ENGINEERING TRANSACTIONS • *Engng. Trans.* • **63**, 3, 361–376, 2015
Polish Academy of Sciences • Institute of Fundamental Technological Research (IPPT PAN)
National Engineering School of Metz (ENIM) • Poznan University of Technology

Calculations of the Strains and Thicknesses of Pipe Elbows on the Basis of Expressions Resulting from the EU Directive for the Case of Large Strains

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The relations to calculate the maximum value of relative strains, which occur in a process of bending of tubes on benders, in stretched layers of tubes, are presented in this work on the basis of the EU Directive concerning production of pressure equipment. It has been shown that for large deformations that occur during bending of the pipes on knees, logarithmic strain measures (real) and relative strain measures give different values of strain and equal wall thicknesses in the bending zone. Reverse expressions were also derived to calculate the required initial wall thickness of the tube to be bent, in order to obtain the desired wall thickness of a knee after bending.

Key words: EU Directive, required thicknesses of pipe elbows, pipe bending processes, relative and logarithmic measures of deformation, required initial thickness of a bent tube.

1. Introduction

According to the EU Directive on requirements in manufacturing of pressure equipment presented in [1, 2] the measure of relative strain is used for calculating the value of deformation [3–9] and this is a measure of the maximum value of the longitudinal component of the strain state for the case of first-order simplification [10, 11]. In this paper, suitable equations will be derived for the three main components of relative strains, which according to the formula from [1, 2], reach their maximal values. Then, the expression for the minimum value of the wall thickness in knee bending layer will be obtained. During tube bending on knees of pipelines or other piping systems, great deformations of several tens of percent are created [10–14]. Then, the used logarithmic measures (logarithmic measures are frequently used to analyze large and small deformations in engineering practice) of strain will not be equal to the measures of the relative strain. Different values will be obtained from the use of these measures. It will cause significant differences in the calculations, which will be demonstrated in the respective examples.

But when for calculating of the state of deformation the expression contained in the EU Directive is used for the purpose of calculating the minimum (acceptable) wall thickness, according to simplification of the first order [10, 11], this results in thicknesses equal to those respectively calculated in the measures of logarithmic strain. Conversely, the required minimum initial thicknesses of the pipes to be bent will be also equal to those calculated in measures of logarithmic strain.

The EU Directive [1, 2] also contains an empirical expression to calculate the required minimum wall thickness in stretched and compressed layers of bent knees. These expressions are identical to the expressions used in practical applications given in the papers [15, 16], except that instead of calculated thickness, the actual thickness is used. The expressions may be useful and can provide some criteria for the selection of appropriate method (technology) of pipe bending for components of pressure equipment (for example, with using drawing or pushing, with or without a mandrel [17–23]). This applies in particular to the conditions and requirements used to qualify the process of tube bending and to assessment of bent pipes and elbows designed for elements such as water-tubular boilers, see [1, 2]. Further discussion and development of this problem will be presented in Sec. 6.

The aim of this work is to draw attention to the possible consequences that may arise from the use of expressions for calculating the deformation in tube bending process according to the EU Directive. Given relationship for calculating the deformation is expressed in measures of relative deformations, depending on the d_{ext} and refers to the stretched layers. According to the given classification, this expression is equal to the modified expression for the longitudinal component for the first-order simplification given in [10, 11], when at the points of stretched layers the state of maximum deformation is reached, namely when $\alpha = \beta = 0^\circ$ and $k\alpha_b = 180^\circ$, see [10–13]. In the paper [12] the derivation of expressions for generalized scheme of deformations was presented, taking into account the shift of neutral axis y_0 and using the concept of kinematically admissible plastic strain fields. Since the bent tube is a spatial element, three components of strain state: longitudinal, circumferential and radial (in thickness) were derived. The use of kinematically admissible fields of plastic deformation is a simplified, commonly applied method in the technological theory of plasticity. Tube bending is treated as a process dependent on the angle of bending α_b as a parameter. The equations obtained in this way very well describe the experimental results presented in [17], see [12].

The use in this work of the first-order simplification derived in [10] instead of generalized model of strain [12, 13] is due to the fact that the expression in the

EU Directive corresponds to the expression for maximum elongation component of deformation for this simplification. The other two components of deformation (circumferential and radial), which are not included in the EU Directive, are the same as in the generalized model [11].

The examples of calculation results in our work show that the values of relative strain are different than those obtained with the use of logarithmic strain measures (real measures), but the wall knees and the required minimum wall thickness of bent tube are equal. This may cause some problems in assessment of pipe bending technology and estimation of manufacturing of a knee.

On the other hand, logarithmic measures of strain, due to their practical meaning, are most often used in description of strain state of many plastic forming processes, in engineering practice and for tube bending [3–7, 13, 14, 21].

2. Basic assumptions and relations

In the papers [1, 2] the expression to calculate the deformation in the stretched layers in the process of tube bending in accordance with EU Directive is presented

$$O = \frac{d_{\mathrm{ext}}}{2R_m}, \tag{2.1}$$

where O – longitudinal (axially) tensile strain in relative terms, R_m is a mean bending radius, $R_m \in \langle R - y_{0\,\max};\ R\rangle$ (Śloderbach [11]).

