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PLANE SHOCK WAVES IN MONOATOMIC GAS. COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL RESULTS*)

Dedicated to the memory of Sir Geoffrey Ingram Taylor

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A review of the reeent theoretical and experimental results on the structure of the shock waves in monoatomic gas is presented. It follows from this review, that the research in this field has reached a very high level of perfection. However there are still some questions unanswered and further research is necessary mainly because understanding of the shock-wave structure can give deeper insight into the fundamentals of gas dynamics.

1. Introduction

The beginning of the story is worth recalling. French artillerymen noticed that the velocity of propagation of the noise produced by gun blast did not agree with the theory of sound propagation. Then, G. G. STOCKES from Cambridge in his communication of November 1868 in the Philosophical Magazine entitled *On the Difficulty in the Theory of Sound* proposed an explanation based on the assumption that a discontinuity propagates through the gas. It was then an extremely unusual idea and Stokes wrote, "It doeas not follow that the discontinuous motion considered can even take place in nature..."

However, Stokes was able to determine some conditions on both sides of this discontinuity. A further step was made by Lord Rayleigh in his *Theory of Sound*. He mentioned that in the general case the conservation of energy cannot be satisfied simultaneously with the equation of conservation of mass and momentum. Later, in a note published in 1908 he added that it is conceivable to admit that in the discontinuity the mechanical energy is lost and converted into heat. However, it was only in G. I. TAYLOR'S paper *The Conditions Necessary for Discontinuous Motion in Gases*, which appeared in the Proceedings of the Royal Society in 1910 that the firm ground of all further developments of the theory of shock waves was established.

The irreversibility of the phenomenon was properly taken into account and the physical picture given in this short paper has remained unchanged up to now. Consid-

*) A revised and abbreviated version of the paper ..The structure of a plane shock wave of monoatomic gas. Theory and experiment" published in the „Rarefied Gas Dynamics", Proceedings of the IX International Symposium 1974, edited by M. Becker and M. Fiebig, (DFVLR-Press, Porz-Wahn, Germany, 1974).

ering the motion of a gas in the frame of reference in which the discontinuity is at rest, u_1 and u_2 being the velocity of gas in front and behind the discontinuity respectively, Taylor, wrote "... in fact, it is only the front of a compression that can possibly travel unchanged. For, if for an instant the sharp discontinuity were to disappear, leaving a small transition layer in which the velocity might vary continuously from u_1 to u_2, then the back part of the layer would travel forward relatively to the front part with a velocity $u_1 - u_2$. Hence if u_1 exceeds u_2, any such transition layer will become obliterated owing to the greater velocity behind, and the discontinuity will thus be maintaned. ...If, however, the wave is a wave of rarefaction, that is, if u_1 is less than u_2, then the layer of transition will get wider and the sharp discontinuity will not be re-established. ...

"It is evident that a plane of absolutely sharp or mathematical discontinuity cannot occur in any real gas. When, owing to change of type, there is a sudden compression or rarefaction of the material in crossing any boundary, modified physical laws must come into operation whose effect is to prevent abrupt discontinuity from being formed. Some clue to the nature of the process involved in this case is afforded by the kinetic theory of gases; for, when the change in velocity is very sudden, the molecules which are moving faster will penetrate among those which are moving more slowly, and an irreversible redistribution of velocities will ensue. This suggests that heat conduction and viscosity are, in the case of a real gas, the causes of the production of dissipative heat...".

As will be seen later on a more detailed look at the molecular structure of the shock wave gives better understanding of what really happens in the small region where the transition from one state of eqiulibrium to another takes place in accordance with the qualitative description given above.

The shock wave as a research object has two important advantages. The first is, so to say, its physical cleanliness, as for a rather wide range of Mach numbers the shock waves in noble gases are not unduly influenced or disturbed by accompanying phenomena; hence, at least in principle, the experiments are easier. The second advantage is the simplicity of mathematical description, as no wall effects are involved and the conditions at $\pm\infty$ are described by known Maxwellian distribution functions which satisfy the conservation conditions.

Thus, the shock wave has been and still remains the most useful test case for both new theoretical approaches and experimental techniques. In this paper we are not going to describe the whole growing field of applications of shock waves, however it should be stressed that a thorough knowledge of the problem provides a sound basis for the investigation of more complex physical and chemical phenomena.

The extent of the problem forced us to abandon the attempt of covering the whole range of shock wave phenomena and in this paper we limit ourselves to the classical case of shock wave in monoatomic (mainly noble) gases. In this manner we avoid all problems in which the inner degrees of freedom must be taken into account. We aim at presenting only the main lines of the development of the theory and we assume that the reader is familiar with the basic ideas of continuum description as well as with those based on Boltzmann theory and their approximations. Instead of giving either introductory remarks or technicalities we offer a rather full comparison between the theory and the experiment.

2. Remarks on experimental techniques

Low density supersonic wind tunnels and shock tubes are used to produce sufficiently thick shock waves under laboratory conditions. Unfortunately, the flows obtained suffer from different imperfections that influence the structure of the shock waves and thus increase the scatter of experimental results.

During the last ten years or so the store of measuring techniques has become significantly richer. New powerful methods have been added to the old ones based on wire probes or on optical reflectivity and the older methods have been considerably improved. This, above all, is true with respect to the electron beam techniques. A new and very accurate ion time of flight method was proposed by Bütefisch and Vennemann from Göttingen; the technique of excited luminescence (GADAMER, SCHUMACHER, MUNTZ) gives the possibility of measuring simultaneously more than one quantity and, at the same time, provides more data on the molecular level, particularly the so-called directional distribution function $F(c_x) = \int f dc_y\, dc_z$. The possibilities of using laser particularly for laser differential interferometry as developed by Smeets is of special interest. Doubtless, the bulk of available experimental data is now much larger and more accurate than those of a few years ago. But it is also true that the new techniques mentioned above have not as yet found their way to many laboratories and that their effects are still to come.

As we will see in many cases the scatter of experimental data is so large, that often they do not give the necessary support for the verification or rejection of proposed theories. Perhaps one of the main deficiencies of the available data is that the great majority of experimental results give a very limited amount of simultaneous information and the repeatability of the tests is far from being perfect.

The scatter of experimental data can be seen on the figures describing the thickness of shock waves. This integral characteristic of the shock wave can be treated as the first check for each theory. As it is known there are many ways of defining the shock-wave thickness. The most appropriate one to use is the classical Prandtl definition in which the shock-wave thickness is determined by the maximum slope of the density distribution times the density jump. This definition is not only the simplest one but also most appropriate because it is related directly to the maximum rate of change of hydrodynamic quantities.

3. Shock-wave thickness

We shall start by comparing experimental results with the continuum theory firmly established in the work of G. I. TAYLOR. As it so often happens in his papers, the assumptions involved in the analysis and, above all, the limitations to weak shock waves are exactly those that are satisfied without leaving the safe ground of continuum theory. Becker's work, in which a similar approach was extended to strong shock waves, has shown that at higher Mach numbers the shock wave thickness becomes smaller than the mean free path. Hence, the continuum theory becomes meaningless, at least of one assumes, as Becker did, that the viscosity and the heat conductivity are constant.

This remark of BECKER's led to two series of investigations. One, in which the viscosity is allowed to vary with temperature, was first analysed by Thomas in 1944. Later, MORCHUCHOW and LIBBY showed that the asymptotic values of the shock wave thickness, for Mach numbers tending to infinity, become zero, or infinity, or take some finite positive value depending on the magnitude of the temperature exponent in the viscosity variation law ($\mu \sim T^{\alpha}$) being smaller, larger or equal to 1/2. However, the asymptotic value as mainly of theoretical interest and we are obliged to GILBARG and PAOLUCCI for the full investigation of the shock wave structure using the Navier-Stokes description: in this work both viscosity and heat conductivity were allowed to vary with temperature over the the whole range of Mach numbers. Their results concerning the aspect of the problem of interest here are shown in Fig. 1 where shock-wave thickness curves which correspond to different empirical laws of viscosity temperature dependence are compared.

