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SHOCK WAVE DISPERSION IN FLUIDS WITH LOOSELY DISTRIBUTED RIGID BODIES

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G. I. TAYLOR'S [1,5] probabilistic approach to blast waves in turbulent air is extended for shock waves propagating through randomly distributed solids in gas. The shock wave suffers dispersion and is changed to a train of weak waves spread over a longer positive time duration. If frictional losses are small, the impulse remains approximately constant and the standard deviation of the attenuated wave has been found to be of the form

$$\sigma = [A \exp(-k\psi)+B]\sqrt{\psi-1}\sqrt{\frac{x}{t}}\,.$$

This is for baffles with randomly aligned perforations. A similar expression has been found for granulated materials in gas. The constants A, B and h have been experimentally determined. The above expression for standard deviation has been maximized with respect to the inverse of void fraction of granules. Thus for thickness of baffles of 0.178 cms, $\psi=1.18$ and for an equivalent diameter of granules of 0.068 cms, $\psi=4.1$ for maximum dispersion. Frictional losses neglected in this study have been dealt with by the authors in a different paper [8].

1. INTRODUCTION

Recent years have witnessed a growing interest in the propagation of waves in non-homogeneous media. Stratified solids, foams and solids in gases belong to this category. Shock wave attenuation is of considerable interest when waves move in these media. An attempt has been made here to extend G.I. Taylor's theory of diffusion by continuous movements [1] to predict shock overpressure and positive time duration for shocks propagating through randomly distrbuted solids in a gas. Taylor's theory [1] has been found to be very fruitful in many fields of fluid mechanics, especially in understanding the behaviour of shock waves in turbulent atmosphere [5] as well as atmospheric and temperature diffusion [2, 3, 4, 6].

Consider a plane one-dimensional shock-wave propagating through randomly distributed solids in a gas. After a few collisions, the main wave is broken down into a train of weaker waves by the reflection and diffraction at each solid, intermixed with vortices formed at the boundary of each solid. Onviously, after a few collisions a Lagrangian description of the wave front is impossible and only a statistical one seems feasible. Figure 1 shows a normal shock wave thus reduced to a train of weak waves after proceeding down a cylindrical vessel filled with spherical objects. The train of weak waves is generated by random reflections and diffractions. The shape of such a wave as it moves down the media is shown on Fig. 3, the crest receding slower than the base, as shown on Fig. 2. This situation is better termed as dispersion rather than diffusion

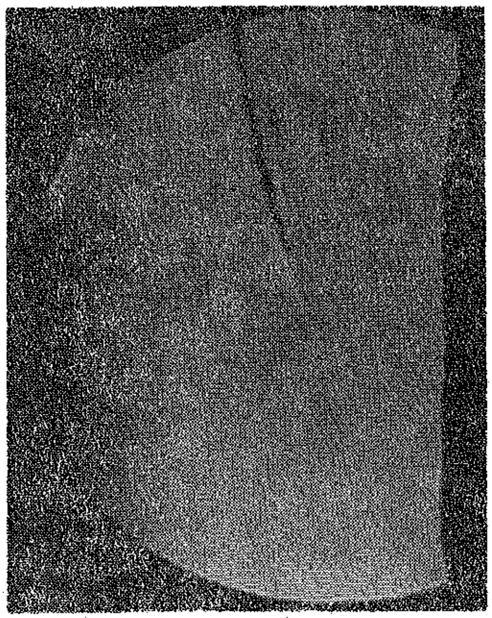

FIG. 1. Schlieren photograph of a train of waves.