The parameters of bending process are presented in Fig. 1. As it results from Fig. 1, Eq. (2.1) takes into consideration the shift of the neutral bending axis by the mean bending radius R_m, since in general $R \neq R_m$. If $y_0 = 0$ then $R = R_m$. There is no unique definition in the Directive for the R_m. This could be for example the arithmetic or geometric mean of bending radius R and $(R - y_{0\,\max})$ or other value in the range $R_m \in \langle R - y_{0\,\max};\ R\rangle$.

In Fig. 1, where d_{ext}, d_{int} are respectively the external and internal diameter of a bent tube, $r_{\mathrm{int}} = d_{\mathrm{int}}/2$.

g_0 – initial thickness of a bent tube, g_i – actual thickness of a bend within the bending zone ($i = 1$ for elongated layers, $i = 2$ for compressed layers), r_{ext} and r_{int} – external and internal radius of a bent pipe, R – bending radius, R_0 – radius of the neutral surface following bending, where $R_0 = R - y_0$, R_i – larger actual radius of a bend associated with longitudinal strain, y_0 – displacement of the neutral surface (axis) with respect to the initial position, α – actual angle of the bending zone determined at the principal bending plane and at planes parallel to it, $\alpha \in \left\langle 0^\circ;\ \frac{\alpha_b}{2}\right\rangle$, where α_b – active bending angle measured over the bending zone, $\alpha_b \in \langle 0^\circ;\ 180^\circ\rangle$, β is actual angle determined at the planes perpendicular to the bending plane, that is $\beta \in \langle 0^\circ,\ 90^\circ\rangle$.

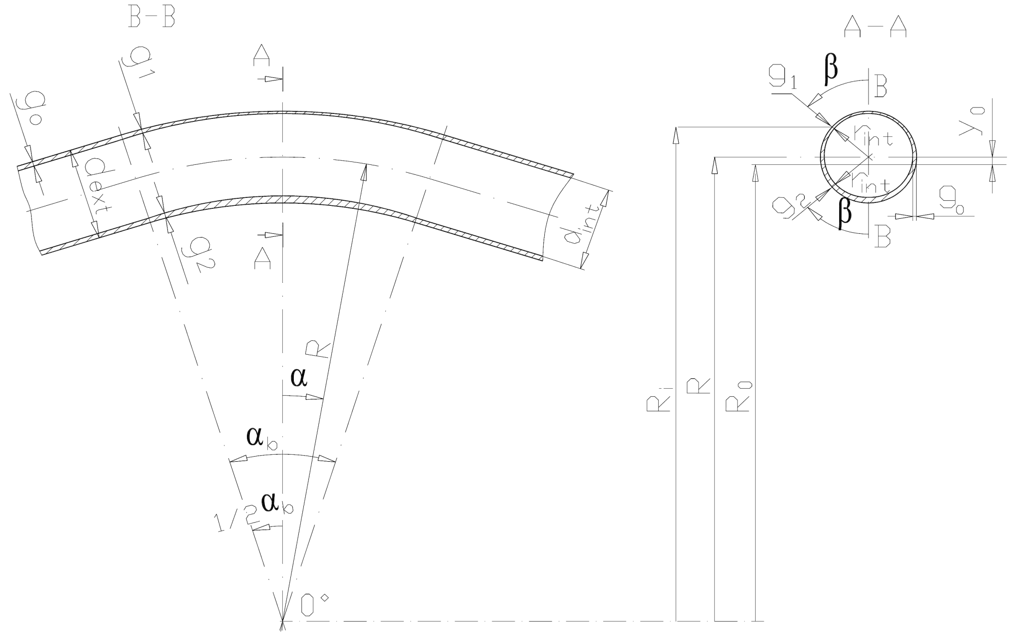

Fig. 1. Geometrical and dimensional quantities pertaining to tube bending processes.

In this paper, the author considers only cold bending of metal tubes of the assumed technological wall thickness $s^* \leq 0.10$ and maximal $d_{\text{ext}} = 160$ mm, (where $s^* = g_0/d_{\text{ext}}$, g_0 and d_{ext} – initial thickness and external diameter of the bent tube, respectively). In the EU Directive [1, 2] the pressure tubes are assumed as thin-walled when $s^*_w \leq 0.05$. When $s^*_w = g_0/d_{\text{int}}$, $d_{\text{int}} = d_{\text{ext}} - 2g_0$, then $s^*_w = s^*/(1 - 2s^*)$.

3. Expression for the displacement of the neutral axis

In [24] the author derived the following approximate expression for displacement of the neutral axis:

$$y_0 = \frac{0.42}{\widetilde{r}} r_m. \tag{3.1}$$

The extended expression, determining displacement of the neutral axis, valid for bending zones, obtained by the author and presented in [12], is

$$y_0 \cong \lambda_0 \frac{0.42}{\widetilde{r}} r_m \left(\cos(k\alpha) - \cos\left(k\frac{\alpha_b}{2}\right)\right), \tag{3.2}$$

where r_m – mean radius of the bent tube, $r_m = r_{\text{int}} + g_0/2$ (Fig. 2), $\widetilde{r}$ – relative radius of bending, $\widetilde{r} = R/d_{\text{ext}}$, k – technological-material coefficient dependent

