180°\rangle$.

In the bending zone there is the equality of the bending and bend angles, so $\alpha_0 = \alpha_b$. When the *plateau* zone has been formed in the bending zone, then $\alpha_0 = \alpha_b + \alpha_{pl}$, where β is the actual angle determined in the plane perpendicular to the bending plane, $\beta \in \langle 0°,\ 90°\rangle$, φ_i are the plastic logarithmic generalised strains in the bending zone (logarithmic equivalent strain) derived in [8–10] ($i = 1, 2, 3$) with the following form:

- $\varphi_1 \cong \lambda_j \ln \dfrac{2\,(R - y_0) \pm (d_j \cos\beta_j \pm 2\,y_0)\left(\cos(k\alpha) - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)}{2(R - y_0)}$ is the longitudinal strain (along the axis),

- $\varphi_2 \cong \ln \frac{d_j}{d_{\text{ext}}}$ is the logarithmic circumferential strain in the plane perpendicular to the main plane of bending,
- $\varphi_3 \cong \ln \frac{g_j}{g_0}$ is the logarithmic radial strain (along thickness), where $j = 1, 2$ ($j = 1$ and sign $(+)$ for elongated layers, $j = 2$ and sign $(-)$ for compressed layers [5–11]).

Expressions on φ_i $(i = 1, 2, 3)$ were derived using concepts of kinematically admissible fields of plastic deformations, see e.g. [20–23]. In this paper on the basis of experimental data and analytical calculations [2–8] we assume, that $y_0 \cong 0$.

The values of elastic deformations are decimal parts of the per cent (a few degrees of bending angle) and are neglected as very small in comparison with the plastic deformations (the maximum theoretically value exceeds 50%), see [2–6, 9–11].

Then $\varphi_1 = \varphi_1^p$, $\varphi_2 = \varphi_2^p$, $\varphi_3 = \varphi_3^p$, $\varphi_i = \varphi_i^p$, where the superscript p means plastic strains. λ_j are the coefficients of correction (modification) of the strain distribution (especially important for description and analysis of the compressed layers) in the bending zone, dependent on technological parameters of bending ($j = 1$ for elongated and $j = 2$ for compressed layers).

3. The basic relationships for calculating g_0 depenging on d_{ext}

In this chapter, the external diameter of the tube subjected to bending d_{ext} is applied during the calculations as a geometrical parameter, and only the top points of the elongated layers ($j = 1$) of the elbow are considered, so $\alpha = \beta = 0°$ and $\lambda_1 = 1$.

a) Generalised model of the strain field

Let us substitute equations for φ_1, φ_2, and φ_3 (see *Greek letters* in Sec. 2) with the condition of plastic incompressibility ($\varphi_1 + \varphi_2 + \varphi_3 = 0$). After transformations and the assumption that $y_0 \cong 0$ we obtain the following algebraic equation of 2nd degree related to g_0:

$$a_1 g_0^2 + b_1 g_0 + c_1 = 0, \tag{3.1}$$

where

$$\begin{aligned} a_1 &= g_1 \left[1 - \cos\left(k \frac{\alpha_b}{2}\right)\right], \\ b_1 &= -\left(\frac{d_{\text{ext}} + 2g_1}{2}\right)\left[R + 2g_1\left(1 - \cos\left(k\frac{\alpha_b}{2}\right)\right)\right], \\ c_1 &= g_1 \left(\frac{d_{\text{ext}} + 2g_1}{2}\right)\left[R + \left(\frac{d_{\text{ext}} + 2g_1}{2}\right)\left(1 - \cos\left(k\frac{\alpha_b}{2}\right)\right)\right], \end{aligned} \tag{3.2}$$

$g_0 \equiv g_{0\text{ext}}$ are the indices (0 ext) mean: 0 is the initial thickness, ext is the dependence on the external diameter, g_1 is the required (calculation) thickness of the elbow wall at the top point of the elongated layers.