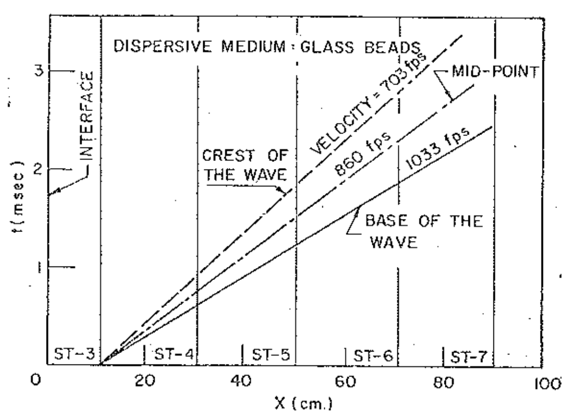

FIG. 2. An $x-t$ diagram obtained from a shock wave moving between glass beads.

If there is no appreciable friction at the boundaries, the impulse of the train of weak waves will be approximately constant, but the maximum overpressure reduces steadily, while the positive time duration increases Is. this paper a standard deviation σ is applied to the initial wave as proposed by G.I. TAYLOR [5] and an attempt

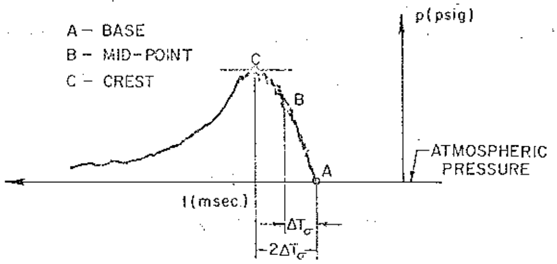

FIG. 3. A $p-t$ diagram of a shock wave propagating between concentric perforated baffles.

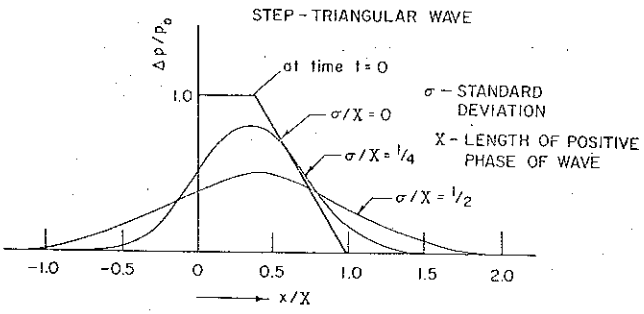

FIG. 4. G. I. Taylor's method applied to a step-triangular wave.

is made to predict changes in the standard deviation σ as the wave moves down the new media. An example of such calculation in terms of σ/X is shown on Fig. 4, where X is the positive length of the wave and x/X is the relative distance.

2. Discussion of Taylor's approach

Taylor's approach is briefly outlined here so that it can be applied to shock waves propgating through solids in a gas. The key to his solution [5] was to estimate the standard deviation σ of a blast wave due to atmospheric eddies as a function of distance. Using this method he botained an approximate shape of the diffused wave at any position. In his previous paper [1], he showed that

$$\sigma^2 = 2[u^2] \int_0^T \int_0^t R_\xi \, d\xi \, dt, \tag{2.1}$$

In an evenly turbulent medium $[u'^2]$ is constant and represents the mean energy of rubulent motion. If a particle starts moving with a uniform velocity u' and after a time τ suddenly makes a fresh start either forward or backward, this particle will move a distance $d=u'\tau$. The coefficient of correlation R between such two motions is a function of time only and falls to zero when time $\xi \to \infty$. For very short times, i.e. $\xi \to 0$, $R \to 1$. The following important cases may be distinguished:

a) The total time T is small R_ξ does not appreciably differ from unity, then

$$\sigma^2 = [u'^2] T^2. \tag{2.2}$$

b) The total time is comparatively large in the limit $\int_0^\infty R_\xi d\xi$ is finite, say I, then

$$\sigma^2 = 2[u'^2] IT. \tag{2.3}$$

As $u'^2 = (d/\tau)^2$ and T/τ is the number of events n, it follows that

$$\sigma^2 = 2d^2 (I/!)\, n. \tag{2.4}$$

c) There is no correlation between two subsequent motions and $I = \tau/2$, which, inserted into Eq. (2.4), gives

$$\sigma^2 = d^2 n. \tag{2.5}$$

This special case also represents the deviation in the „random walk" problem.

d) A certain functional value is assumed for the correlation coefficient R to fit the observed data better. Thus SUTTON [4] assumed $R = [a/|u|\,\xi]^2$ and showed that for large scale turbulence the size of the "effective eddy" is not constant, as suggested by Taylor, but grows with distance.