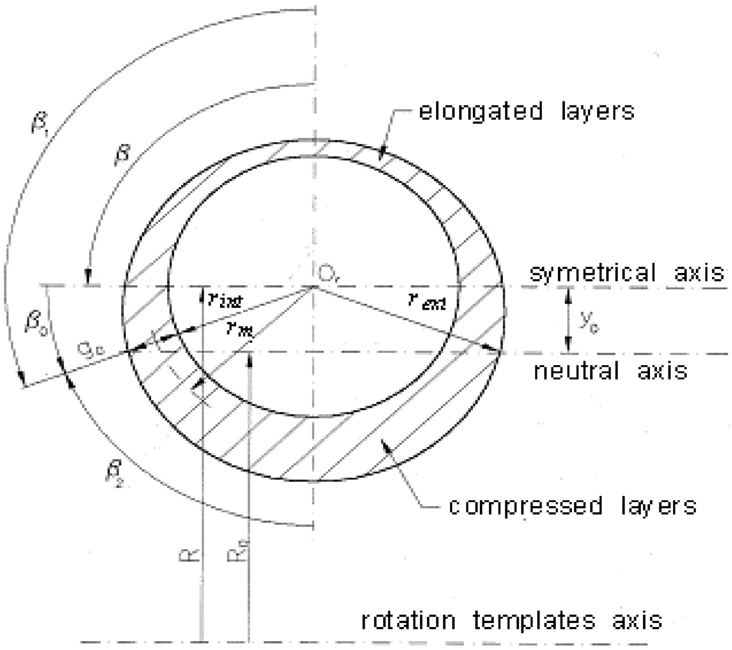

FIG. 2. Schematic of the elbow cross section and its characteristic parameters.

on the bent tube material and the applied bending technology, determining a bending zone range in the bent zone. This coefficient is defined during experiments, theoretically $k \in \langle 1\,;\ \infty)$. It seems that in the case of majority of metallic materials it is sufficient when $k \in \langle 1;\ 6\rangle$. From the recognized tests and calculations it even appears that $k \in \langle 1;\ 3\rangle$, (see, e.g., [10–13, 17, 20]). The case of more ductile, soft, plastic materials bent at elevated temperatures (hot, semi-hot or preheated bending) and bent with a greater radius R, and at a more fitted expanding mandrel (segment with an adjusted external diameter) with rich lubrication of the mandrel and the tube interior, results in coefficient k being lower (tends to the unit, $k \to 1$). Thus, it appears that coefficient k allows to include (indirectly and in part) some effects of friction between the mandrel and the bent tube wall. For elbows bent to 180°, coefficient k expresses a ratio of the bending angle α_0 to a real value of the bending angle α_b, i.e., $k = \alpha_0/\alpha_b$. When the bent angle $\alpha_0 = k\alpha_b = 180°$, for example as in [9–12], then $k = 180°/\alpha_b$. If $\alpha_0 = 90°$, then $2\alpha_0 = k\alpha_b = 180°$, when $\alpha = 60°$, then $3\alpha_0 = k\alpha_b = 180°$, etc., where α_0 – bend angle (the angle by which a template or a former is rotated). In theory for spirals $\alpha_0 \in \langle 0°;\ \infty)$ but for the analyzed method $\alpha_0 \in \langle 0°,\ 180°\rangle$. Obviously, within the bending zone the two angles are equal ($\alpha_0 = \alpha_b$). When the plateau zone was formed, then ($\alpha_0 = \alpha_b + \alpha_{\mathrm{pl}}$), where α_{pl} – angle of a plateau zone [10–12].

Coefficient λ_0 determines characteristic technological-material parameters of the tube bending process such as type of mandrel, tube material, shape of the template and the flatter, strip pressure, clearances, forces of friction between the bent tube and the bending machine device, rigidity of the bending machine, bending method (cold, hot, self-hot, with preheating). From Eqs. (3.1) and (3.2) it appears that for very small bending radii $R \in \langle 0.5 \times d_{\text{ext}};\ 1 \times d_{\text{ext}} \rangle$ and more thin-walled tubes ($s_w^* \ll 0.05$), the maximum displacement of the neutral axis can be equal to $\sim$25% of a diameter value of the tube which is going to be bent. Greater displacements of the neutral axis may be caused by another bending technology because, in the considered ranges $\widetilde{r}$ and s_w^*, tubes are often bent with use of a force which is opposite to the force rotating the template so as to obtain a suitable stress distribution in the cross section. From the extended Eq. (3.2) it also appears that displacement of the neutral axis is influenced not only by the bending radius and the tube thickness (thin-walled) but also by a suitable technology, bending parameters and the tube material. In Eq. (3.2) it is also shown that there are three additional parameters determining displacement of the neutral axis and its position in the bending zone: the bending angle and the angle determining a position of the point in the bending zone, and the coefficient k. Thus, if ($\cos(k\alpha) = 1$ and $\cos\left(k\frac{\alpha_b}{2}\right) = 0$) then $y_0 = y_{0\,\max} \cong \lambda_0 \frac{0.42}{\widetilde{r}} r_m$, see Eq. (3.2).

In Fig. 2 β_1 and β_2 are the angles determined in elongated and compressed layers, $\sin\beta_0 = y_0/r_{\text{ext}} \approx y_0/r_m$. Taking some additional calculations into account, in practice it is recommended to limit the considered bending method to the radii ($R \geq 1.5 \times d_{\text{ext}}$). Pressure tubes which are most often used in pipelines for power industry and other tube installations of power engineering devices are usually in the range ($0.00 < s_w^* \leq 0.125$) or ($0.00 < s^* \leq 0.10$).

