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TRANSIENT RESPONSE OF A PLATE-FLUID SYSTEM TO STATIONARY AND MOVING PRESSURE LOADS(1)

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Bernoulli-Euler plate theory, in conjunction with Laplace and Fourier transforms, is utilized to investigate possible wave motion in a fluid half-space which supports an infinite plate. The surface of the plate is subjected to time and space dependent loadings. Two specific loading conditions are considered, one stationary and one moving, and wave front approximations for the pressure disturbance in the fluid are obtained. Limitations on previous steady-state analysis are pointed out. Results are compared with those obtained in the absence of the plate. Within the framework of this theory, the presence of the plate weakens the pressure wave front over that in the plane fluid half-space.

1. Introduction

Interest in hardening underground structures to withstand surface shocks has led to studies of the dynamic response of plates in contact with a supporting fluid to transient and moving loads [1, 2, 3]. Of principal concern in these analyses is the nature of the pressure wave which is transmitted through the plate into the fluid. The case of a supporting fluid gives the main features of the solution for a general elastic foundation, and is simpler to treat than the general case. Limitations on the analysis by assuming an acoustic fluid rather than an elastic media have been detailed in [3].

Previous investigations of this plate-fluid interaction problem for conditions of a moving load have been based upon a pseudo steady state assumption. This assumes that the moving load has been travelling with a constant velocity for sufficiently long time that, to an observer moving along with the load, the material response is independent of time. As shown in this analysis, this steady state assumption suppresses all but one of the possible wave fronts, and indeed suppresses the major contribution to the pressure wave propagating into the fluid. Also, two very significant limitations on the steady state analysis are, firstly, that it is not possible to define how long, if at all, it takes to achieve the steady-state motion, and, secondly, it prohibits an investigation of the physically significant problem of a moving load whose amplitude at the load front is changing with time.

To circumvent the limitations associated with the moving load steady-state solution, this paper considers the full-field behaviour of the plate-fluid system. The Bernoulli-Euler plate theory is assumed to govern the plate behaviour, while the fluid

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is assumed to be an acoustic medium. This assumption restricts the range of validity of the present analysis to loading velocities less than the plate seismic velocity. For the application considered here, the plate seismic velocity will be much greater than the acoustic velocity of the fluid, so some interesting cases of loading velocities super-seismic to the fluid are within the scope of this analysis. The geometry is two--dimensional and the applied pressure is defined in terms of one space variable and time. By the application of a double Laplace-Fourier transform to the field equations and boundary conditions, and making use of a change of variable employed previously by NORWOOD [4], an expresssion for the pressure in the acoustic medium is obtained. The complexity of the result prohibits an exact transform inversion for the pressure for all time, however, using the techniques of [4 – 7], one readily shows the existence and the behaviour of pressure wave fronts propagating into the fluid. Within the spirit of this investigation, RUSSELL and HERRMANN [8] obtain the complete solution to the problem of a submerged infinite cylindrical shell subjected to a moving load. They also utilize the Bernoulli-Euler theory for the sheel, but this analysis is restricted by their assumption of an incompressible fluid (infinite wave speed).

As examples of this analysis, two specific loading conditions are considered. The first is a stationary load suddenly applied over half of the plate surface, and the second is an expanding load spreading out from the origin in both directions with a velocity u and exhibiting exponential decay with respect to both time and distance. In the second problem, an additional wave occurs if u is greater than the fluid velocity, and is the only wave obtained in the steady-state moving load analysis [1]. It is found that, for both of these problems, the strongest pressure wave front arises from a pole of the double transform of the loading function. Analogously, the weakest pressure wave front arises from the zeros of a denominator representing the plate-fluid coupling. This latter front is parallel to the plate and moving in a direction normal to the plate surface. This result is in accordance with the well-known parabolic nature of the Bernoulli-Euler plate equation which provides for an instantaneous disturbance everywhere along the plate surface.

Results are compared with those obtained in the absence of the plate for both loading conditions considered. The presence of the plate greatly reduces the sharpness of the propagating wave fronts in the fluid. For the stationary load, the discontinuity in pressure is reduced by the plate to a discontinuity in the pressure rate at the wave front. A similar reduction occurs in the moving load case.

2. STATEMENT OF THE PROBLEM AND GENERAL SOLUTION

In a rectangular coordinate system consider a plate supported by a fluid half--space. A pressure load is suddenly applied, at time $t=0$, to the plate, as shown in Fig. 1. The governing equation for the fluid is

$$c^2 \nabla^2 \varphi = \varphi_{,tt}, \quad c^2 = K/\rho, \tag{2.1}$$

where t is the time, K is the bulk modulus, ρ is the fluid density, and φ is the displacement potential function. This potential function is related to the fluid pressure p and displacement $\mathbf{u}$ by