One notes from the above discussion that i) the reasoning behind this approach is very appropriate to a moving shock wave interacting with randomly dispersed solids, ii) the treatment can be adapted to a variety of circumstances, iii) the cases b) and c) reduce to a general statement:

$$\sigma_e \sqrt{n}\,, \tag{2.6}$$

where σ_e = deviation due to one event and σ = (deviation due to one event) × (square root of number of events). If both the expressions on the right hand side of Eq. (2.6) could be defined for the case of randomly distributed solids, then the standard deviation σ in conjunction with a Gaussian distribution would provide the answer.

G.I. TAYLOR [5] in the study of a shock wave progressing in turbulent atmosphere assumed that there was no correlation between motions (case c). If the turbulent atmosphere is composed of eddies of diameter L, the wave progressing at the speed of sound c will take a time L/c to cross one eddy. If the oscillating velocity produced by the eddy is $\pm u'$, then the deviation per eddy on one event is $\sigma_e = Lu'/c$. The number of events in proceeding a distance is x/L. The standard deviation in crossing a distance x is then

$$\sigma = \frac{Lu'}{c}\sqrt{\frac{x}{L}}\,. \tag{2.7}$$

Figure 4 shows the result of a computation using Taylor's method for a flat top shock wave in terms of $\sigma/cT_p = \sigma/X$ where T_p is the positive time duration of the wave (or X positive length). The impulse of the wave remains constant, which is physically correct and mathematically implies that the probability integral is unity between $\pm\infty$.

3. Experimental apparatus and procedure

Figure 5 is a schematic of the apparatus used. It consisted of a 2″ diameter L-shaped shock tube with the driver section in the horizontal arm. The solids to be tested were distributed in the vertical arm. Pressure signals from Kistler pressure transducers were amplified by Kistler Charge amplifiers Model 504-A and recorded on a 14 track Hewlett-Packard magnetic recording system HP 3955 at a tape speed of 60 inches per second. The data was reproduced at tape speed of 3-3/4 inches per second on a Honeywell 1508, 6 channel Visicorder. The direct print photographic paper speed was 20 inches/second. Time marks, a milli-second apart, were also printed on the paper from an external source.

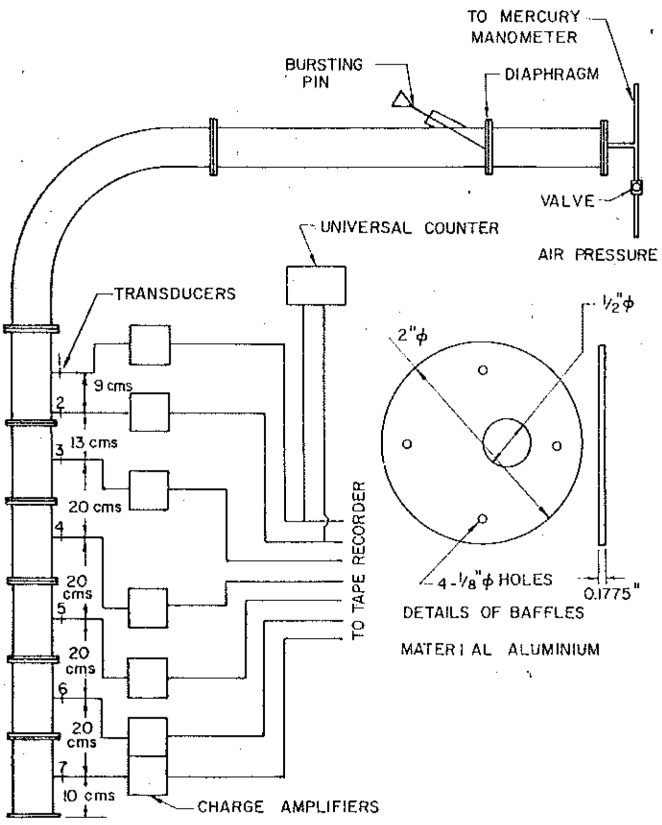

FIG. 5. Shock tube layout.