The introduced limitations concerning the tube bending parameters cause that, for example, the maximum (for instance for $R = 1.0 \times d_{\text{ext}}$, $s_w^* = 0.03$ and $\lambda_0 = 0.5$) – relative (related to the external diameter of the bent tube) displacement of the neutral axis is $y_0/d_{\text{ext}} \approx 10\%$. However, for some ranges (R and s_w^*), bending technologies and tube materials, relationships which do not include displacement of the neutral axis y_0 can be applied to strain analysis. Thus, they were applied in [13] for precise description of fundamental experiments presented in [17]. The estimated maximum value y_0, can be in practice even lower owing to a suitable selection and set up of tooling of the bending machine, removal of clearances, more plastic material for the bent tube, application of bending at elevated temperatures, increase of rigidity of the bending machine and so on/etc. In the compressed layers, effects resulting from non-unbounded upsetting may be smaller. They are more intense along the perimeter of displacement of the bent tube material to the sides, upward and along the bent axis. This can cause lower values of the coefficient λ_0, see expressions (3.1) and (3.2).

According to the assumptions that the derived expressions for strain components in tube bending processes are identified with plastic strains (it appears that in the angular measure the elastic strains are related to the main bending angle equal to some degrees [10–13]) we conclude that ε_1', ε_2', ε_3' are relative components of plastic deformations for the first-order simplification and φ_1', φ_2', φ_3' are logarithmic components of plastic deformations for the first-order simplification. Since the bent pipe is spatial, a proper analysis of the plastic strain requires the determination of three major components of strain. These components in relative and logarithmic measures according to symbolism accepted in mechanics of solids and according to designations used in papers of Śloderbach [10–13] when ($y_0 \neq 0$ and exchanging R on R_m), after formal transformations, for the case of first-order simplification [10, 11], have the following forms:

$$\begin{aligned}
&\varepsilon_1' = \frac{d_{\text{ext}} \cos \beta_1 \left(\cos (k\alpha) - \cos \left(k \frac{\alpha_b}{2} \right) \right)}{2R_m}, \\
&\varepsilon_2' = \frac{d_{1r}' - d_{\text{ext}}}{d_{\text{ext}}}, \qquad \varepsilon_3' = \frac{g_{1r}' - g_0}{g_0},
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
&\varphi_1' = \ln \frac{2R_m + d_{\text{ext}} \cos \beta_1 \left(\cos (k\alpha) - \cos \left(k \frac{\alpha_b}{2} \right) \right)}{2R_m}, \\
&\varphi_2' = \ln \frac{d_{1l}'}{d_{\text{ext}}}, \qquad \varphi_3' = \ln \frac{g_{1l}'}{g_0},
\end{aligned} \tag{3.4}$$

where d_{1r}' and are the outer minimum diameters of knee in stretched layers determined for the relative and logarithmic measures of strain, respectively, where $d_{1r}' = d_{\text{int}} + 2g_{1r}'$ and $d_{1l}' = d_{\text{int}} + 2g_{1l}'$, g_{1r}, and g_{1l}, are minimum wall thickness of bent knee in tension layers determined for the relative and logarithmic measures of strain, respectively.

When ($\alpha = \beta_1 = 0°$ and $k\alpha_b = 180°$), then Eqs. $(3.3)_1$ and $(3.4)_1$ take their maximum values as

$$\varepsilon_1' = \frac{d_{\text{ext}}}{2R_m}, \qquad \varepsilon_2' = \frac{d_{1r}' - d_{\text{ext}}}{d_{\text{ext}}}, \qquad \varepsilon_3' = \frac{g_{1r}' - g_0}{g_0}, \tag{3.5}$$

and

$$\varphi_1' = \ln \frac{2R_m + d_{\text{ext}}}{2R_m}, \qquad \varphi_2' = \ln \frac{d_{1l}'}{d_{\text{ext}}}, \qquad \varphi_3' = \ln \frac{g_{1l}'}{g_0}. \tag{3.6}$$

Expressions (3.3), (3.4), (3.5) and (3.6) are empirical relationships mutually arising from their engineering definitions measures of strains and adoption of incompressibility condition of plastically deformed materials. Incompressibility condition is valid for majority of metallic materials [25].

The equations for intensity of plastic strain for strain measures (3.3), (3.4), (3.5) and (3.6), for the case of great deformations, are the followings:

$$
\begin{aligned}
\varepsilon'_{(i)} &= \exp\sqrt{\frac{2}{3}\left(\ln^2(1+\varepsilon'_1) + \ln^2(1+\varepsilon'_2) + \ln^2(1+\varepsilon'_3)\right)} - 1,\\
\varphi'_{(i)} &= \sqrt{\frac{2}{3}\left({\varphi'_1}^2 + {\varphi'_2}^2 + {\varphi'_3}^2\right)}.
\end{aligned}
\tag{3.7}
$$

The conditions of plastic incompressibility of the material have the following form:

$$
\begin{aligned}
\varepsilon'_1 + \varepsilon'_2 + \varepsilon'_3 + \varepsilon'_1\varepsilon'_2 + \varepsilon'_1\varepsilon'_3 + \varepsilon'_2\varepsilon'_3 + \varepsilon'_1\varepsilon'_2\varepsilon'_3 &= 0,\\
\varphi'_1 + \varphi'_2 + \varphi'_3 &= 0.
\end{aligned}
\tag{3.8}
$$