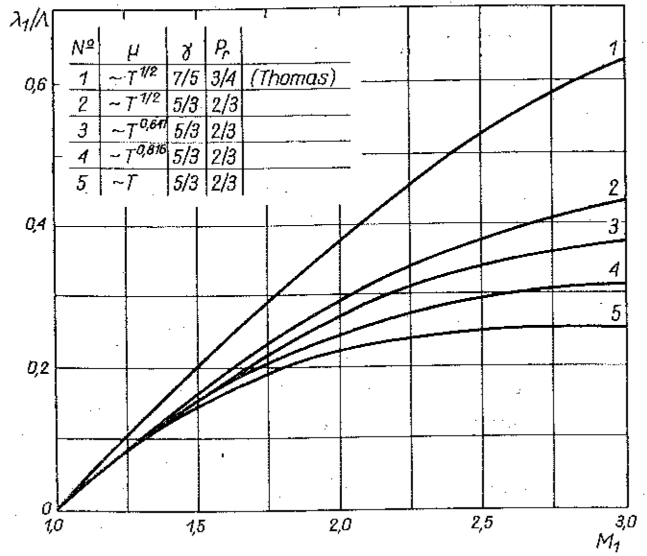


FIG. 1. Shock-wave thickness for different viscosity-temperature dependences (GILBARG, PAOLUCCI 1953).

The second method of investigation is based on the use of the Boltzmann equation for the determination of the transport coefficients retaining the continuum description or, what seems more important, for obtaining higher order hydrodynamic approximations such as the BURNETT and super-Burnett equations or GRAD's 13 moment method. Many ingenious attemps are worth mentioning, for example, those of GRAD, SHERMAN, SCHWARTZ and HORNING and others.

Unfortunately, the numerical computations made and, recently also FOCH's mathematical analysis have shown that, whereas the Navier-Stokes description gives results for all Mach numbers, the higher order hydrodynamic equations fail at Mach numbers some where below 2. Some of these difficulties can be overcome in the case of Grad's method by using a different weighting function as shown by BUTLER and ANDERSON a few years ago.

From Fig. 2, correlating the experimental results and the theoretical continuum theory ones, it can be seen that in spite of the asymptotic predictions an appropriate value of the exponents of the interaction potential can always be found to give a fair agreement with experimental results for shock-wave thickness, however, for any given value of the exponent the range of this correlation is limited.

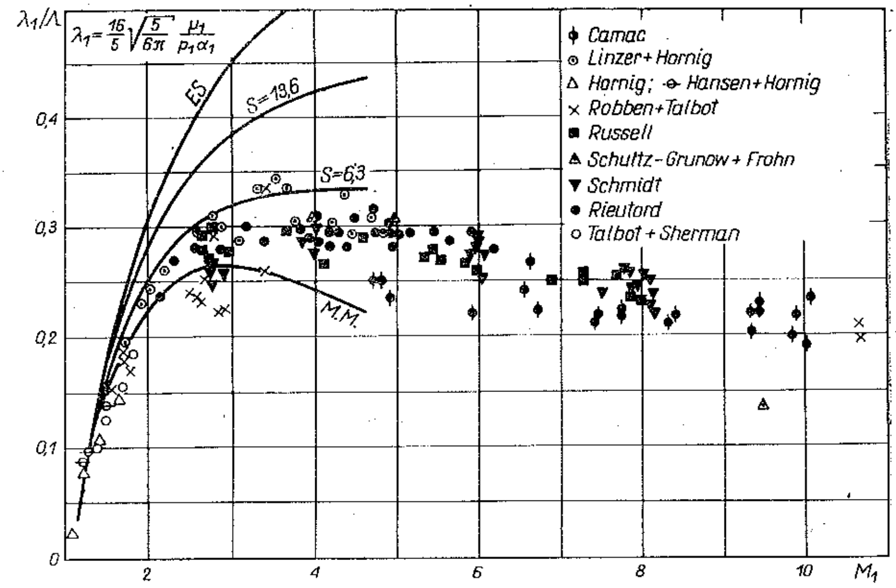


FIG. 2. Comparison of shock-wave thickness from continuum gas theories with experiment

For Mach numbers less than 1.6 the agreement between the continuum model and experimental results is good, as shown in Fig. 3; the best one is for $\mu \sim T$. It is noticed that for the same $\mu = \mu(T)$ relation the results of the 13 moment method fail to fit the experimental data even for M numbers less than, say, 1.3.

Of course results which are better than those obtained from the continuum theory can be expected by using Boltzmann's equation directly. The saying that "due to difficulties connected with the direct solution of Boltzmann's etc..." has become a standard one but this is still true in spite of recent successes in the mathematical understanding of its behaviour. Thus, the bulk of directly useful results are due either to the solution of the model equations in which the collision integral is replaced by a simpler expression or to the approximate solution of the Mott-Smith type. Only fairly recently have some reliable results of numerical solutions of the full Boltzmann equation become available. No analytical solution of the equation for the shock-wave structure problem is known so far in spite of the relative simplicity of the problem.

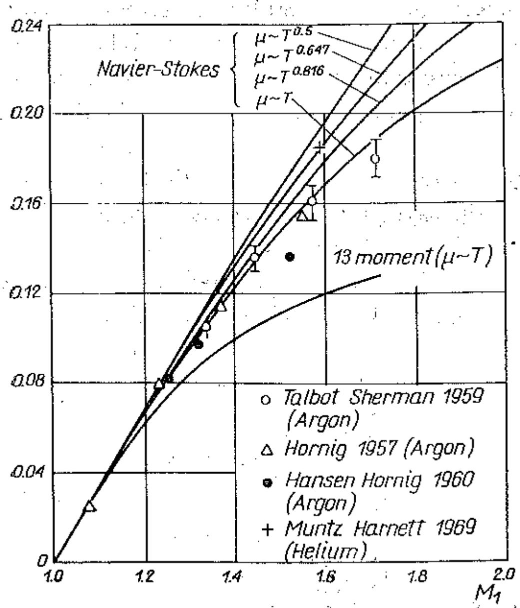


FIG. 3. Shock-wave thickness for different viscosity--temperature deependences at low Mach numbers.

The shock-wave thickness calculated by means of the BGK model is shown in Fig. 4 where it can be compared with some experimental results. Similar results for the Mott-Smith (and also the SALWEN, GROSCH, ZIERING) are shown in Fig. 5. Finally, the two available curves — for elastic spheres and Maxwellian molecules obtained by YEN and NG from the Monte Carlo solutions of the Boltzmann equation are shown in Fig. 6. Only one point available from the Bird's Monte Carlo direct simulation method is also shown in this figure together with a curve passing through this point obtained by using MUCKENFUSS relation between the shock-wave thickness and the Mach number $\Delta/\lambda_1 \sim M^{4/5}$, with the power exponent of the potential $s=11$.

Conclusions on the shock-wave thickness can be summarized as follows:

1) The Navier-Stokes theory gives good results for Mach numbers up to 1.5 or 1.8 and this is true for all other theoretical attempts made using hydrodynamic theories. No doubt this is due to the fact that all these theories converge at M close to 1.

2) For all theories the extreme cases of Maxwellian molecules and elastic sphere molecules enclose the experimental points and hence it is always possible to find such an interaction law that will suit the experimental data at least for not too wide a range of Mach numbers. All theories concerning shock-wave thickness are very sensitive to the variation of intermolecular potential.

3) If one assumes that the potential remains constant within the range of Mach numbers considered, then it is clear that the NS and BGK descriptions are far away from the experimental points; it is necessary to note that the range of validity of the assumption concerning the constant value of the interaction potential is still not fully determined and varies for different gases.