(2.2) $$p=-\rho\varphi_{,tt}, \quad \mathbf{u}=\nabla\varphi.$$

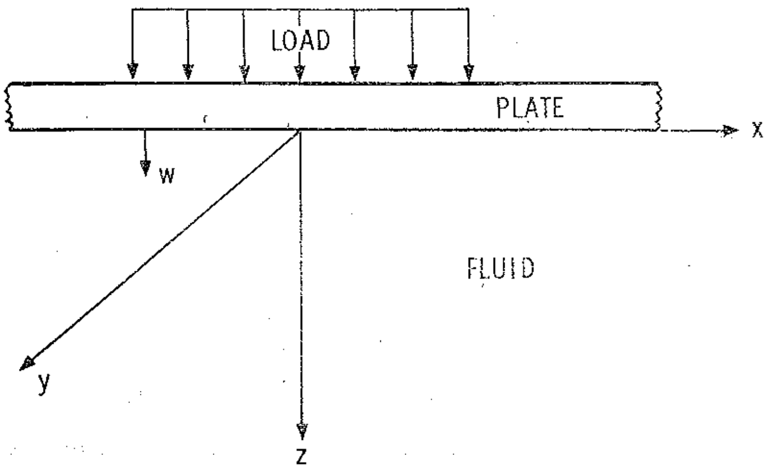

FIG. 1. Problem of a pressure loaded plate supported by a fluid.

The plate displacement w satisfies the Bernoulli-Euler equation

(2.3) $$EIw_{,xxxx}+\mu w_{,tt}=q-p_0,$$

where EI is the flexural rigidity of the plate ([2]), μ is density per unit area, q is the applied pressure, and p_0 is the pressure at the fluid-plate interface. At the fluid-plate interface, the following conditions must be satisfied:

(2.4) $$p=p_0=-\rho\varphi_{,tt}, \quad w=\frac{\partial\varphi}{\partial z} \quad \text{at} \quad z=0.$$

The potential φ is required to vanish for large z, and zero initial conditions are assumed for φ and w.

Formal solution. The Laplace and Fourier transforms to be used here are defined, respectively, by the equations

(2.5) $$\mathscr{L}\{v\}=\hat{v}(x,z,s)=\int_0^\infty v(x,z,t)e^{-st}\,dt,$$

(2.6) $$v^*(k,z,t)=\int_{-\infty}^{\infty} v(x,z,t)\,e^{-ikx}\,dx.$$

([2]) For plates, $EI=Eh^3/12(1-\nu^2)$, where h is the plate thickness, E is the elastic modulus and ν is Poisson's ratio.

The assumption of zero initial conditions leads to the double transforms

(2.7) $$\frac{d^2 \hat{\varphi}^*}{dz^2} - (k^2 + a^2 s^2)\hat{\varphi}^* = 0\,, \qquad ac = 1\,,$$

(2.8) $$(EIk^4 + \mu s^2)\,\hat{w}^* = \hat{q}^* - \hat{p}_0^*\,.$$

The solution for $\hat{\varphi}^*$ which vanishes for large z is given by

(2.9) $$\hat{\varphi}^*(k, z, s) = A(k, s)\,e^{-\eta(k,s)z}, \qquad \eta(k, s) = (k^2 + a^2 s^2)^{\frac{1}{2}},$$

where the required branch is $\operatorname{Re}\eta(k, s) > 0$. By the first of conditions (2.4), one has that

(2.10) $$\hat{p}_0^* = -\rho s^2 A(k, s)\,,$$

and the second of conditions (2.4) yields the relation

(2.11) $$\hat{w}^* = -\eta(k, s)\,A(k, s)\,.$$

The substitution of (2.10) and (2.11) into (2.8) leads to

(2.12) $$[(EIk^4 + \mu s^2)\,\eta(k, s) + \rho s^2]\,A(k, s) = -\hat{q}^*(k, s)\,.$$

From the equations (2.9) and (2.12), and the inversion expression for the Fourier transform, it follows that the Laplace transform of φ may be written as

(2.13) $$\hat{\varphi}(x, z, s) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{q}^*(k, s)\,\frac{e^{-\eta(k,s)z + ikx}\,dk}{(EIk^4 + \mu s^2)\,\eta(k, s) + \rho s^2}\,.$$

Following NORWOOD [4], the change of variable $ik = -s\sigma$ is now introduced in (2.13) to obtain

(2.14) $$\hat{\varphi}(x, z, s) = \frac{i}{2\pi}\int_{-i\infty}^{i\infty} \hat{q}^*(is\sigma, s)\,\frac{e^{-s[\eta(i\sigma, 1)z + \sigma x]}}{D(s, \sigma)}\,d\sigma\,,$$

(2.15) $$D(s, \sigma) = (EIs^4\sigma^4 + \mu s^2)\,\eta(i\sigma, 1) + \rho s\,.$$

Before introducing a specific form of $q(x, t)$, it seems appropriate to consider the Eq. (2.14) for $z > 0$ in its present form(3). From the work of NORWOOD [4, 5], it follows that the wave character of φ is embodied in the exponent $\eta(i\sigma, 1)\,z + \sigma x$, while the information about the load is found in $\hat{q}^*(is\sigma, s)$, and the denominator $D(s, \sigma)$ determines the effect of the plate on the solution for the fluid. Thus, in accordance with [4, 5], the cylindrical wave front information at $t^2 = a^2(x^2 + z^2)$ will result from the saddle point of (2.14), and the information at other possible wave fronts will be deduced from the poles of $\hat{q}^*(is\sigma, s)$ and the zeros of $D(s, \sigma)$. In the absence of the plate, it is easy to show that the solution is given by the Eq. (2.14) by setting μ and EI equal to zero.