Formulas for calculating the coefficients of the maximum thinning of tube wall with respect to its initial thickness are the following [15, 16, 20]:

$$
\begin{aligned}
K_{g'r} &= \frac{g_0 - g'_{1r}}{g_0},\\
K_{g'l} &= \frac{g_0 - g'_{1l}}{g_0}.
\end{aligned}
\tag{3.9}
$$

4. Calculation of the required minimum wall thickness

Substituting components of relative plastic strain (3.5) and (3.6) respectively to incompressibility conditions $(3.8)_1$ and $(3.8)_2$, after transformations we obtain the following expression for the appropriate minimum wall thickness of the knee in the apex points of tension layers: ($\alpha = \beta = 0°$ and $k\alpha_b = 180°$), for the case when ($y_0 \neq 0$ and $R \neq R_m$). Hence,

$$
\begin{aligned}
&\frac{d_{\text{ext}}}{2R_m} + \frac{d'_{1r} - d_{\text{ext}}}{d_{\text{ext}}} + \frac{g'_{1r} - g_0}{g_0} + \frac{d_{\text{ext}}}{2R_m}\cdot\frac{d'_{1r} - d_{\text{ext}}}{d_{\text{ext}}} + \frac{d_{\text{ext}}}{2R_m}\cdot\frac{g'_{1r} - g_0}{g_0}\\
&\qquad + \frac{d'_{1r} - d_{\text{ext}}}{d_{\text{ext}}}\cdot\frac{g'_{1r} - g_0}{g_0} + \frac{d_{\text{ext}}}{2R_m}\frac{d'_{1r} - d_{\text{ext}}}{d_{\text{ext}}}\cdot\frac{g'_{1r} - g_0}{g_0} = 0,\\
&g'_{1l} = -\frac{(d_{\text{ext}} - 2g_0)}{4} + \sqrt{\left(\frac{d_{\text{ext}} - 2g_0}{4}\right)^2 + \frac{R_m d_{\text{ext}} g_0}{2R_m + d_{\text{ext}}}},
\end{aligned}
\tag{4.1}
$$

where g'_{1r} and g'_{1l} are the minimum wall thicknesses of bent knee in tension layers determined for the relative and logarithmic measures of strain, respectively and $d'_{1r} = d_{\text{int}} + 2g'_{1r}$.

Example 1. Let the mean bending radius $R_m = 80$ mm ($R_m \approx 1.8 \times d_{\text{ext}}$), and the dimensions of bent pipe: $\phi 44.5 \times 4.5$ mm. Based on Eqs. (3.5), (3.6) and (4.1) we obtain $\varepsilon_1' \cong 0.2781$, $\varepsilon_2' \cong -0.0378$, $\varepsilon_3' \cong -0.1869$, $\varphi_1' \cong 0.2454$, $\varphi_2' \cong -0.0385$, $\varphi_3' \cong -0.2069$ and $g_{1r}' \cong 3.659$ mm, $g_{1l}' \cong 3.659$ mm.

These are calculated minimum wall thicknesses of the bent knee in stretch layers expressed in real (logarithmic) and relative measures of strain, respectively, obtained on the basis of the EU Directive [1, 2]. The thinning coefficients, corresponding to the above calculated thicknesses, have the following values $K_{g'r} \cong 0.187$, $K_{g'l} \cong 0.187$.

Based on the above results and on the data from Fig. 4 we obtain the following equality:

$$g_{1r}' = g_{1l}'. \tag{4.2}$$

Thus, on the basis of formulas (3.9) it results that

$$K_{g'r} = K_{g'l}. \tag{4.3}$$

For the case of large strains, see, e.g., [3, 10–13, 26–31], such strains occur during bending of tubes in stretched layers (which are valid for each bending radius R or R_m and geometric dimensions of bent tube) and based on equalities (4.2), (4.3) and expressions (3.5), (3.6) and (3.7), the following inequalities in strains occur:

$$\varepsilon_1' > \varphi_1', \qquad |\varepsilon_2'| < |\varphi_2'|, \qquad |\varepsilon_3'| < |\varphi_3'| \tag{4.4}$$

and

$$\varepsilon_{(i)}' > \varphi_{(i)}'. \tag{4.5}$$

The above examples of computational results show that the values of relative strains calculated according to the EU Directive are different than those obtained with the use of logarithmic (real) strain measures. On the other hand, it is known that logarithmic measures of strain, due to their practical properties, are usually used to describe the state of strain in several plastic forming processes, including the pipe bending [10–13, 17–20, 26]. This fact may cause some problems in designing and technology, and also in operating and resistance.

The values of thinning coefficient of the wall thickness, calculated in example 1, in all cases exceed the value of acceptable thinning, which, according to Korzemski [20] for the outer diameter of tube bent with radius $R_m \geq 3 \times d_{\text{ext}}$ is equal to $K_{g'\text{all}} = 0.08$. This results from the fact that bending with the radius $R_m \cong 1.8 \times d_{\text{ext}}$ (as in example 1) is "sharper". For knees made of thin-walled metal on appropriate benders (with rotating template and during upsetting [10–13, 17–24] and bent with the radius $R_m \geq 1.25 \times d_{\text{ext}}$, the thinning of wall can be smaller than 15% ($K_{g'} < 0.15$). From formulas (4.1) it also results that if the bending radius R_m tends to infinity, then the values g'_{1r} and g'_{1l} tend to value g_0, respectively, and that means the lack of bending effect.