Two roots are a solution of Eq. (3.1). Only one of them (positive) satisfies real conditions of bending. The other root is negative. Thus,

$$g_0 \equiv g_{0\text{ext}} = \frac{-b_1 + \sqrt{\Delta}}{2a_1}, \qquad \Delta = b_1^2 - 4a_1c_1. \tag{3.3}$$

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (the plateau zone forms), then $g_0 = g_{0\,\max}$. Expressions (3.1) and (3.2) have the following simple form:

$$a_1 g_0^2 + b_1 g_0 + c_1 = 0, \tag{3.4}$$

where

$$\begin{aligned} a_1 &= g_1, \\ b_1 &= -\left(\frac{d_{\text{ext}} + 2g_1}{2}\right)[R + 2g_1], \\ c_1 &= g_1\left(\frac{d_{\text{ext}} + 2g_1}{2}\right)\left[R + \left(\frac{d_{\text{ext}} + 2g_1}{2}\right)\right]. \end{aligned} \tag{3.5}$$

b) Simplification of the 1st order

The suitable quantities for component strains and the expressions obtained under the simplification of the 1st type were noted by the symbol ($'$). The simplification of the 1st order concerns the form of the numerator of expression φ_1 for elongated layers ($j = 1$), see (*Greek letters*, Sec. 2), $2(R - y_0) + (d_1 \cos\beta_1 + 2\,y_0)\big(\cos(k\alpha) - \cos\big(k\frac{\alpha_b}{2}\big)\big)$ in the bending zone, where the diameter d_j is formally replaced by the diameter d_{ext} and $\varphi_2' = \varphi_2$, $\varphi_3' = \varphi_3$, see [5, 8, 10].

Let us substitute the strain components for the simplification of the 1st order expressed versus d_{ext} with the condition of plastic incompressibility ($\varphi_1' + \varphi_2' + \varphi_3' = 0$), after transformations and the assumption that $y_0 \cong 0$ we obtain the following expression:

$$g_0' \equiv g_{0\text{ext}}' = \frac{g_i\,(d_{\text{ext}} + 2g_1)\left[2R + d_{\text{ext}}\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}{2\left[R\,(d_{\text{ext}} + 2g_1) + d_{\text{ext}}g_1\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}, \tag{3.6}$$

where $g_{0\text{ext}}'$ – indices (0ext) and g_1 like in subpoint a.

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (formation of the plateau zone), and then $g'_{0\text{ext}} = g'_{0\max}$. Equation (3.6) takes a simple form

$$g'_{0\max} \equiv g'_{0\text{ext}} = \frac{g_i \left(d_{\text{ext}} + 2g_1\right)\left(2R + d_{\text{ext}}\right)}{2\left[R\left(d_{\text{ext}} + 2g_1\right) + d_{\text{ext}} g_1\right]}. \tag{3.7}$$

c) Simplification of the 2nd order

In the simplification of the 2nd type it is assumed that the value of the circumferential strain φ_2 is negligibly low in relation to two other strain components. From the available experimental data [2, 4, 6, 10] it results that those strains can be almost five times smaller than the other strain components. Physically, it means the application of a certain "quasi-plane" strain state. In this case, let as denote suitable quantities as $('')$, then $\varphi''_1 = \varphi_1$, $\varphi''_2 \cong 0$, and $\varphi''_3 = \varphi_3$.

Let us substitute the strain components for the simplification of the 2nd order from related to d_{ext} with the condition of plastic incompressibility ($\varphi''_1 + \varphi''_2 + \varphi''_3 = 0$), after transformations we obtain the following expression:

$$g''_0 \equiv g''_{0\text{ext}} = \frac{g_1 \left[2R + \left(d_{\text{ext}} + 2g_1\right)\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}{2(R + g_1)}, \tag{3.8}$$

where $g''_{0\text{ext}}$ indices (0ext) and g_1 have the same meaning as at subpoints a, b.

When $k\alpha_b = 180°$ the maximum strains are reached (the plateau zone occurs) at the top point, then $g''_{0\text{ext}} = g''_{0\max}$. The expression (3.8) takes a simple form

$$g''_{0\max} \equiv g''_{0\text{ext}} = \frac{g_1(2R + d_{\text{ext}} + 2g_1)}{2(R + g_1)}. \tag{3.9}$$

d) Simplification of the 3rd order

This simplification is assumed to be a composition of simplifications of the 1st and 2nd orders. Physically, it also means the application of a certain "quasi plane" state of strain. In the case of this simplification, suitable quantities are denoted as $(''')$. Thus, we have that $\varphi'''_1 = \varphi'_1$, $\varphi'''_2 = \varphi''_2 \cong 0$ and $\varphi'''_3 = \varphi_3$.