For simplicity, consider the region $x > 0$. In this region the integration path of Fig. 2 will be used, where C_1 and C_2 are arcs of a circle, and Path I will be selected

(3) The solution scheme for $z = 0$ is easier and will be outlined at the end of this section.

as a path of steepest descent. It is easy to show by Jordan's lemma that, for the $\hat{q}^*$ under consideration here, the arcs C_1 and C_2 give zero contribution. The saddle point of (2.14) is located at

(2.16) $$\sigma_s = xa(x^2+z^2)^{-\frac{1}{2}},$$

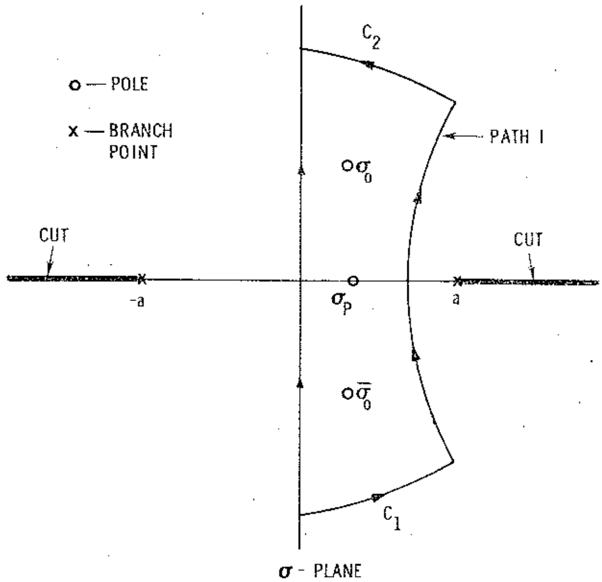

FIG. 2. Integration Paths in the σ-plane.

and the path of steepest descent through this saddle point is given by

(2.17) $$\sigma = \frac{x\lambda}{x^2+z^2} \pm \frac{iz}{x^2+z^2}(\lambda^2-t_0^2)^{\frac{1}{2}}, \qquad t_0 = a(x^2+z^2)^{\frac{1}{2}} \leqslant \lambda < \infty,$$

where the + sign (−sign) holds for the upper (lower) portion of Path I. By residue theory it follows that

(2.18) $$\hat{\phi}(x,z,s) = \hat{F}_1(x,z,s) + \hat{F}_2(x,z,s) + \hat{F}_3(x,z,s),$$

(2.19) $$\hat{F}_1(x,z,s) = \Sigma \text{ Residues at the poles of } \hat{q}^*(is\sigma,s),$$

(2.20) $$\hat{F}_2(x,z,s) = \frac{i}{2\pi}\int_{\text{Path I}} \hat{q}^*(is\sigma,s)\frac{e^{-s[\eta(i\sigma,1)z+\sigma x]}d\sigma}{D(s,\sigma)},$$

(2.21) $$\hat{F}_3(x,z,s) = \Sigma \text{ Residues at the zeros of } D(s,\sigma),$$

where it is understood that only residues from points lying in the closed contour of Fig. 2 are to be included.

The expressions (2.18) – (2.21) are not in general readily invertible in the Laplace transform variable s, so consideration will be directed to the investigation of possible discontinuities. In accordance with the Tauberian thoerem for hyperbolic equations

[6], the behaviour of equations (2.18) – (2.21) as $s \to \infty$ gives the jumps at wave fronts and also the approximate behavior of the disturbance near the arrival time. These wave front asymptotics will be found by combining the techniques utilized in references [4 – 7].

The first step in deducing the behavior of $\hat{F}_3$ for large values of s is to find approximately the zeros of $D(s, \sigma)$. From (2.15) it follows that

$$D(s,\sigma) = \overline{D(s,\bar{\sigma})} = D(s,-\sigma) = \overline{D(s,-\bar{\sigma})}, \tag{2.22}$$

where the bar denotes the complex conjugate of the quantity. Therefore, if D vanishes at σ_0, then it also vanishes at $-\sigma_0$, $\bar{\sigma}_0$, and $-\bar{\sigma}_0$. Also, by the branch cut selected for $\eta(i\sigma, 1)$, there are no purely real or imaginary zeros of D. The function

$$\mathscr{D}(s,\sigma) = D(s,\sigma) - 2ps \tag{2.23}$$

satisfies relations equivalent to (2.22), but admits the possibility of two purely imaginary or purely real roots between $\sigma = a$ and $\sigma = -a$. This follows from the fact that, for σ on the imaginary axis and also for $-a<\sigma<a$, the function $\eta(i\sigma, 1)$ is real and positive, approaching zero as σ approaches $\pm a$. Thus, as σ varies along the imaginary axis and between $-a$ and a,

$$0 < (EIs^4\sigma^4 + \mu s^2)\,\eta(i\sigma,1) < \infty, \tag{2.24}$$

and, by the evenness on σ, there are two points $\pm\sigma$ at which the quantity in the inequality equals ρs, so that $\mathscr{D}(s, \sigma)$ vanishes at these points. The polynomial in σ

$$\mathscr{D}(s,\sigma)\,D(s,\sigma) = 0 \tag{2.25}$$

has ten roots. In the approximation as s approaches infinity, $D(s, \sigma)$ has a complex zero at