5. Determination of the required initial thickness of bent tube

This is the inverse problem to the one considered in the previous section. Now, the required (desired) wall thickness g_1 will be given that satisfies, for example, resistance and construction conditions, technological and operational requirements, requirements of EU-PN (PN – Polish Standards) or regulations of the UDT (Office of the Polish Technical Supervision, see [15, 16]). The initial (starting) required thickness g_0 of wall of the tube to be bent is studied/examined/investigated/researched.

Substituting the components of plastic strains, relative (3.5) and logarithmic (3.6) respectively, to incompressibility condition $(3.8)_1$ and $(3.8)_2$, after transformations we obtain the following expression for appropriate initial (starting) required thickness of the wall of tube to be bent in the stretched layers:

$$\frac{d_{\text{ext}}}{2R_m} + \frac{2(g_1 - g'_{0r})}{d_{\text{ext}}} + \frac{g_1 - g'_{0r}}{g'_{0r}} + \frac{d_{\text{ext}}}{2R_m} \cdot \frac{2(g_1 - g'_{0r})}{d_{\text{ext}}} + \frac{d_{\text{ext}}}{2R_m} \cdot \frac{g_1 - g'_{0r}}{g'_{0r}} + \frac{2(g_1 - g'_{0r})}{d_{\text{ext}}} \cdot \frac{g_1 - g'_{0r}}{g'_{0r}} + \frac{d_{\text{ext}}}{2R_m} \cdot \frac{2(g_1 - g'_{0r})}{d_{\text{ext}}} \cdot \frac{g_1 - g'_{0r}}{g'_{0r}} = 0, \tag{5.1}$$

and

$$g'_{0l} = \frac{g_1\,(d_{\text{ext}} + 2g_1)(2R_m + d_{\text{ext}})}{2\,[R_m\,(d_{\text{ext}} + 2g_1) + d_{\text{ext}} g_1]}, \tag{5.2}$$

where g'_{0r} and g'_{0l} are required initial wall thickness values expressed through relative and logarithmic strains, g_1 is a required (desired) minimum wall thickness of the knee in the apex point of stretched layers.

Example 2. Let the mean bending radius $R_m = 80$ mm ($R_m \approx 1.8 \times d_{\text{ext}}$), bending angle $k\alpha_b = 180°$, outer diameter of tube $d_{\text{ext}} = 44.5$ mm and the required wall thickness of the knee in the apex (middle) point of stretched layers $g_1 = 4.5$ mm. Then, on the basis of formulas (5.1) and (5.2) after calculations we obtain $g'_{0r} = 5.459$ mm and $g'_{0l} = 5.495$ mm.

These are computed required initial wall-thickness values, obtained respectively for logarithmic and relative measures of strain, depending on values of outer diameter of bent tube based on EU Directive.

On the basis of these results and the data from Fig. 5 we obtain the following equality:

$$g'_{0r} = g'_{0l}. \tag{5.3}$$

The method of using the relations derived in this section is the following: for given parameters of bending described with average bending radius R_m and for given geometric dimensions of the tube to be bent ($l \times d_{\mathrm{ext}}$) and for required value of wall thickness of bent knee g_1, required value of initial thickness of the tube to be bent is determined on the basis of expressions (5.1) and (5.2). From formulas (5.1) and (5.2) it also results that when bending radius R_m tends to infinity, then the values of $g'_{0r\,\mathrm{req}}$ and $g'_{0l\,\mathrm{req}}$ tend to g_1 and that means the lack of results of bending.

6. Results and discussion

Changes in the minimum thickness of the bent knee (g'_{1r} and g'_{1l}) depending on the average bending radius R_m, for bent tube with dimensions ϕ44.5×4.5 [mm], with the use of relative and logarithmic measures of strain, respectively, are presented in Fig. 3. As can be visible from the plots, when