4) Although there are not enough theoretical results using the Monte Carlo method is seems most probable that for real interaction potentials the calculated results will be closer to the experimental ones because they correspond to the Boltz-

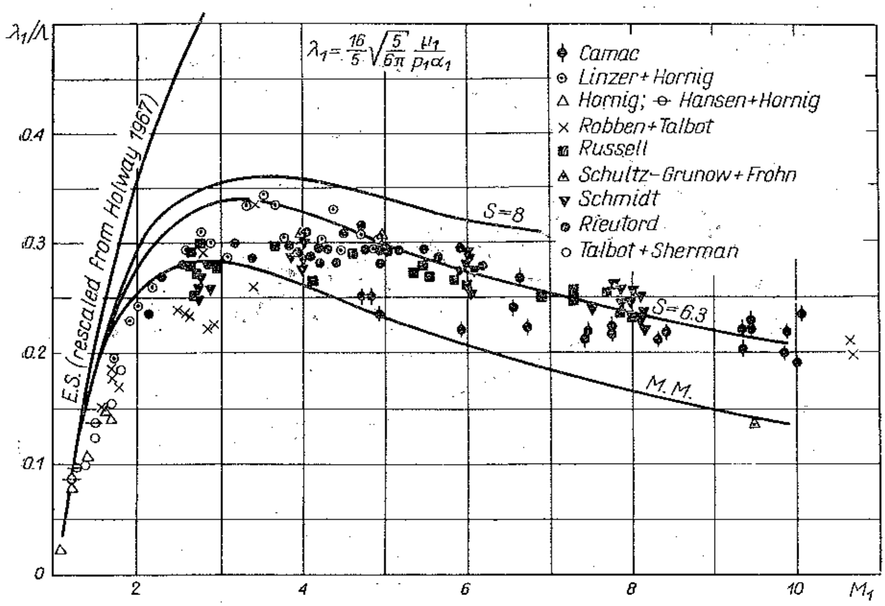


FIG. 4. Comparison of shock-wave thickness, calculated for the BGK model, with experiment

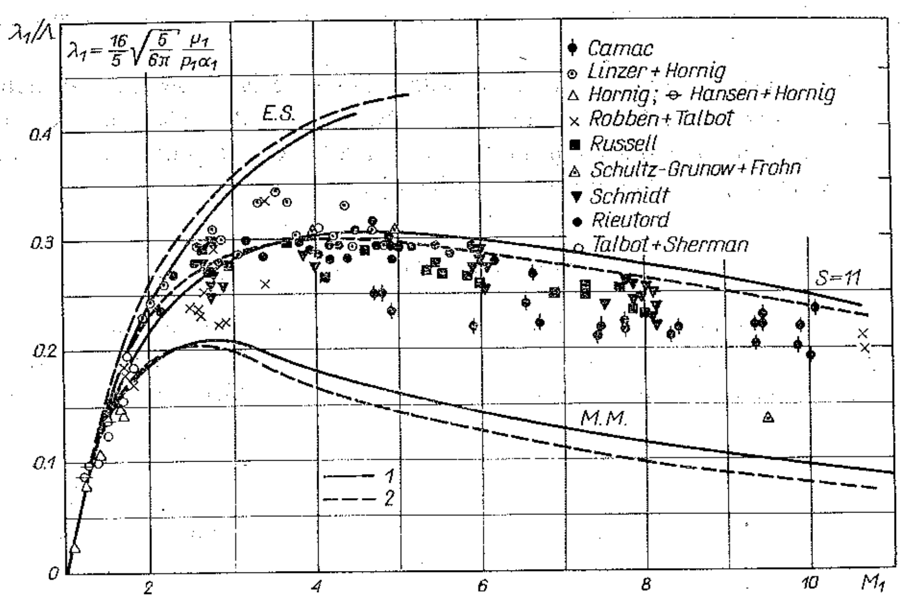


FIG. 5. Comparison of shock-wave thickness, calculated for different models of distribution function, with experiment:

1 — Mott-Smith, *2* — Salwen, Grosch, Ziering, V_x^2, $V_x V^2$ (For *ES* and $S=11$ rescaled from *MM* after Holway).

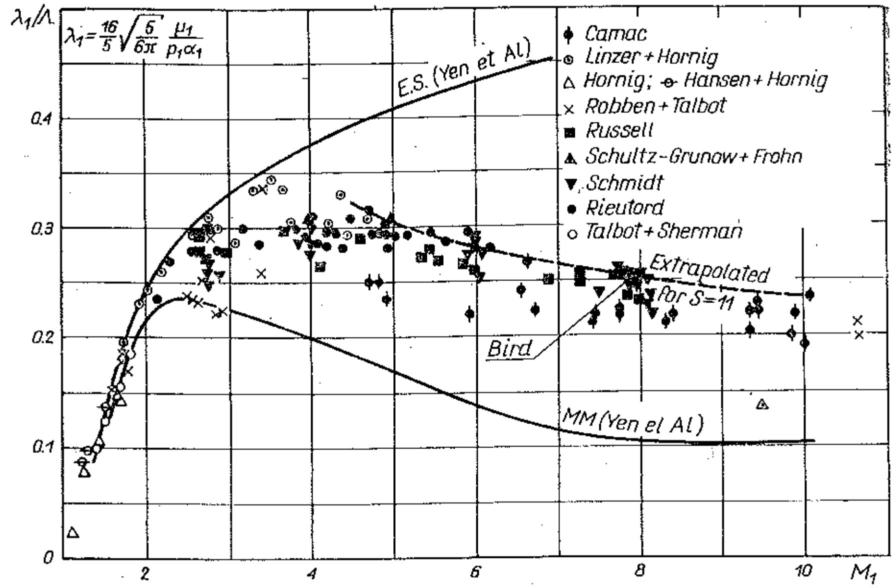


FIG. 6. Comparison of shock-wave thickness, calculated using Monte-Carlo methods, with experiment.

mann description. In this context it would be of special interest to check if in the Mach number range running from 2 to 5 the Monte Carlo method will give as uniformly valid results (with the same inetraction law) as it does for large Mach numbers.

5) No extensive comparison is made with other models as the data available are scarce for a proper evaluation. However, one can expect that models that are more sophisticated than the Mott-Smith model will give a better agreement over a wider range of Mach numbers but at the cost of losing simplicity.

4. Shock-wawe structure

Of course, the agreement between the theoretical and experimental values of one integral parameter, i.e., the shock-wave thickness, cannot be treated as a verification of the theory because this coincidence does not necessarily lead to a correct description of the shock-wave structure.

A typical example of this discrepancy is the structure of shock-waves as predicted by the Navier-Stokes theory for large Mach numbers. As mentioned before it is always possible to find such a value of the exponent of the interaction potential to make the theoretical and experimental shock-wave thickness equal. However, the predicted shock-wave structures would then disagree considerably with experimental observations. For small Mach numbers, say up to 1.8., Taylor's hyperbolic tangent formula is in very good agreement with experiments as shown in Fig. 7. Fortunately, thanks to the work of MUNTZ and HARNETT we can state safely that