$$\sigma_0(\rho) = \frac{b}{s^{1/2}}\,e^{i\pi/4}\left\{1 + \left(\frac{\rho}{4\mu a}\right)\frac{1}{s} + \dots\right\}, \qquad b^4 = \frac{\mu}{EI} \tag{2.26}$$

and, by property (2.22), it also has zeros at $-\sigma_0(\rho)$, $\overline{\sigma_0(\rho)}$, and $\overline{-\sigma_0(\rho)}$. $\mathscr{D}(s,\sigma)$ has zeros at $\sigma_0(-\rho)$, $\overline{\sigma_0(-\rho)}$, $-\sigma_0(-\rho)$, and $\overline{-\sigma_0(-\rho)}$. By the foregoing argument, $\mathscr{D}(s, \sigma)$ has two more zeros either between $-a$ and $+a$ or along the imaginary axis. This accounts for all the zeros of (2.25). Thus, it has been shown that $D(s, \sigma)$ has four zeros in the σ-plane which are deduced, approximately, from the Eq. (2.26). It follows that $\hat{F}_3$ may be written as

$$\hat{F}_3(x,z,s) = \{\text{Residue at } \sigma_0(\rho) + \text{Residue at } \overline{\sigma_0(\rho)}\}. \tag{2.27}$$

To simplify the work in subsequent sections, one now assumes $\hat{q}^*(is\sigma, s)$ to be a ratio of polynomials in σ and s^4. By this assumption one can now proceed to the details of the integration along the path of steepest descent. Let σ_1 be defined by

$$\sigma_1(\lambda) = \frac{x\lambda}{x^2+z^2} + \frac{iz}{x^2+z^2}(\lambda^2 - t_0^2)^{\frac{1}{2}}, \tag{2.28}$$

(4) If $q^*(is\sigma, s)$ is not of this form, one might have to redefine Path I.

and denote by $f(\sigma)\exp(-s\lambda)$ the integrand of (2.20). Then the integration along Path I gives

$$\hat{F}_2(x,z,s)=\frac{i}{2\pi}\left\{\int_{\infty}^{t_0} f\left(\overline{\sigma_1(\lambda)}\right)\overline{\frac{d\sigma_1(\lambda)}{d\lambda}}\,e^{-s\lambda}d\lambda+\int_{t_0}^{\infty} f\left(\sigma_1(\lambda)\right)\frac{d\sigma_1(\lambda)}{d\lambda}\,e^{-s\lambda}d\lambda\right\}, \tag{2.29}$$

which is reduces to

$$\hat{F}_2(x,z,s)=-\frac{\mathrm{Im}}{\pi}\int_{t_0}^{\infty} f\left(\sigma_1(\lambda)\right)\frac{d\sigma_1(\lambda)}{d\lambda}\,e^{-s\lambda}d\lambda, \tag{2.30}$$

$$\frac{d\sigma_1(\lambda)}{d\lambda}=\frac{i\eta(i\sigma_1,1)}{(\lambda^2-t_0^2)^{\frac{1}{2}}},\qquad \eta(i\sigma_1,1)=\frac{z\lambda}{x^2+z^2}-\frac{ix}{x^2+z^2}(\lambda^2-t_0^2)^{\frac{1}{2}}. \tag{2.31}$$

Introduce the change of variables $\tau=\lambda-t_0$ into (2.30) to obtain

$$\hat{F}_2(x,z,s)=-e^{-st_0}\frac{\mathrm{Im}}{\pi}\int_{0}^{\infty} f\left(\sigma_1(\tau+t_0)\right)\frac{d\sigma_1(\tau+t_0)}{d\tau}\,e^{-s\tau}d\tau. \tag{2.32}$$

To obtain the approximation for large s, one takes the contribution about the saddle point to obtain as the first term

$$\hat{F}_2(x,z,s)\approx -e^{-st_0}\frac{\mathrm{Im}}{\pi}\int_{0}^{\infty} f\left(\sigma_1(t_0)\right)\frac{i\eta\left(i\sigma_1(t_0),1\right)}{\tau^{\frac{1}{2}}(2t_0)^{\frac{1}{2}}}\,e^{-s\tau}d\tau. \tag{2.33}$$

At the saddle point, $\eta(i\sigma,1)$ is real and therefore $D(s,\sigma)$ is also real. By the definition of $\hat{q}^*(is\sigma,s)$ via (2.5) and (2.6), $\hat{q}^*(is\sigma,s)$ is also real at the saddle point. Consequently, one can write (2.33) as

$$\hat{F}_2(x,z,s)=-\frac{e^{-st_0}}{\pi}\frac{f\left(\sigma_1(t_0)\right)}{(2t_0)^{\frac{1}{2}}}\eta\left(i\sigma_1(t_0),1\right)\int_{0}^{\infty}\frac{e^{-s\tau}}{\tau^{\frac{1}{2}}}d\tau= \tag{2.34}$$

$$=-\frac{e^{-st_0}}{\pi}\frac{f\left(\sigma_1(t_0)\right)}{(2t_0)^{\frac{1}{2}}}\eta\left(i\sigma_1(t_0),1\right)\left(\frac{\pi}{s}\right)^{\frac{1}{2}}.$$

In this equation, one now recalls equation (2.5) to deduce that $\exp(-st_0)$ is the Laplace transform of $\delta(t-t_0)$, so that (2.34) represents a cylindrical wave front at $t=t_0=a(x^2+z^2)^{\frac{1}{2}}$.