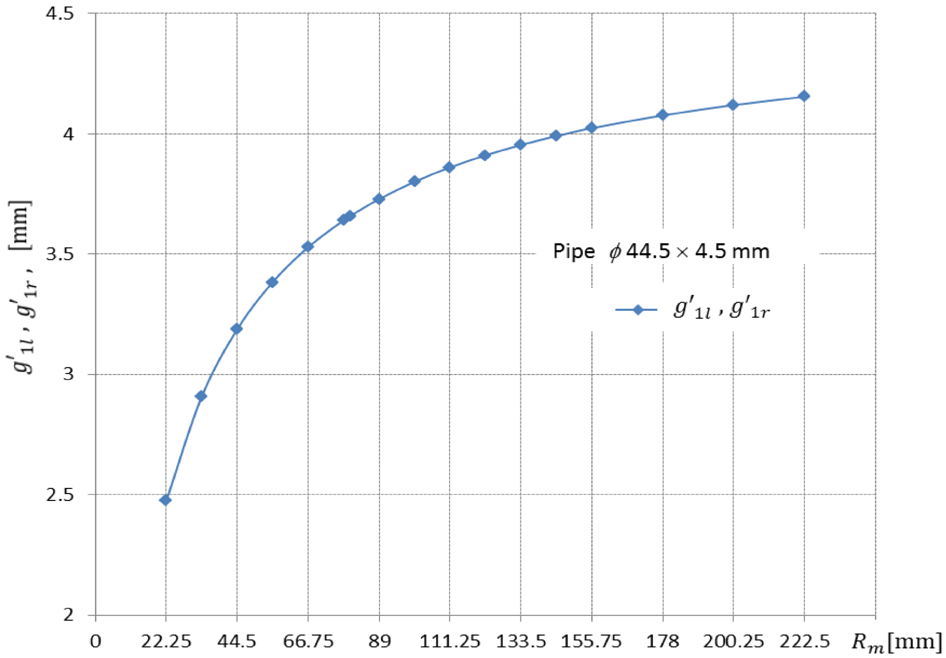

Fig. 3. Variation of the minimum wall thickness values ($g'_{l\,l}$, g'_{lr}) of a bend with bending radius R_m for ϕ44.5×4.5 mm pipe.

bending radius R_m decreases, the differences in thicknesses are increased (differences between in the initial thickness g_0 and actual thickness g'_{1r} or g'_{1l}). The use of relative and logarithmic measures of strain causes the differences in the calculations of intensity of plastic strains, see Fig. 5. It is just the opposite when the bending radius R_m increases. When bending radius R_m strives to infinity then calculated thicknesses (g'_{1r} and g'_{1l}) strive respectively to thickness g_0 ($g_0 = 4.5$ mm) and that means the lack of bending.

Graphs of initial thicknesses of the bent pipes depending on the value of the average bending radius R_m, when the required (desired) thickness of the wall of the bent knee is $g_1 = 4.5$ mm, are presented in Fig. 4.

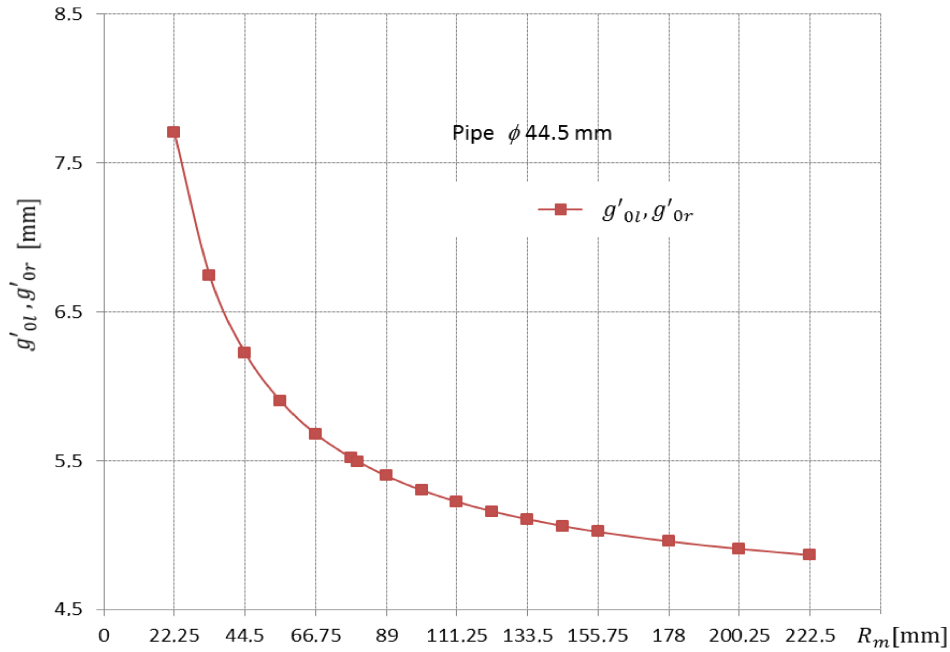

FIG. 4. Variation of the initial wall thickness (g'_{0r} , g'_{0l}) with bending radius R_m, if a required minimum wall thickness of a bend for the pipe $d_{\text{ext}} = 44.5$ mm is $g_1 = 4.5$ mm.

Analogously to the graph presented in Fig. 3, when the bending radius R_m decreases, the differences in thicknesses are increased (differences between the initial thickness g'_{0r} or g'_{0l} and a required thickness g_1) and vice versa. When bending radius R_m increases, strains decrease and both measures of strain become nearly equal. When using the reverse dependencies, derived at this point (which are used to determine the initial thickness of the pipe to be bent), application of the relative and logarithmic strain measures determines that $g'_{0l} = g'_{0r}$. At the end it should be mentioned that equalities (4.2), (4.3), (5.3) and inequalities (4.4), (4.5), derived in this work, will be also met for each bending radius R_m and all geometric dimensions of bent tube and for $(R > y_{0\,\max})$.

It was shown in the papers [10, 12, 13] that the use of logarithmic (real) measures of strain very well describes (even with accuracy of about 1 %) experimental data found in [17–21] and author's own data, in both layers, stretched and compressed. In graphs in Figs. 3 and 4 the results of respective calculations for (g'_{1r}, g'_{1l}) and (g'_{0l}, g'_{0r}) were presented; these obtained with the use of relative measures of strain in stretched layers (according to the EU Directive [1, 2]) with those obtained with the use of logarithmic measures. It means that, in fact computational strains resulting from expression included in the EU Directive were compared with experimental data included in [17, 20] producing accuracy of a few percent. This comparison leads to inequalities (4.4), (4.5) and final conclusion that intensity of plastic strains calculated using the EU Directive are greater than the logarithmic ones, see Fig. 5.