Let us substitute the strain components to the simplification of the 3rd order related to d_{ext} with the condition of plastic incompressibility ($\varphi'''_1 + \varphi'''_2 + \varphi'''_3 = 0$), after the transformations and assumption that $y_0 \cong 0$ we obtain the following expression:

$$g'''_0 \equiv g'''_{0\text{ext}} = \frac{g_1 \left[2R + d_{\text{ext}}\left(1 - \cos(k\dfrac{\alpha_b}{2})\right)\right]}{2R}, \tag{3.10}$$

here g'''_{ext} indices (0ext) and g_1 have the same meaning as at subpoints a, b, c.

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (the plateau zone occurs). Then $g'''_{0\text{ext}} = g'''_{0\max}$. The expression (3.10) takes the following simple form:

$$g'''_{0\max} \equiv g'''_{0\text{ext}} = \frac{g_1(2R + d_{\text{ext}})}{2R}. \tag{3.11}$$

The expression (3.11) has already been published in the materials of the Polish Technical Inspection Office and applied in the industrial practice, see [24–26].

As it was mentioned above, in all the formulas in this chapter g_1 means a determined and acceptable thickness of the elbow wall at the top point of the elbow which satisfies the criteria of strength and working safety as well as conditions given by the European [13] and of other countries' standards (or any other existing and acceptable criteria and conditions).

Example 1. Let the bending radius be $R = 80$ mm, the external diameter of the tube $d_{\text{ext}} = 44.5$ mm, and the required thickness of the elbow wall at the top point of the elongated layers is $g_1 = 4.5$ mm. Then, from Eqs. (3.3)–(3.5), (3.7), (3.9), and (3.11) and calculations we obtain:

$$g_{0\text{ext}} \cong 5.45 \text{ mm}, \quad g'_{0\text{ext}} \cong 5.5 \text{ mm}, \quad g''_{0\text{ext}} \cong 5.68 \text{ mm}, \quad g'''_{0\text{ext}} \cong 5.75 \text{ mm}.$$

3.1. Initial and boundary conditions in tube bending processes

The introduced expressions for generalised φ_1, φ_2, and φ_3 (see *Greek letters* in Sec. 2) and simplificated models φ'_i, φ''_i and φ'''_i, where $i = 1, 2, 3$, satisfy the required initial and boundary conditions of the tube bending process:

a) when $\alpha = \left(\dfrac{\alpha_b}{2}\right) = 0$ – beginning of the bending process (no bending),

b) when $\alpha = \left(\dfrac{\alpha_b}{2}\right) \neq 0$ – beginning and end of the bending zone,

c) when $\beta = 90°$ – location of neutral surface, then it follows that $\varphi_1 = \varphi'_1 = \varphi''_1 = \varphi'''_1 = 0$, $\varphi_2 = \varphi'_2 = \varphi''_2 = \varphi'''_2 = 0$, $\varphi_3 = \varphi'_3 = \varphi''_3 = \varphi'''_3 = 0$,

d) when $k\alpha_b = 180°$ – the maximum strains (formation of the plateau zone) and $\alpha = \beta = 0°$ – top point of the the bending zone of the elongated layers, then suitable initial thickness reaches the maximum values.

4. Basic relationships for calculations g_0 versus d_{int}

In this chapter, the internal diameter d_{int} is used for calculations and, as previously, only the top points of the elongated layers ($j = 1$) of the elbow are taken into account. Thus, $\alpha = \beta = 0°$ and $\lambda_1 = 1$.