For $z=0$, the analysis proceeds by setting $z=0$ in the Eq. (2.14)–(2.21), but now Path I collapses to an integration path along the branch cut from a delta to ∞.

3. Load over half of the plate

Assume that the applied pressure is given by

$$q(x,t)=H(t)H(-x)e^{\alpha x},\qquad \alpha>0, \tag{3.1}$$

as shown in Fig. 3; when $\alpha=0$ this represents a uniform pressure over half of the plate. The Eq. (3.1) leads to

$$\hat{q}^*(is\sigma, s)=\frac{1}{s}\,\frac{1}{s\sigma+\alpha}, \tag{3.2}$$

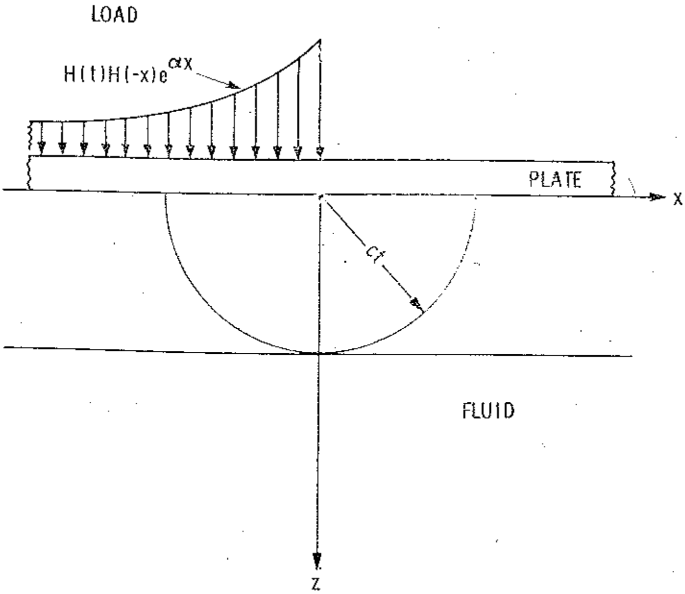

FIG. 3. Problem of a stationary load over half of the plate.

which has a pole at $\sigma=-\alpha/s$. For $x>0$ the contour in (2.14) is closed to the right and one finds that only $\hat{F}_2$ and $\hat{F}_3$ contribute. For large s, at $\sigma=\sigma_1(t_0)$,

$$\hat{q}^*(is\sigma_1, s)\approx\frac{1}{\sigma_1 s^2}, \quad D(s,\sigma_1)\approx EIs^4\sigma_1^4\eta(i\sigma_1, 1). \tag{3.3}$$

Thus the Eq. (2.34) becomes

$$\hat{F}_2(x,z,s)\approx-Ae^{-st_0}s^{-13/2}, \tag{3.4}$$

$$A=[(2t_0\pi)^{\frac{1}{2}}EI\sigma_1^5(t_0)]^{-1}. \tag{3.5}$$

Therefore, for $T_0=t-t_0$ sufficiently small, it follows that

$$F_2(x,z,t)\approx-AH(T_0)T_0^{11/2}\Gamma^{-1}(13/2), \tag{3.6}$$

where Γ is the gamma function. The Eq. (3.6) represents a wave emanating from the point ($x=0$, $z=0$) and travelling into the fluid with the fluid velocity c, as shown in Fig. 3.

From (2.26), (2.27), and (3.2), it follows that

$$\hat{F}_3(x,z,s)=2\,\mathrm{Re}\big(\text{Residue at }\sigma_0(\rho)\big). \tag{3.7}$$

For large values of s, this equation becomes

$$\hat{F}_3(x,z,s) \approx -\frac{e^{-saz}}{2a\mu s^4} \operatorname{Re} e^{ib^2 z/2a} e^{-bxs^{1/2} e^{i\pi/4}}. \tag{3.8}$$

In the case of F_2, one was able to show in the Eq. (3.6) the power of T_0 which holds at the wave front $t-t_0=0$. In the case of F_3 this is not possible, and one must then provide some corresponding information about F_3. This can be done by identifying $\exp(-saz)=\mathscr{L}\{\delta(t-az)\}$, and noting the decaying exponential in the last factor: $\exp(-bxs^{1/2}\,2^{-1/2})$. The Eq. (3.8), for T sufficiently small, leads to

$$F_3(x,z,t) \approx -\frac{H(T)}{2a\mu} G(x,z,T), \quad T=t-az, \tag{3.9}$$

where

$$\hat{G}(x,z,s) = \operatorname{Re} e^{ib^2 z/2a} e^{-bxs^{1/2} e^{i\pi/4}} s^{-4}. \tag{3.10}$$

Using the relations

$$\mathscr{L}\{f^{(n+1)}(t)\} = s\mathscr{L}\{f^{(n)}(t)\} - f^{(n)}(+0), \tag{3.11}$$

$$\lim_{s\to\infty} s\hat{f}(s) = f(0), \tag{3.12}$$

repeatedly in the Eq. (3.10), one concludes that $G(x,z,T)$ and all of its partial derivatives with respect to T vanish at $T=0$. By these results one concludes that (3.9) represents a wave which is infinitely smooth at the front. This result is in accordance with the diffusive behavior of the plate equations, the effect of which is inherent in $D(s,\sigma)$ as pointed out in the paragraph following the Eq. (2.15). This wave motion parallel to the surface of the plate and travelling with the fluid velocity c results from the Bernoulli-Euler plate equation which provides for any disturbance to be felt instantaneously everywhere in the plate.