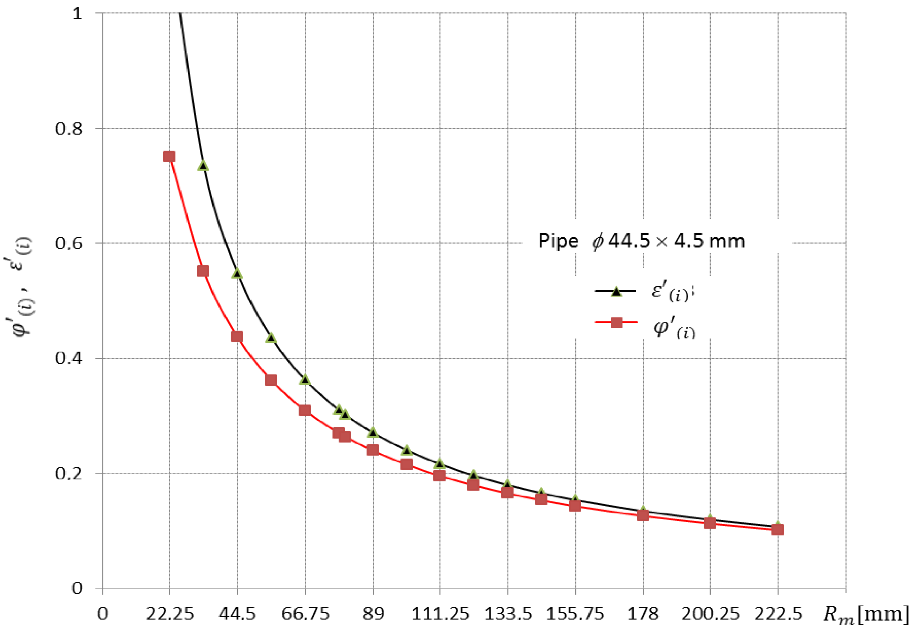

FIG. 5. Variation of intensity of plastic deformation values $(\varphi'_{(i)}, \varepsilon'_{(i)})$ of a bend with bending radius R_m for ϕ44.5 × 4.5 mm pipe.

Selection first-order simplifications of logarithmic and relative measures of strain provides, in addition to the advantages mentioned previously, very good accuracy of description of experimental data. These simplifications also take into account, during real processes of bending tubes on benders, the effect of even lowering with the angle of bending and mutually proportional (due to the effects of thinning and ovality of cross section) of the outer stretched layers and simultaneous shifting "downwards" (in the direction of the center of curvature and rotation), (see, e.g., [10–13, 17–24]) the inert layer of plastic bending. During the bending of tubes on a mandrel with a trackpad and using a profiled strip with an adjustable clamp and with minimum clearances between the tools and the walls of the bent pipe, plastically deformed material of pipe will move more "sideways" and less swell in compression layers, which will cause that actual position of the inert layer will be less moved "downwards", (see, e.g., [10–13, 17–24]). Due to the occurrence of these effects, in real technological processes of bending pipes, we have some physical justification for the use of simplified expressions of first degree. The simplifications in the formulas for the longitudinal (axial) strain thus contain in numerator the value of d_{ext} instead of d_i.

7. Remarks and conclusions

1. The above examples of computational results show that the relative intensity of plastic strains are higher (for small and large strains) than those obtained with the use of logarithmic strain measures (real), but the minimum wall thicknesses are equal. On the other hand, logarithmic measures of strain, due to their practical properties, are most often applied to description of strain state in many forming processes, including tube bending (see, e.g., [10–13, 17–21]). This fact may cause some problems in designing and technology and also in strength and operation.
2. From inequalities (4.4), (4.5) and from graphs in Figs. 3–5 results that different values of components of plastic deformations and intensity of plastic deformations counted in relative and logarithmic measures give the same walls thicknesses in bent pipes.
3. In order to obtain the required (in accordance with the Directive concerning pressure equipment and its production, included in [1]) values of strain and the thickness of bent knees and the initial wall thickness of tubes to be bent for large deformations, we should use the relations (3.5), $(4.1)_1$ and (5.1) derived in this work and $(3.8)_1$. However, for solution for small deformations we should use other relations, see [11].
4. This work can be treated as the first step and the next steps could be development of nomograms and tables for bending tubes of various dimensions ($d_{\mathrm{ext}} \times g_0$) or ($d_{\mathrm{int}} \times g_0$) for various R_m applying expressions (3.5), $(4.1)_1$ and (5.1) valid for large deformations. When initial thicknesses g_{0l} or g_{0r} are calculated depending on d_{ext} or d_{int}, the results are different, see [11]. The UE Directive contains dependence on d_{ext} and not on d_{int}.

5. The solution of the problem of pipe bending on benders in the framework of nonlinear solid mechanics is difficult due to complex relations between strains and deformations, see [26–31] and is open to further studies. We should remember that tube bending is not a free process but depends on bender, its stiffness, shape of working tools, type of applied mandrels, bending parameters such as R_m, tube dimensions ($d_{\text{ext}} \times g_0$), tube material and other factors [1, 2, 9–14, 17–24].

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Received March 16, 2015; accepted April 23, 2015.