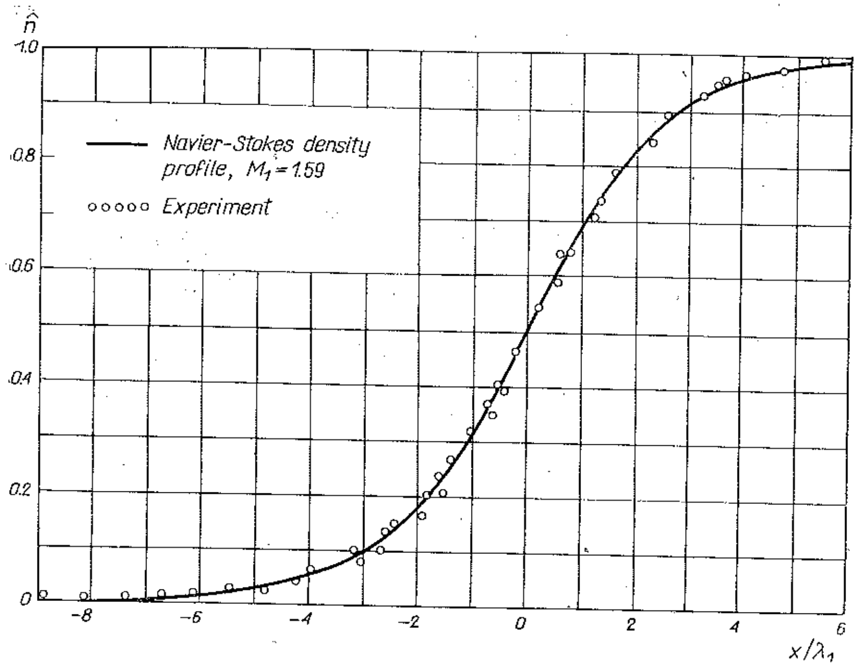


FIG. 7. Comparison of the shock-wave density profile calculated from Navier-Stokes equations with experiment (MUNTZ, HARNETT, 1969)

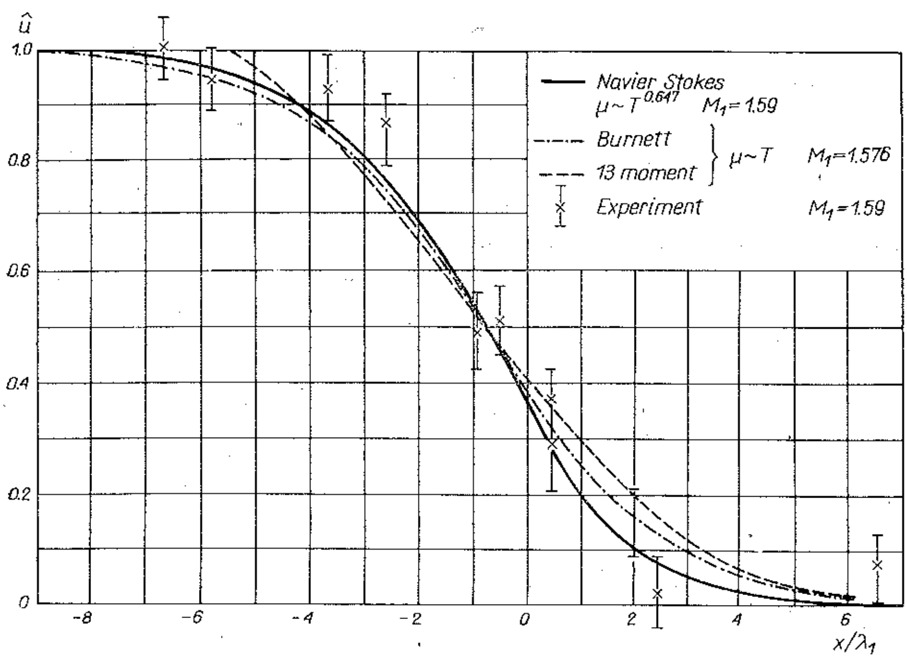


FIG. 8. Comparison of different theoretical shock-wave velocity profiles with experiment (TALBOT, SHERMAN, 1959, MUNTZ, HARNETT, 1969)

also the velocity profile, Fig. 8, calculated form the Navier-Stokes theory correlates well with experimental measurements, better than the 13 moment and BURNETT results. Also, the temperature profile agrees fairly well with the NS-theory (Fig. 9). Hence we can conclude, as also confirmed by LIEPMANN et al, that for weak shock-waves the assumption of local Maxwellian equilibrium is quite justified.

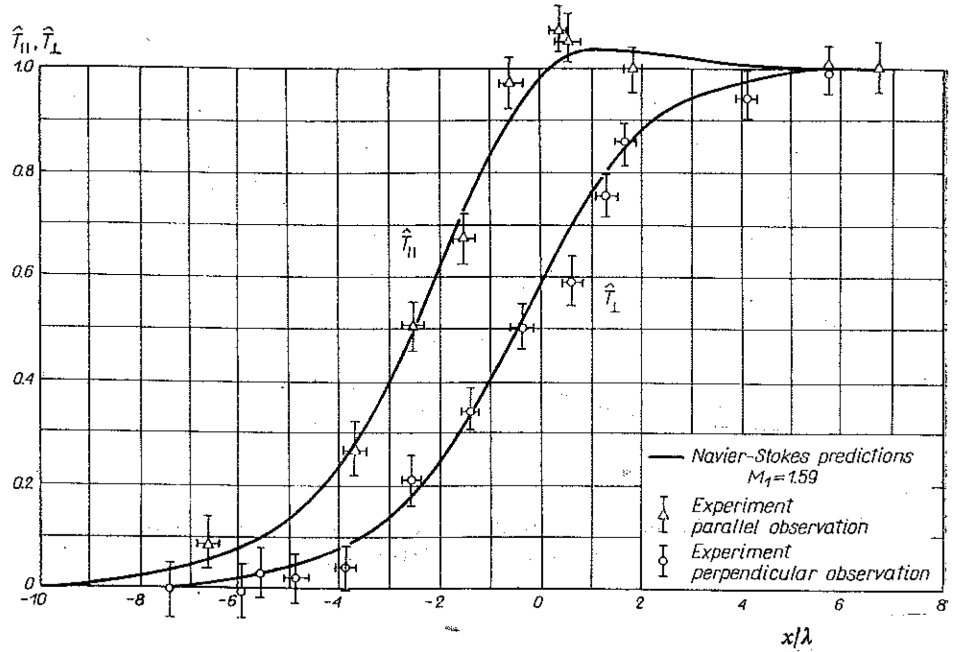


FIG. 9. Comparison of the shock-wave temperature profile calculated from Navier-Stokes equations with experiment (MUNTZ, HARNETT, 1969)

Whithin the frame of kinetic models of Boltzmann's equation many extensive numerical calculations of the shock-wave structure were made by CHAHINE (1963) and by ANDERSON (1966) who used the BGK model.

An analysis of the shock-wave structure problem based on a deep physical insight is given by LIEPMANN, NARASIMHA and CHAHINE in the (1962) and (1964) papers. A comparison of Navier-Stokes (N.S.) temperature profiles with the BGK ones at $M=10$ and $M=5$ indicates the coincidence at low Mach numbers and large differences at higher M in the calculated shock-wave structure, particularly on the cold side; the differences in the density profiles are less pronounced. Liepmann proposed a heuristic explanation for this asymmetric behaviour based on the difference in the characteristic length in the upstream and downstream parts of the shock-wave, connected with the different local mean free paths and the Mach numbers. It is interesting to note that the solution of the BGK equation points at the pronounced bimodal character of the distribution function, as can be seen in Fig. 10.

The simple BGK model of the collision operator, which leads in the hydrodynamic limit to the Navier-Sotkes equations, contains one free parameter which can be chosen to give a correct value either of the coefficient of viscosity μ, or heat conductivity κ. The Prandtl number in both cases is one, whereas the correct value for a monoatomic gas is close to 2/3. To remove this deficiency of the simple BGK model several improvements were proposed with more complicated expressions. SEGAL and FERZIGER (1971, 1972) systematized the construction of these models and obtained a sequence of BGK, ellipsoidal and polynomial models.