a) General scheme of the strain field

Let us substitute $d_j = d_{\text{int}} + 2g_j$ and $d_{\text{ext}} = d_{\text{int}} + 2g_0$ with the expressions for φ_1, φ_2, φ_3 (see *Greek letters* in Sec. 2) and next introduce them to the condition of plastic incompressibility ($\varphi_1 + \varphi_2 + \varphi_3 = 0$). After transformations and assumption that $y_0 \cong 0$ and $j = 1$ we obtain the following algebraic equation of the 2nd degree related to g_0:

$$a_1 g_0^2 + b_1 g_0 + c_1 = 0, \tag{4.1}$$

where

$$\begin{aligned} a_1 &= 4R, \\ b_1 &= 2R d_{\text{int}}, \\ c_1 &= -g_1 \left(d_{\text{int}} + 2g_1\right)\Big[2R + \left(d_{\text{int}} + 2g_1\right)\Big(1 - \cos\Big(k\frac{\alpha_b}{2}\Big)\Big)\Big]. \end{aligned} \tag{4.2}$$

After solving equation (4.1) we receive

$$g_{0\text{int}} = \frac{-2R d_{\text{int}} + a^*}{8R}, \tag{4.3}$$

where

$$a^* = \sqrt{(2R d_{\text{int}})^2 + 16R\Big[2R + \left(d_{\text{int}} + 2g_1\right)\Big(1 - \cos\Big(k\frac{\alpha_b}{2}\Big)\Big)\left(d_{\text{int}} + 2g_1\right)g_1\Big]},$$

$g_{0\text{int}}$ indices (0int) mean: 0 – initial thickness, int – dependence on the internal diameter, g_1 – required thickness of the elbow wall at the top point of the elongated layers.

Two roots are a solution of Eq. (4.1). Only one of them (4.3) is positive and satisfies real conditions of bending. The other root is negative.

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (formation of the plateau zone), and $g_{0\text{int}} = g_{0\,\max}$. The expressions (4.1) and (4.2) have the following simple form:

$$a_1 g_0^2 + b_1 g_0 + c_1 = 0, \tag{4.4}$$

where

$$\begin{aligned} a_1 &= 4R, \\ b_1 &= 2R d_{\text{int}}, \\ c_1 &= -g_1 \left(d_{\text{int}} + 2g_1\right)[2R + \left(d_{\text{int}} + 2g_1\right)]. \end{aligned} \tag{4.5}$$

Thus

$$g_{0\mathrm{int}} = \frac{-2Rd_{\mathrm{int}} + \sqrt{(2Rd_{\mathrm{int}})^2 + 16R[2R + (d_{\mathrm{int}} + 2g_1)(d_{\mathrm{int}} + 2g_1)g_1]}}{8R}. \tag{4.6}$$

b) Simplification of the 1st order

Let in the numerator of expression φ_1 (similar as in simplifications of the 1st order, chapter 2) the diameter d_j be formally replaced with the diameter d_{ext} (where $d_{\mathrm{ext}} = d_{\mathrm{int}} + 2g_0$) and in the denominator of expressions φ_2, diameter d_{ext} be replaced with ($d_{\mathrm{ext}} = d_{\mathrm{int}} + 2g_0$). Let us substitute the strain components for this simplification expressed versus d_{int} with the condition of plastic incompressibility ($\varphi_1' + \varphi_2' + \varphi_3' = 0$), we obtain as $y_0 \cong 0$ and $j = 1$ the following expression:

$$a_1 {g_0'}^2 + b_1 g_0' + c_1 = 0, \tag{4.7}$$

where

$$\begin{aligned} a_1 &= 4R, \\ b_1 &= 2\left[Rd_{\mathrm{int}} - g_1(d_{\mathrm{int}} + 2g_1)\left(1 - \cos\left(k\frac{\alpha_b}{2}\right)\right)\right], \\ c_1 &= -g_1\,(d_{\mathrm{int}} + 2g_1)(2R + d_{\mathrm{int}})\,. \end{aligned} \tag{4.8}$$

Thus

$$g'_{0\mathrm{int}} = \frac{-b_1 + \sqrt{\Delta}}{2a_1}, \qquad \Delta = b_1^2 - 4a_1c_1, \tag{4.9}$$

where $g_0' \equiv g'_{0\mathrm{int}}$ is the meaning of the indices (0int) and g_1 is the same as at subpoint a.

Two roots are a solution of Eq. (4.7), one of them is positive (4.9) and it satisfies real conditions of bending. The other root is negative.