Incident shock velocity was determined by timing the shock arrival at transducers 1 and 2 which were 58.8 cms apart. A Hewlett Packard Universal counter 5325A was used for this purpose.

Normal shock tube techniques were followed for the experiments. About 100 cms of the vertical arm of the shock tube was filled with randomly distributed solids. Transducers 3 to 7 were mounted in this medium. A 0.005″ cellophane was used as the diaphgram. The pressure in the driver section was about 800 mm of mercury (gauge) and that in the driver section atmospheric (average 660 mm Hg), this combination of pressures gave a time interval of shock arrival at transducers 1 and 2 of about 232 μsecs. Experiments on various solids were repeated and Figs. 1, 2 and 3 show typical results.

4. Geometrical aspect

In discussing the case of a shock passing through a medium of solids distributed in a gas, the following simplified geometrical model clarifies some of the ideas used.

Consider a vessel of uniform cross-section, volume V and height h. It is filled with a solid to a height q and the remaining $h-q=p$ with the fluid. The relative amount of fluid is

$$\alpha = p/(p+q) \tag{4.1}$$

and solid

$$1-\alpha = p/(p-q), \tag{4.2}$$

α is termed the void fraction and $(1-\alpha)$ the solidity ratio.

Now if the solid content in the vessel is distributed randomly throughout the vessel instead of being in one place, the void fraction or the solidity ratio is not changed. In particular, if the solid is in N slices, distributed in the vessel so that the average thickness of the slices is t and the average gap g, then g/h is proportional to α/N and t/h to $(1-\alpha)N$. Furthermore it can be stated that the number of events n is proportional to N and

$$n \sim N \sim \alpha \frac{h}{g} \sim (1-\alpha)\frac{h}{t}. \tag{4.3}$$

This is a more general expression than the one used in Eqs. (4.1) and (4.2) and stresses the importance of α and introduces t, g, n and N into the argument.

When the solid material is very densly packed, i.e. $\alpha \to 0$, the number of events $n \sim h/t$; also the gap decreases to a very small value and the ratio α/g remains finite. Similar reasoning holds for the other extreme when $\alpha \to 1$. From the experimental point of view, for a densly packed material, say $0<\alpha \lesssim 0.5$, like for example granulated solids, it is easier to assess the diameter of the particles corresponding to this than the distance between them. For a loosely arranged solid, say $(0.5\text{–}0.57) \lesssim \alpha < 1$, like a series of perforated baffles, it is more convenient to measure the distance between the obstacles corresponding to the gap g. In all cases it is comparatively simple to estimate the void fraction α by immersion in a liquid. Thus, during the experiments the ratio of gas to solid for each case was determined by inserting a number of the solids in a graduated jar and pouring a predetermined volume of water into it. If the volume of solids and air was V_1, that of water poured in was V_2 and the final volume of water and solids was V_3, then the ratio of gas to (gas+solid) is $\alpha = (V_1 - V_3 + V_2)/V_1$.

It is experimentally possible to estimate the standard deviation of the dispersive wave. Figure 3 shows a dispersive wave plotted in the $p-t$ coordinates. As explained before (Sect. 1), the crest of the wave C recedes with time in relation to A and the deviation in time is measured by the mean value AT_σ between A and B. Knowing the velocity of propagation W, $\sigma = W\Delta T_\sigma$.