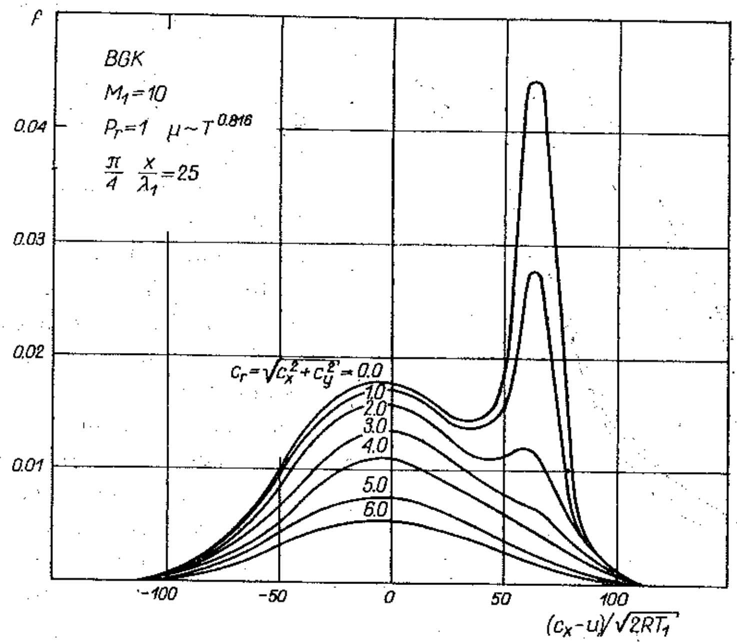


FIG. 10. Distribution function calculated from the BGK equation (CHAHINE, NARASIMHA, 1965).

A similar series of models followed from the generalized formula proposed by SHAKHOV (1968) and also given by ZHUK, RIKOV and SHAKHOV (1973).

The reduced density and temperature shock-wave profiles for the different models are shown in Fig. 11 indicating the particularly large influence of the model chosen on the distance between the points of maximum slope of temperature and density profiles. It can also be noticed that the difference in the upstream parts of the shock-wave profile for the polynomial and so-called trimodal models differs appreciably from the BGK and ellipsoidal ones. In spite of the numerous theoretical results obtained by means of the BGK model, very accurate experimental results obtained by SCHMIDT (1969) indicate that there can be a better agreement or fit with kinetic models of the distribution function, i.e., the MS type approach. This can be seen in Fig. 12 where the theoretical values obtained by means of the NS, BGK and MS discriptions are compared with Schmidt's data.

The first and most frequently used kinetic model was introduced in 1951 by Mott-Smith and, independently, in a 1965 publication based on an earlier work (1947) by TAMM. To stress the non-equilibrium aspects of the shock wave the bimodal form was proposed using the Maxwellian distribution functions far upstream and downstream; in this well-known expression only the number density n is an unknown function.

It can easily be checked that this bimodal function cannot be an exact solution of the Boltzmann equation and hence only approximate methods can be relevant when using it. The most natural method is to find the minimum of the functional

$$S=\int\left\{c_x\frac{\partial f}{\partial x}-J[f,f]\right\}^2 dx\,d\mathbf{c}$$

with a trial function in the bimodal form. This approach was used in 1947 by TAMM. OBERAI (1967) proposed a similar approach and also NARISIMHA et al, proceeded on somewhat similar lines.

In all these cases the hard sphere model was employed.

TAMM showed that under certain assumptions the condition for the minimum of the functional leads to the shock-wave structure known from the continuum theory. There are clearly several ways in which one can proceed using the variational method. It is possible to use a wide class of intermolecular interaction potentials and introduce weighting functions to the functional S. However, no such extensive attempts are known to the authors.

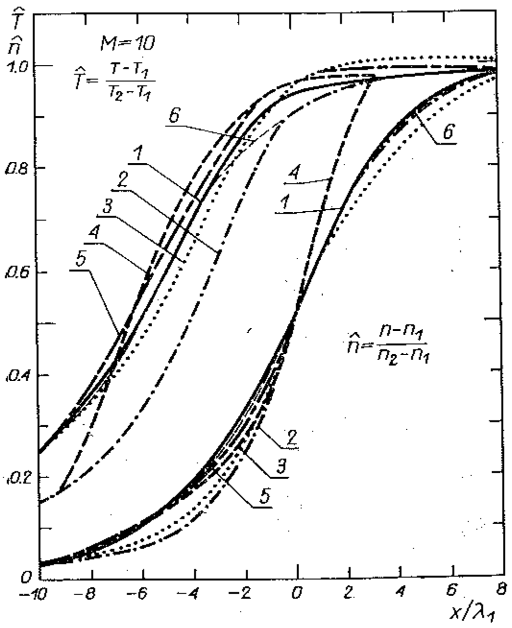


FIG. 11. Shock-wave density and temperature profiles for different theoretical models (SEGAL, FERZIGER, 1971)

1 — Monte Carlo, 2 — BGK, 3 — Ellipsoidal, 4 — Polynomial $\kappa=3/5$, 5 — Polynomial $\kappa=1$, 6 — Trimodal Gain function

In the MS type approach unlike Tamm's method conservation equations are not sufficient to determine the space dependence of the number density function. Therefore, an additional transport equation of a non-conserved function of velocity is required. According to the choice of this moment equation, different results are obtained. This points out the necessity of having additional criteria for selecting the additional relation whenever the bimodal model is used.

The bimodal model was also criticized as an isolated approximation without the possibility of further improvements. To improve these features and to avoid the experimentally unjustified symmetry of the MS shock profiles, particularly at high M, many modifications of that model were proposed. Most of them can be written in the form

$$f=\sum_i a_i(x,\mathbf{c})f_i,$$

where the f_i — s are Maxwellian type distribution functions. The parameters in f_i as well as the a_i functions are derived from an appropriate number of independent