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (formation of the plateau zone), then $g'_{0\mathrm{int}} = g'_{0\,\mathrm{max}}$. The expressions (4.7) and (4.8) have the following simple form:

$$a_1 {g_0'}^2 + b_1 g_0' + c_1 = 0, \tag{4.10}$$

where

$$\begin{aligned} a_1 &= 4R, \\ b_1 &= 2\,[Rd_{\mathrm{int}} - g_1(d_{\mathrm{int}} + 2g_1)], \\ c_1 &= -g_1\,(d_{\mathrm{int}} + 2g_1)(2R + d_{\mathrm{int}})\,. \end{aligned} \tag{4.11}$$

After solving equation (4.10) we receive

$$g'_{0\mathrm{int}} = \frac{-2\left[Rd_{\mathrm{int}} - g_1(d_{\mathrm{int}} + 2g_1)\right] + b^*}{4R}, \tag{4.12}$$

where

$$b^* = \sqrt{4\left[Rd_{\mathrm{int}} - g_1(d_{\mathrm{int}} + 2g_1)\right] + 4R(2R + d_{\mathrm{int}})(d_{\mathrm{int}} + 2g_1)}.$$

c) Simplification of the 2nd order

Let in the numerator of expression φ_1 the diameter d_i be formally replaced with $d_j = d_{\mathrm{int}} + 2g_j$. In the stretched layers $j = 1$, and then $d_1 = d_{\mathrm{int}} + 2g_1$. Let us assume (similar as in the simplifications of the 2nd order, Sec. 3 point c) that $\varphi''_2 \cong 0$ and $\varphi''_3 = \varphi_3$.

Let us substitute the strain components for the simplification of the 2nd order versus d_{int} with the condition of plastic incompressibility $(\varphi''_1 + \varphi''_2 + \varphi''_3 = 0)$, after transformations when $y_0 \cong 0$, we obtain the following expression:

$$g''_{0\mathrm{int}} = \frac{g_1\left[2R + (d_{\mathrm{int}} + 2g_1)\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}{2R}, \tag{4.13}$$

where $g''_{0\mathrm{int}}$ are the meanings of the indices (0int) and g_1 is the same as at subpoints a, b.

When $k\alpha_b = 180^\circ$ the maximum strains are reached at the top point (formation of the plateau zone), then $g''_{0\mathrm{int}} = g''_{0\max}$. The expression (4.13) has the following simple form:

$$g''_{0\max} = g''_{0\mathrm{int}} = \frac{g_1\left[2R + (d_{\mathrm{int}} + 2g_1)\right]}{2R}. \tag{4.14}$$

d) Simplification of the 3rd order

This simplification is assumed to be a composition of simplifications of the 1st and 2nd orders. Thus, we have that $\varphi'''_1 = \varphi'_1$, $\varphi'''_2 = \varphi''_2 \cong 0$ and $\varphi'''_3 = \varphi_3$ related to d_{int}.

Let us substitute the strain components to the simplification of the 3rd order with the condition of plastic incompressibility $(\varphi'''_1 + \varphi'''_2 + \varphi'''_3 = 0)$, after transformations and assumption that $y_0 \cong 0$ we obtain the following expression:

$$g'''_{0\mathrm{int}} = \frac{\left[2R + d_{\mathrm{int}}\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}{2\left[R - g_1\left(1 - \cos\left(k\dfrac{\alpha_b}{2}\right)\right)\right]}, \tag{4.15}$$

here $g'''_{0\text{int}}$ are the meanings of the indices (0int) and g_1 is the same as at subpoints a, b.

When $k\alpha_b = 180°$ the maximum strains are reached at the top point (formation of the plateau zone), then $g'''_{0\text{int}} = g'''_{0\max}$. The expression (4.15) has the following simple form:

$$g'''_{0\max} = g'''_{0\text{int}} = \frac{g_1\left[2R + (d_{\text{int}} + 2g_1)\right]}{2R}. \tag{4.16}$$

The expressions determined in Secs. 3 and 4 satisfy the required initial and boundary conditions (see Subsec. 3.1) for the tube bending process.