For $x<0$, one closes the contour in (2.14) to the left along the mirror images of C_1, C_2, and Path I. By residue theory, one obtains

$$\hat{\varphi}(x,z,s) = \hat{f}_1(x,z,s) + \hat{f}_2(x,z,s) + \hat{f}_3(x,z,s), \tag{3.13}$$

$$\hat{f}_2(x,z,s) = \hat{F}_2(x,z,s), \tag{3.14}$$

$$\hat{f}_3(x,z,s) = -\hat{F}_3(-x,z,s), \tag{3.15}$$

and $\hat{f}_1(x,z,s)$ is the contribution from the pole of $\hat{q}^*(is\sigma,s)$ at $\sigma=-\alpha/s$. The behavior of $f_2(x,z,t)$ and $f_3(x,z,t)$ follows from (3.6) and (3.9). For $\hat{f}_1(x,z,s)$, one has

$$\hat{f}_1(x,z,s) = -\frac{e^{-s[\eta(i\alpha/s,\,1)z-\alpha x/s]}}{s^2 D(s,\alpha/s)}. \tag{3.16}$$

For large s, this reduces to

$$\hat{f}_1(x,z,s) \approx -\frac{e^{\alpha x} e^{-saz}}{\mu a s^4}, \tag{3.17}$$

leading to

$$f_1(x,z,t)\approx -\frac{e^{\alpha x}}{\mu a}H(T)\frac{T^3}{3!}. \tag{3.18}$$

By the first of relations (2.2), the pressure may be obtained from

$$\hat{p}(x,z,s)=\hat{P}_1(x,z,s)+\hat{P}_2(x,z,s)+\hat{P}_3(x,z,s), \tag{3.19}$$

$$\hat{P}_i(x,z,s)=-\rho s^2 \hat{f}_1(x,z,s). \tag{3.20}$$

Thus, from (3.4) and (3.8), it follows that for $x>0$,

$$P_2(x,z,t)\approx \rho A H(t_0)T_0^{7/2}\Gamma^{-1}(9/2), \tag{3.21}$$

$$P_3(x,z,t)\approx \rho\frac{H(T)}{2a\mu}\frac{\partial^2 G(x,z,T)}{\partial T^2}. \tag{3.22}$$

For $x<0$, (3.14) and (3.15) yield corresponding relations for the pressure contributions. In addition, one has the contribution from the Eq. (3.17) which gives

$$p_1(x,y,t)\approx \rho\frac{e^{\alpha x}H(T)T}{\mu a}. \tag{3.23}$$

In the absence of the plate, the solution for the potential φ may be obtained from the Eq. (2.14) by setting μ and EI equal to zero. In the case when α is identically zero, the technique used to generate the Eqs. (2.14) – (3.23) would give the exact solution. However, the present purpose is to exhibit the asymptotic form for non-zero α. This form is given by

$$p(x,z,t)\approx e^{\alpha x}H(-x)H(T)+H(T_0)\frac{z}{\pi x}\left(\frac{2T_0}{t_0}\right)^{\frac{1}{2}}, \qquad x\neq 0. \tag{3.24}$$

Comparing [(3.21) – (3.23) to (3.24)], one concludes that the presence of the plate has prevented the load discontinuity from propagating into the fluid. Also, the presence of the plate has created a wave front at $z=ct$ for all x.

4. Moving pressure load

Consider a pressure load spreading over the plate, and assume that the intensity decreases with time and distance from the origin. The specific form to be considered is (see Fig. 4)

$$q(x,t)=[H(x)H(t-vx)e^{-\alpha x}+H(-x)H(t+vx)e^{\alpha x}]e^{-\beta t}, \tag{4.1}$$

where $vu=1$ and u is the travelling speed. Taking the Laplace transform on time and the Fourier transform on x, one obtains the expression

$$\hat{q}^*(is\sigma,s)=\frac{1}{s+\beta}\left[\frac{1}{s\sigma+(\beta+s)v+\alpha}-\frac{1}{s\sigma-(\beta+s)v-\alpha}\right], \tag{4.2}$$

with poles located at $\pm\sigma_p$, $s\sigma_p=(\beta+s)\,\nu+\alpha$.