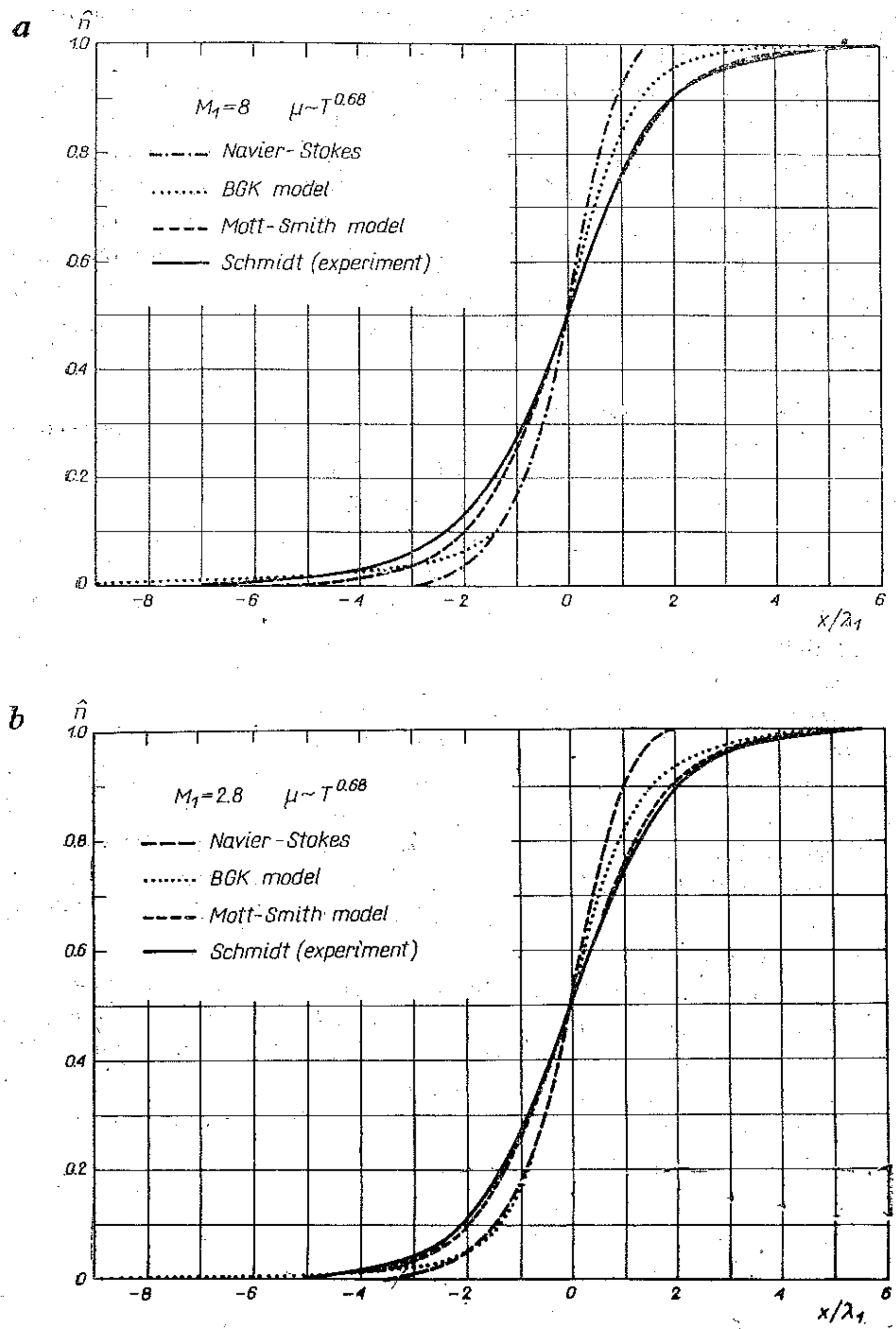


FIG. 12. Comparison of different theoretical shock-wave density profiles with experiment (SCHMIDT, 1969)

transport equations based on **c** and c_x. All these models suffer from the absence of uniqueness in the choice of the additional moment equations necessary for determining the increased number of parameters involved.

However, in comparison with, say, the Mott-Smith model the additional parameters provide better possibilities to satisfy the existing physical conditions and hence should give a better correlation. Many other models or methods have also been proposed by SALWEN, GROSCH, ZIERING, HOLWAY (1965), KROOK (1969), MACOMBER (1964), BARANTSEV (1962), KOGAN (1967). BAGANOFF and NATHENSON (1970) advocated the use of a constitutive equation instead of the additional moment equation. The outcome was highly successful in the cases they considered.

The great success of the simplest Tamm and Mott-Smith model and some of its refinements indicate clearly that we must give up the attempts to build models based on the assumption that within the shock wave the gas is close to the thermal equilibrium at Mach numbers, say above 2. At least, we cannot expect to be successful when we use such theories for greater Mach numbers and again we are forced to try to obtain a solution for the full Boltzmann equation.

At present it seems that the most appropriate way is to use the Monte Carlo method. As this method is rather a stochastic computational procedure there are essentially two different ways of using this MC procedure, in the first place as a tool for solving Boltzmann's equation and secondly, for direct simulation of the motion of particles. It must, however, be noted that there is no sharp division between these two ways and a successful use of MC method even in the first case depends strongly on the ability of simulating the process involved.

One of the most fruitful methods is perhaps the one proposed originally by NORDSIECK (NORDSIECK and HICKS, 1967), and used effectively by HICKS et al (1967), (1969), (1972) and YEN et al., (1973), (1974) for solving the shock-wave structure. In these reports the number density is taken as an independent variable. Using a special iterative procedure a large bulk of important results was obtained; some will be recalled here.

The direct simulation method, due to BIRD (1965), can be regarded as a numerical experiment. The simulated molecules are followed concurrently and simultaneously. The actual collision probability depends on the molecular model. BIRD (1970) showed that his assumptions about the molecular structure are similar to those used in the derivation of Boltzmann's equation and hence his procedure can be directly related to this equation.

The exellent agreement of both Yen et al., and Bird's method with experimental data is shown in Fig. 13 and 14, taken from Holtz's University of California 1974 Ph.D. thesis and BARCELO'S 1971 Cal Tech Ph.D. thesis.

It is worth recalling a few other attempts to solve Boltzmann's equation. CHEREMISIN (1970) used the integral form of Boltzmann's equation and the space variable as the independent one. An iterative method of calculation was used at Mach number 2 for the E.S. model. CHORIN (1971, 1972) proposed a new numerical method of solving the full Boltzmann's equation based on the use of Hermite polynomials.

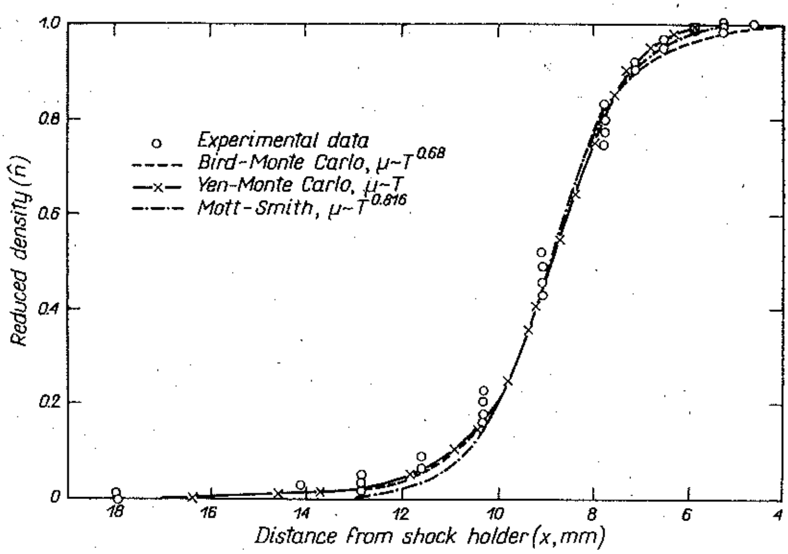


FIG. 13. Comparison of different theoretical shock-wave density profiles with experiment (HOLTZ, 1974).

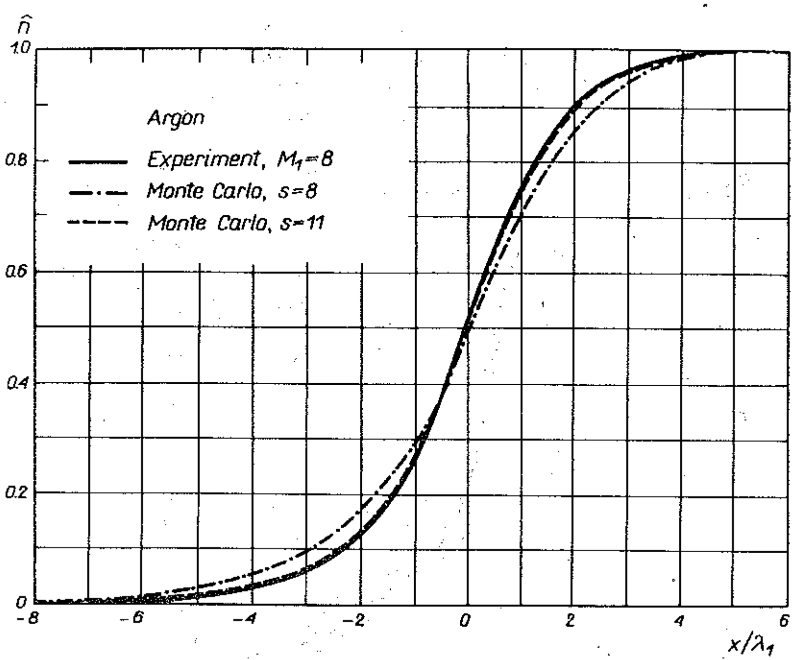


FIG. 14. Comparison of different theoretical shock-wave density profiles with experiment (BARCELO, 1971).