Example 2. Let the bending radius be $R = 80$ mm, internal diameter $d_{\text{int}} = 35.5$ mm, and the required thickness of the elbow wall at the top point of the elongated layers $g_1 = 4.5$ mm. Then, according to Eqs. (4.6), (4.12), (4.14), and (4.16), and after transformations we obtain

$$g_{0\text{int}} \cong 5.50\ \text{mm}, \quad g'_{0\text{int}} \cong 5.55\ \text{mm}, \quad g''_{0\text{int}} \cong 5.75\ \text{mm}, \quad g'''_{0\text{int}} \cong 5.83\ \text{mm}.$$

According to the above example and analysis of the calculation results obtained from expressions (4.1)–(4.16) for the same parameters of bending, internal diameter of the tube subjected to bending, and the required thickness, we can derive the following inequalities:

$$\begin{gathered} g_0(d_{\text{int}}) < g'_0(d_{\text{int}}) < g''_0(d_{\text{int}}) < g'''_0(d_{\text{int}}) \\ \text{or} \qquad g_{0\text{int}} < g'_{0\text{int}} < g''_{0\text{int}} < g'''_{0\text{int}}. \end{gathered} \tag{4.17}$$

5. Conclusions

1. In this paper the generalised and simplified relationships for logarithmic measures of strains during bending thin- and thick-walled metallic tubes at bending machines derived and placed in [8–10] were used. Expressions on measures of components of strains φ_i: longitudinal (along the tube axis), circumferential, and along thickness (radial) presented in papers [9–11] were received using concepts of kinematically admissible fields of plastic deformations, see e.g. [20–23]. The strains can be defined in the main bending plane and each parallel and perpendicular plane, i.e. at all points of the bending zone. The derived relationships describing the measures of logarithmic strains depend on the bending radius R, geometrical dimensions of the tube, the bending angle α_b, angular coordinates α and β, which describe the bending zone in the range of the bending angle $k\alpha_b \in [0°;\ 180°]$,

displacement of the neutral axis y_0, and two technological-material coefficients k and λ_i. The calculations included the value of the coefficient of strain distribution in the elongated layers ($\lambda_1 = 1$).

2. The simplified expressions for strains obtained in [5, 8, 10] determine a higher initial thicknesses of the walls of tubes for bending. It means that they are more safe from the point of view of strength, life, and working safety of the elbows in a pipeline [8, 14–19].
3. However, those simplified analytical schemes of strain fields do not satisfy the conditions of continuity of displacements and strains but are a formal mathematical treatment facilitating calculations [5, 8, 10]. The simplified methods of the 1st, 2nd, and 3rd orders determine very close results, similar to those obtained by the method of a general strain scheme. In three cases, the initial equations are algebraic equations of the 1st degree – calculations can be easily done with the use of a calculator.
4. The introduced simplifications of the 1st, 2nd, and 3rd types are not only formal mathematical operations making expressions and calculations simpler [5, 8, 10]. They have a physical sense as well. In the case of simplification of the 1st type, as the wall thickness becomes thinner, the neutral layer of plastic bending proportionally displaces downwards (in the direction of the centre of rotation). In the considered case this displacement is not big, and we can write that $r_1/R_0 \approx \text{const}$ and $R_0 = R_0(\alpha_b)$. From the tests and analytic calculations it also appears that the radius determining the position of the neutral layer is $R_0 \approx R - (g_0 - g_1)$. In the case of simplification of the 2nd type we can state that circumferential strains are sometimes five times smaller than longitudinal and radial ones (along the axes and thickness). Thus, they can be formally neglected ($\varphi_2 \approx 0$). Simplification of the 3rd type is a composition of simplifications of the 1st and 2nd type, [5, 8, 10].

References

1. Von Karman Th., *Festigkeitprobleme im Maschinenbau*, Encyklopedie der Mathematischen Wissenschaften, B.G. Teubner [Ed.], 4, 1910.
2. Franz W.D., *Das Kalt-Biegen von Rohren*, Springer-Verlag, Berlin, 1961.
3. Grunow O., *Praktisches Rohrbiege*, Springer-Verlag, Berlin, 1985.
4. Korzemski J.W., *Bending of thin-walled pipes* [in Polish], WNT, Warszawa, 1971.
5. Śloderbach Z., Strauchold Sz., *Approximate methods of strain calculations in the tube bending process* [in Polish], Technical Inspection, No. 1, pp. 1–6, Warszawa, 1999.
6. Śloderbach Z., *A model for strain geometry evaluation in pipe bending processes*, Engineering Transactions, **47**, 1, 3–20, 1999.