By the symmetry of the problem, it will suffice to consider the region $x>0$, so that the Eqs. (2.14) – (2.34) are directly applicable. In this region there is a pole contribution if the inequality $\sigma_p<\sigma_s$ is satisfied (σ_s is defined by (2.16)). This inequality can be satisfied only if $u>c$, and it implies that

$$x > cz\,(u^2-c^2)^{-\frac{1}{2}} \cdot \tag{4.3}$$

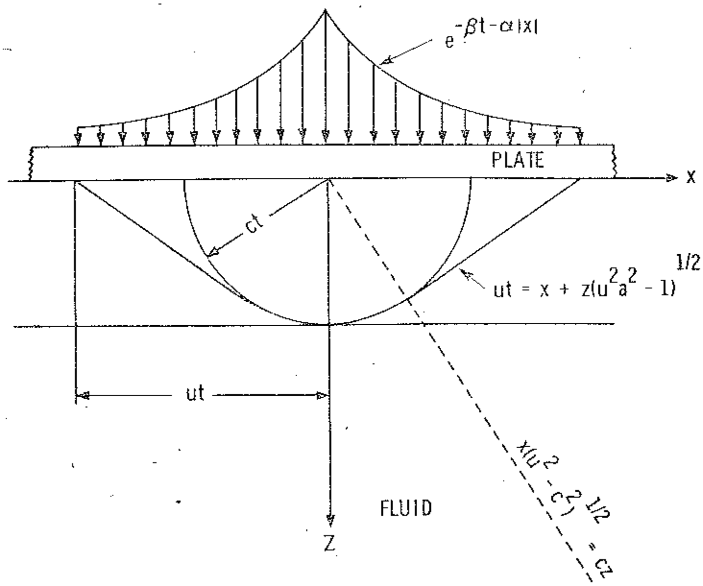

FIG. 4. Problem of a moving pressure load.

That is to say, the contribution is found only for points to the right of the line $x\,(u^2-c^2)^{\frac{1}{2}}=cz$ shown in Fig. 4. One finds that

$$\hat{F}_1\,(x,z,s)=-\frac{1}{s\,(s+\beta)}\,\frac{e^{-s\,[\eta\,(i\sigma_p,\,1)\,z+\sigma_p\,x]}}{D\,(s,\sigma_p)}\,. \tag{4.4}$$

By approximating this equation for large values of s, one finds that

$$\hat{F}_1\,(x,z,s)\approx-\frac{e^{-s\,[(a^2-\nu^2)^{\frac{1}{2}}z+\nu x]}}{EI\nu^4\,(a^2-\nu^2)^{\frac{1}{2}}\,s^6}\;e^{-(\nu\beta+\alpha)\,[x-cz\,(u^2-c^2)^{-\frac{1}{2}}]}, \tag{4.5}$$

$$F_1\,(x,z,t)\approx-\frac{T_B^5\,H(T_B)\,e^{-(\nu\beta+\alpha)\,[x-cz\,(u^2-c^2)^{-\frac{1}{2}}]}}{EI\nu^4\,(a^2-\nu^2)^{\frac{1}{2}}\,5!},\qquad T_B=t-\nu x-(a^2-\nu^2)^{\frac{1}{2}}\,z\,, \tag{4.6}$$

which is valid in the neighborhood of the straight line $t=\nu x+z\,(a^2-\nu^2)^{\frac{1}{2}}$ shown in Fig. 4. This wave front corresponds to the conical wave fronts found in [5]. It does not exist for $u<c$.

For large s, at the saddle point $\sigma=\sigma_1(t_0)$,

$$\hat{q}^*(is\sigma_1, s)\approx -\frac{2\nu}{s^2(\sigma_1^2-\nu^2)}. \tag{4.7}$$

Substituting this into (2.34), one obtains

$$\hat{F}_2(x,z,s)\approx Be^{-st_0}s^{-13/2}, \tag{4.8}$$

$$B=2\nu\{[\sigma_1^2(t_0)-\nu^2]EI\sigma_1^4(t_0)(2\pi t_0)^{\frac{1}{2}}\}^{-1}, \tag{4.9}$$

$$F_2(x,z,t)\approx BH(T_0)T_0^{11/2}\Gamma^{-1}(13/2), \tag{4.10}$$

valid for $T_0=t-t_0$ sufficiently small.

From (2.26), (2.27) and (4.2), one finds that

$$\hat{F}_3(x,z,s)=2\mathrm{Re}\ \big(\text{Residue at } \sigma_0(\rho)\big), \tag{4.11}$$

and, for large values of s, this reduces to

$$\hat{F}_3(x,z,s)\approx ba^{-1}e^{-sza}\,\mathrm{Re}\,e^{-i3\pi/4}e^{ib^2z/2a}e^{-be^{i\pi/4}xs^{1/2}}s^{-9/2}. \tag{4.12}$$

The comments given after the Eqs. (3.8) and (3.12) apply in this case also, mutatis mutandis.

Appealing to the first of the relations (2.2), the results for the pressure follow from (3.19) and (3.20). Thus, from (4.5) and (4.8),

$$P_1(x,z,t)\approx\rho\frac{u^4T_B^3H(T_B)e^{-(\nu\beta+\alpha)[x-cx(u^2-\nu^2)^{-\frac{1}{2}}]}}{6EI(a^2-\nu^2)^{\frac{1}{2}}}, \tag{4.13}$$

$$P_2(x,z,t)\approx -\rho BH(T_0)T_0^{7/2}\Gamma^{-1}(9/2). \tag{4.14}$$

Once again, $P_3(x,z,t)$ represents a wave front at $t=az$ which contains the smoothness properties of $F_3(x,z,t)$.