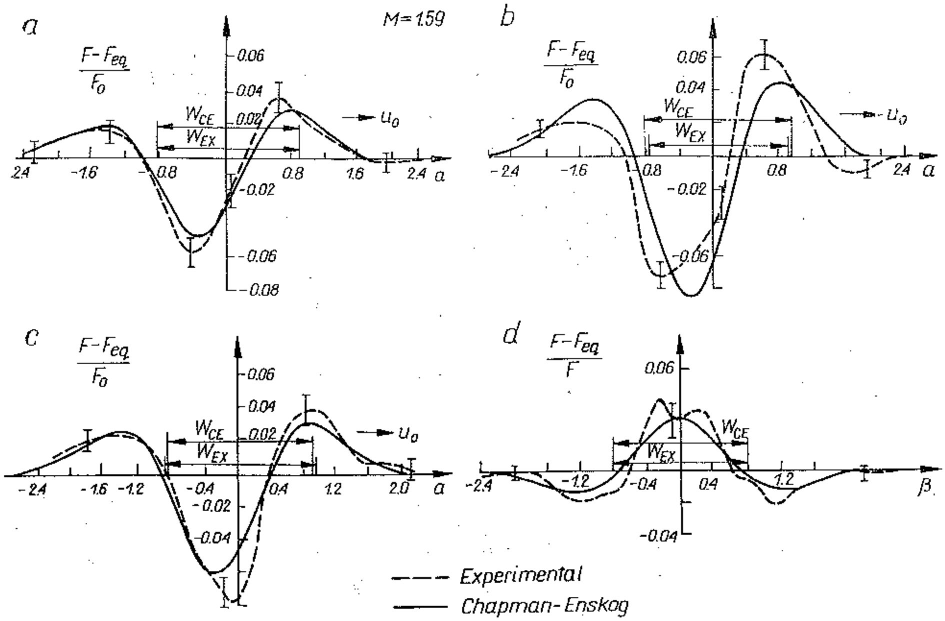
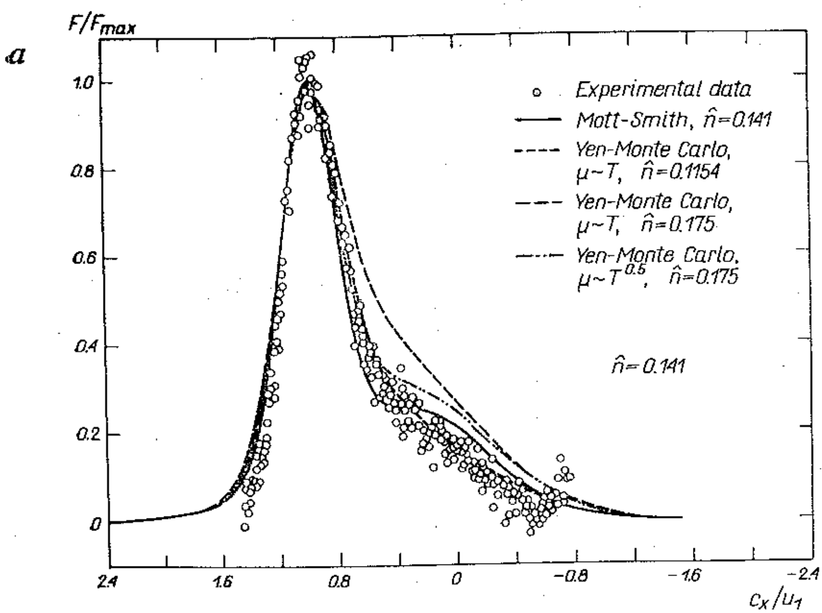
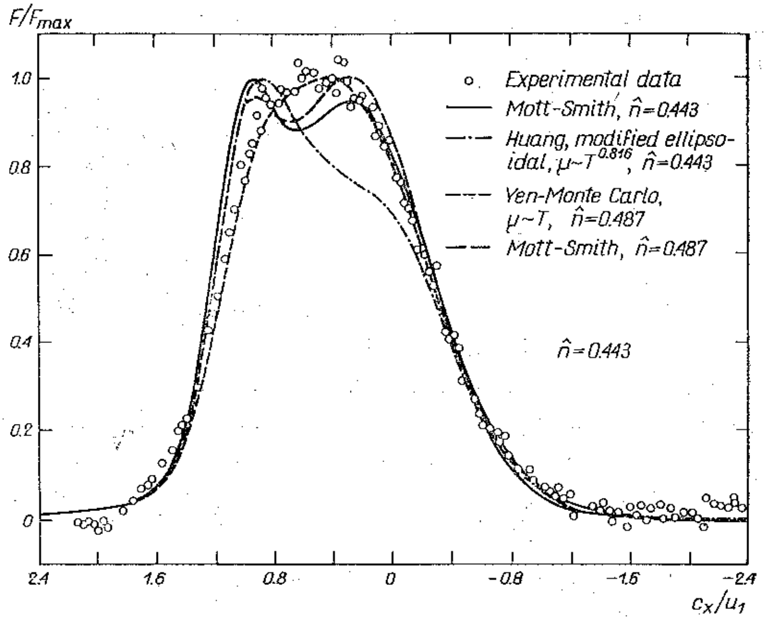


FIG. 15. Comparison of directional moments of the distribution function obtained from theory and experiment (HOLTZ, MUNTZ, YEN, 1971);

	x/λ_1	τ/P	
a	—3.66	0.062	PARALLEL
b	—2.15	0.120	PARALLEL
c	+0.50	0.096	PARALLEL
d	—2.15	0.120	PERPENDICULAR



b



c

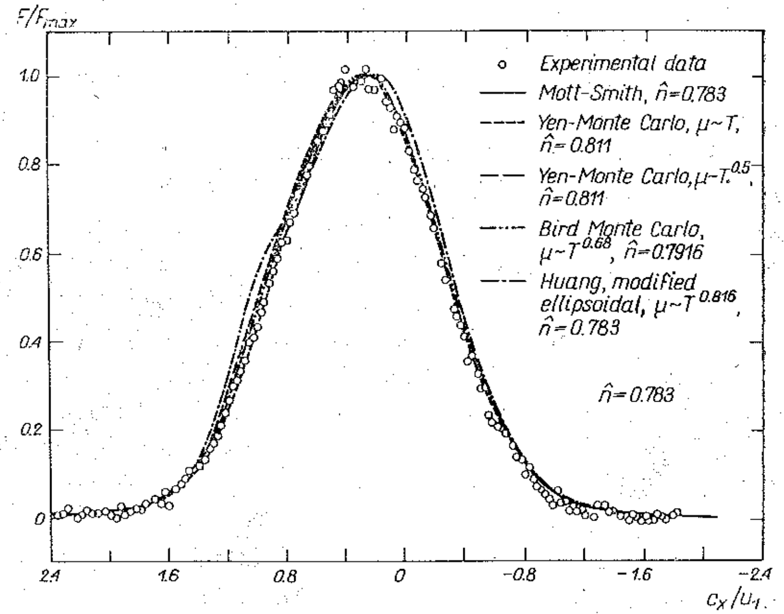


FIG. 16. a, b, c, Directional moments of the distribution function at various positions inside the shock-wave (HOLTZ, 1974).

In this very promising attempt either Gaussain quadratures or MC can be applied to evaluate the collision integral at each step of the proposed iterative procedure.

The numerical results given by the author for the shock-wave structure are limited to one case, namely the E.S. model with Mach number 2. The results obtained are in good agreement with those of other methods. The procedure proposed seems to be well substantiated and may play an important role in the future.

On the boundary of two groups of papers in which MC is used there is the "test-particle method" developed by HAVILAND (1963) and PERLMUTTER (1969). It consists in assuming at first a distribution of "target" particles and then using a different group of "incident" molecules which are followed one by one in their process of random collisions with the target molecules.

An important feature of all methods in which the Boltzmann equation is solved is that they provided an important further step, i.e., a check whether the good agreement on the hydrodynamic level — shock-wave structure — leads to the agreement between the macroscopic theory and experiments on the microscale, i.e., on the scale of the distribution function.