7. Śloderbach Z., Rechul Z., *Effect of strain hardening and normal anisotropy on allowable values of strain and stress in pipe-bending processes*, Journal of Theoretical and Applied Mechanics, **4**, 38, 843–859, 2000.

8. Śloderbach Z., *Some problems of mechanics in pipeline bending processes* [in Polish], Publishing House of Wrocław University of Technology, Wrocław, 2002.

9. Śloderbach Z., *Genaralized model of strains during bending of metal tubes in bending machines*, Journal of Theoretical and Applied Mechanics, **52**, 4, 1093–1106, 2014.

10. Śloderbach Z., *Derivation of relations and analysis of tube bending processes using discontinuous fields of plastic strains. Part I. Derivation of the geometric-analytic relationships. Part II. Discussion and analysis of the obtained results*, International Journal of Applied Mechanics and Engineering, **20**, 2, 417–435, 2015.

11. Śloderbach Z., Pajak J., *Determination of the required thicknesses of pipe elbows on the basis of expressions resulting from EU-Directive*, Archives of Applied Mechanics, **85**, 5, 629–640, 2015.

12. Tang N.C., *Plastic-deformation analysis in tube bending processes*, International Journal of Pressure Vessels and Piping, **77**, 12, 751–759, 2000.

13. EN 13445-4 (2009), Unfired Pressure Vessels – Part 4: Fabrication, Tube Bents, or earlier: Draft Standart EN UFPV, (1996): Unfired Pressure Vessels – Part 4: Manufacture. CEN/TC54/267 JWGB N277, rev. 5.

14. Seyna F., Ginalski J., *Long-term safe exploatation of steam pipelines* [in Polish], Power Industry, No. 6, 214–217, 1987.

15. Seyna F., Ginalski J., *Evaluation of residual life in steam pipelines* [in Polish], Technical Inspection, No. 3, 103–106, 1989.

16. Życzkowski M., Tran L.B., *Interaction curves corresponding to the decohesive carrying capacity of a cylindrical shell under combined loading*, International Journal of Plasticity, **13**, 6–7, 551–570, 1997.

17. Dobosiewicz J., Wojczyk K., *Life assessment in steam pipeline bends* [in Polish], Power Industry, No. 3, 88–90, 1998.

18. Dzidowski E.S., Strauchold Sz., *Effect of technological factors in pipe bending on damage characteristics and reliability of power pipelines* [in Polish], Scientifics Papers of the Opole Technical University, Series Eletric, No. 46, 119–125, 1998.

19. Dzidowski E.S., *Reliability and solving problems in thin-walled pipe bending processes for the case of great manufacturing tolerance* [in Polish], 4th Conference PIRE 2001, pp. 91–101, 2001.

20. Hill R., *Mathematical theory of plasticity*, Clarendon Press, Oxford, 1985.

21. Olszak W., Perzyna P., Sawczuk A. [Ed.], *Theory of Plasticity* [in Polish], PWN, Warszawa, 1965.

22. Calladaine C.R., *Engineering plasticity*, Pergamon Press, Oxford-New York, 1969.

23. Kaczanow L.M., *Foundations of the theory of plasticity*, North-Holland, Amsterdam, 1971.

24. UDT DT-UT-90/WO-W/06, General requirements. Manufacturing. Bending pipes of pressure elements [in Polish], Job Publishing Company Legal, Warszawa, 1991.

25. UDT CONDITIONS (WUDT-UC-WO-O/02:10), Pressure Installations. General Requirements. Bengding Elbows. Strength Calculations [in Polish], Company Legal, Issue I, Warszawa, 2003.

26. Wick Ch., Benedict J.T., Veilleux R.F., *Tool and manufacturing engineers handbook. A reference book for manufacturing engineers, managers and technicans, Forming*, 4th ed., Society of Manufacturing Engineers, One SME Drive, USA, vol.II, Dearbon, Michigan, 2001.

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