As was indicated previously, in the absence of the plate, the solution is given by (2.14) with $D(s,\sigma)$ defined by $D(s,\sigma)=\rho s$. This leads to

$$P_1(x,z,t)\approx H(T_B)e^{-(\nu\beta+\alpha)[x-c(u^2-c^2)^{-\frac{1}{2}}z]}, \tag{4.15}$$

$$P_2(x,z,t)\approx C(x,z)H(T_0)T_0^{\frac{1}{2}}, \tag{4.16}$$

$$C(x,z)=2\nu z(2t_0)^{\frac{1}{2}}[\pi(\nu^2-\sigma_1^2(t_0))(x^2+z^2)]^{-1}. \tag{4.17}$$

Comparing (4.13) to (4.15) and (4.14) to (4.16), one again concludes that the presence of the plate has prevented the load discontinuity from propagating into the fluid, and that the presence of the plate has created a wave front at $z=ct$ for all x.

5. Discussion of results

In both the stationary and moving load problems, three distinct types of waves are generated in the fluid. Two of these have counterparts in the solution with no plate present, while the third wave arises from the plate-fluid interaction. The pressure associated with this last wave, along with all its time partial derivatives, vanishes at the wave front. Also, this wave front is found to be parallel to the plate surface and propagating into the fluid with the fluid velocity. These characteristics of the plate-fluid interaction wave are due to the parabolic nature of the Bernoulli--Euler plate equation which provides for a disturbance to be felt instantaneously everywhere in the plate. The remaining two waves arise from (i) the specific nature of the loading function itself, and (ii) the hyperbolic nature of the wave equation governing the behavior of the acoustic fluid.

The contribution to the wave motion in the fluid arising from the specific loading function yields the strongest wave front. In both the stationary load and moving load problems, the strength of the wave fronts are reduced by the presence of the Bernoulli-Euler plate. For the stationary load, the time discontinuity in pressure is reduced to a discontinuity in pressure rate. A similar reduction occurs in the moving load case.

References

1. J. P. Jones, *The response of a plate on an elastic foundation*, J. Appl. Mech., 35, 186–187, 1968
2. D. H. Y. Yen and C. C. Chou, *Response of a plate supported by a fluid half-space to a moving pressure*, J. Appl. Mech., 37, 1050–1054, 1970.
3. J. W. Miles, *Response of a layered half-space to a moving load*, J. Appl. Mech., 33, 680–681, 1966.
4. F. R. Norwood, *Exact transient response of an elastic half-space loaded over a rectangular region of its surface*, J. Appl. Mech., 39, 516–522, 1969.
5. F. R. Norwood, *Interior motion of an elastic half-space due to a normal finite moving line load on its surface*, Int. J. Solids and Structures, 6, 1483–1498, 1970.
6. L. Knopoff and F. Gilbert, *First motion methods in theoretical seismology*, J. Acoust. Soci. America, 31, 1161–1168, 1959.
7. D. E. Amos and P. J. Chen, *Transient heat conduction with finite wave speeds*, J. Appl. Mech., 37, 1145–1146, 1970.
8. J. E. Russell and G. Herrmann, *Submerged cylindrical shells subjected to moving loads*, SUDAM Report No. 72–2, March 1972, also Scientific Report AFOSR-TR-72-0251.

Streszczenie

PRZEJŚCIOWE ZACHOWANIE SIĘ UKŁADU PŁYTA-CIECZ POD WPŁYWEM CIŚNIENIA STACJONARNEGO I RUCHOMEGO

Teoria Bernoulliego-Eulera oraz metody transformacji Laplace'a i Fouriera zostały wykorzystane do badania możliwego ruchu falowego w półprzestrzeni wypełnionej cieczą i podpierającej nieskończoną płytę. Powierzchnia płyty jest poddana obciążeniom zależnym od czasu i współrzędnych przestrzennych. Rozważono dwa szczególne warunki obciążenia: stacjonarny i ruchomy oraz otrzymano przybliżone rozwiązanie na czole fali dla zaburzenia ciśnienia w cieczy. Podano ograniczenia dla przeprowadzonej wcześniej analizy stanu ustalonego. Otrzymane wyniki porównano z przypadkiem braku płyty. W ramach tej teorii obecność płyty osłabia czoło fali ciśnienia w stosunku do ciśnienia występującego w płaskiej półprzestrzeni wypełnionej cieczą.

Резюме

ПЕРЕХОДНОЕ ПОВЕДЕНИЕ СИСТЕМЫ ПЛАСТИНКИ-ЖИДКОСТИ ПОД ВЛИЯНИЕМ СТАЦИОНАРНОГО И ПОДВИЖНОГО ДАВЛЕНИЯ

Теория Бернулли-Эйлера и методы преобразований Лапласа и Фурье использованы для исследования возможного волнового движения в полупространстве заполненном жидкостью и поддерживающем бесконечную пластинку. Поверхность пластинки подвергнута нагрузкам зависящим от времени и пространственных координат. Рассмотрены два частных условия нагружения: стационарное и подвижное и получено приближенное выражение для возмущения давления на фронте волны. Даются ограничения для проведенного ранее анализа установившегося состояния. Полученные результаты сравнены со случаем отсутствия пластинки. В рамках этой теории присутствие пластинки ослабляет фронт волны давления по отношению к давлению выступающему в плоском полупространстве заполненном жидкостью.

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