The method enabling the independent measure of parallel (in the direction of flow) and perpendicular directional moments using the electron beam luminescence technique proposed by MUNTZ (1968) opened the possibility of more detailed comparisons. The correlation of the experimental results with the Chapman-Enskog theory shown in Fig. 15 seems to be very good. Newer results are due to HOLTZ (1974) for the Mach number 7.18. Again, not only does the structure of the shock wave agree extremely well with the predictions of two Monte Carlo methods, as we have seen earlier, but on the molecular scale the agreement is very good indeed as seen in Fig. 16 with the Yen et al., and Bird's results and not so satisfactory for the Mott-Smith and ellipsoidal model. There is no doubt that the last papers mentioned mark a new stage in the understanding of the shock-wave structure.

STURTEVANT and STEINHILPER (1974) have developed and applied the Monte Carlo method solving the inverse problem: to obtain the molecular interaction potential from the measured shock-wave structure.

The great power of Nordsieck and Hicks' (and Bird's) Monte Carlo method in determining the distribution function can perhaps be better appreciated when looking at the impressive results of YEN and NG (1974) shown in Fig. 17. They compared the distribution function at different positions in the shock wave for MM gas and an ES gas at the Mach number 4 and showed the presence of penetration of high speed molecules from the cold to the hot side in the later case (ES) and the absence of this phenomenon in the first (MM). Perhaps, as a result of this penetration, they observed the pronounced bimodal character of the ES distribution function. Their conclusion that the relaxation toward equilibrium in the downstream wing is completed earlier for Maxwellian molecules can also be explained in a similar way.

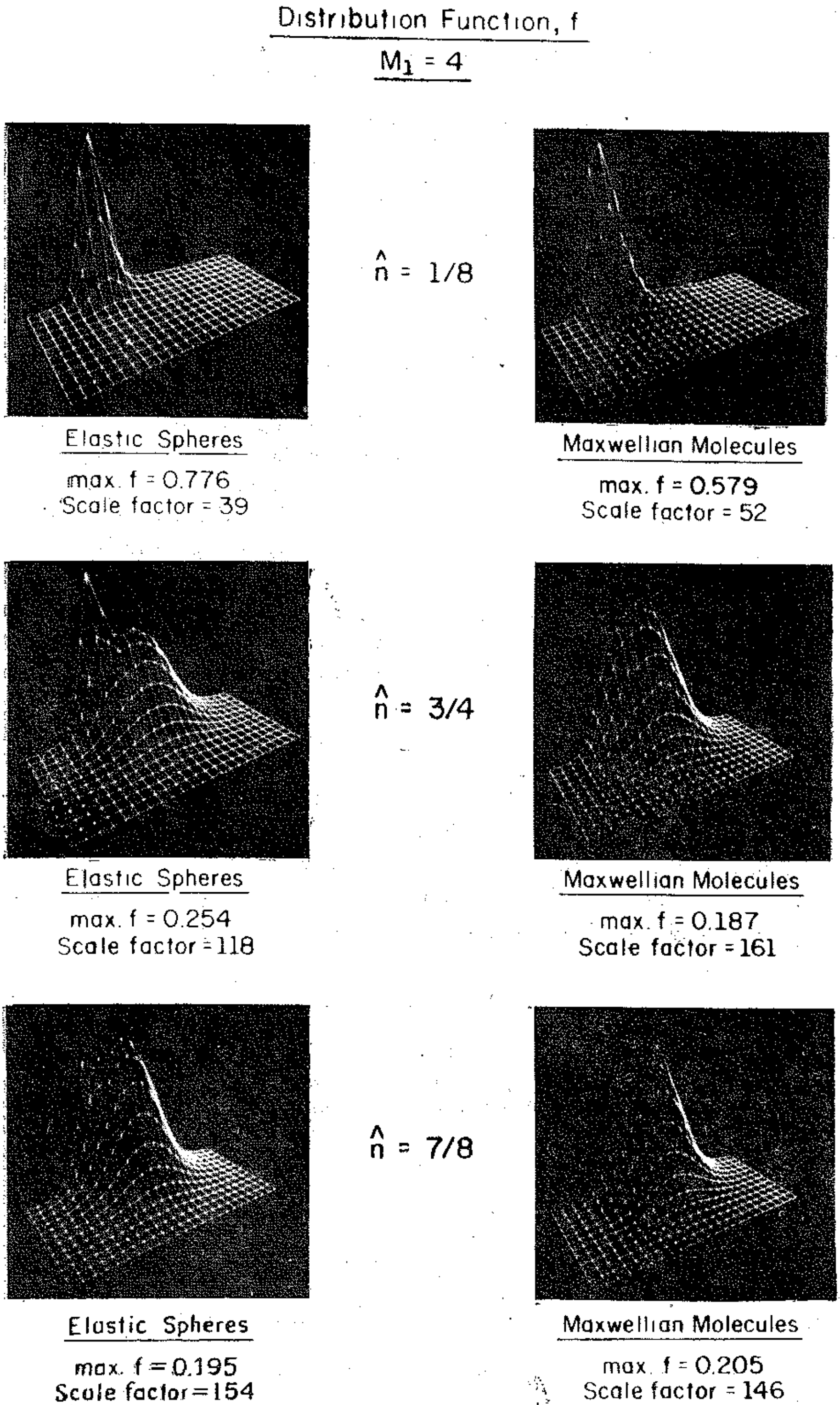


FIG. 17. Distribution function at various positions inside the shock-wave calculated using Monte-Carlo method (YEN, NG, 1973).

5. General conclusions

It is fair to say that the reviewed study of plane shock wave in monatomic gases for the range of Mach numbers in which no additional physical effects must be taken into account has reached a very high level of perfection. The need for further research follows from the fact that the understanding of the shock-wave problem gives a deep insight into the physical phenomena on which the whole structure of gas dynamics is based.

It seems clear that recent developments on the theoretical plane put more emphasis on the solution of the full Boltzmann's equation, superseding the different collision integral models so widely used until recently. This is why the Monte Carlo methods are so important, being as yet the only ones giving the possibility of dealing with the collision integral. However, one must not lose sight of the other mathematical techniques which may be successfully exploited and in which more confidence is placed on the power of reasoning than on the capacity of the computers.

There is still ample space for simple and ingenious theories which, for the limited ranges of parameters involved, can give proper physical but approximate solutions; these methods have great traditions in the field of physical sciences and always provide a guide through yet undiscovered lands. On theese lines much has been done over the last few years and a lot can be expected in the future.

New experimental techniques have given recently a deep insight into the microscopic structure of the shock wave.

Classical measurements have also gained much from the development of advanced techniques and are giving now the accuracy which was not expected a few years ago. But the experiments become sophisticated, lengthy and costly and this is perhaps the reason why the number of experimental data is still not large at least large enough to satisfy our needs.

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Streszczenie

PŁASKIE FALE UDERZENIOWE W GAZIE JEDNOATOMOWYM. PORÓWNANIE WYNIKÓW DOŚWIADCZALNYCH I TEORETYCZNYCH

W pracy podano przegląd wyników badań teoretycznych i doświadczalnych nad strukturą fal uderzeniowych w gazie jednoatomowym. Z przeglądu tego wynika, iż badania w tej dziedzinie osiągnęły bardzo wysoki poziom. Jednakże niekóre zagadnienia są jeszcze nie rozwiązane i dalsze prace wydają się konieczne, przede wszystkim dlatego, że zrozumienie struktury fali uderzeniowej może dać lepszy pogląd na podstawy dynamiki gazów.

Резюме

ПЛОСКИЕ УДАРНФЕ ВОЛНЫ В ОДНОАТОМНОМ ГАЗЕ. СРАВНЕНИЕ ЭКСПЕРИМЕНТАЛЬНЫХ И ТЕОРЕТИЧЕСКИХ РЕЗУЛЬТАТОВ

В работе дается обозрение результатов теоретических и экспериментальных исследований структуры ударных волн в одноатомном газе. Из этого обозрения следует, что исследования в этой области достигли очень высокого уровня совершенства. Однако некоторые вопросы еще не решены и дальнейшие работы кажутся быть необходимыми, прежде всего потому, что понимание структуры ударной волны может дать лучший взгляд на основы динамики газов.

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