If the above reasoning relating the geometrical relations with the number of events is correct and Taylor's argument holds, then, for a given configuration (which includes some finite value of the integral of the correlation coefficient R)

$$\sigma \approx \frac{1}{\sqrt{t}} \quad \text{for a densly packed material}$$

and

$$\sigma \approx \frac{1}{\sqrt{g}} \quad \text{for loosely arranged obstacles.}$$

Figure 6 shows such a plot (corrected for effects of varying α) obtained from typical experiments performed in a shock tube filled with spherical objects and also with randomly perforated concentric baffles. The agreement is satisfactory and the relation

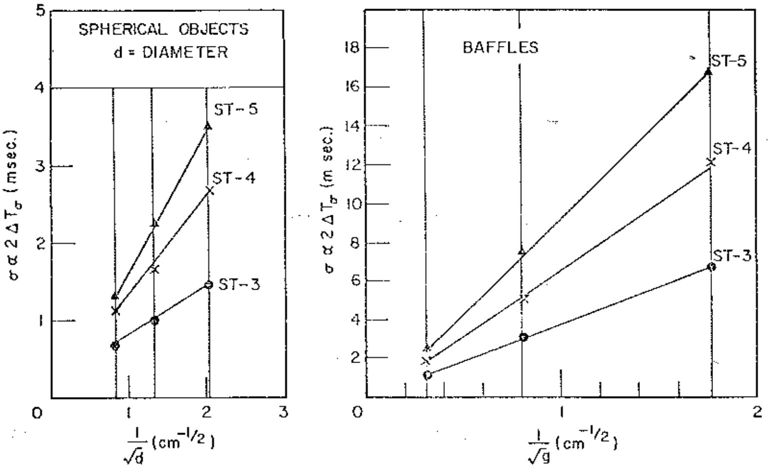

FIG. 6. Standard deviation as function of $1/\sqrt{d}$ (spherical objects) and $1/\sqrt{g}$ (perforated baffles).

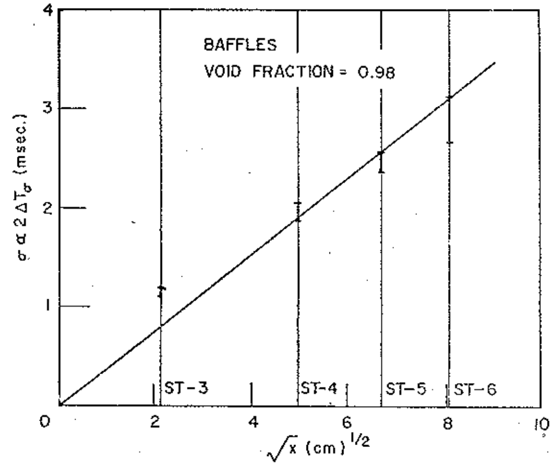

FIG. 7. Standard deviation as function of $\sqrt{x}$.

is linear. Again if the same argument is true for all the cases, $\sigma \approx \sqrt{\text{distance}} \approx \sqrt{x}$. Figure 7 shows such a plot for baffles. Again the agreement is good as it gives a straight line.

5. The deviation functions

The arguments given above must be further generalised to give valid results in a variety of circumstances. It is obvious that both the correlation coefficients integral as well as the void fraction should be included. Following Eq. (2.6), one must try to define some deviation function $S \approx \sigma_e$ such that for $0 < \alpha < 0.5$,

$$\sigma = S\sqrt{\frac{h}{t}} \tag{5.1}$$

and for $0.5 \gtrless \alpha < 1$

$$\sigma = S\sqrt{\frac{h}{g}}\,. \tag{5.2}$$

The function S which is proportional to the standard deviations σ_e of one event would depend upon the void fraction α, the internal geometry of the solid material, and would also have to include the integrated value of the correlation coefficient R. Although the task of defining such a function seem formidable, one is helped with some simplifying assumptions.

As discussed before for a weak correlation (Eq. (2.3))

$$\int_0^{\xi} R d\xi = I, \tag{5.3}$$

where I is some constant value. In the case of a void fraction α, one may assume that $R = f(\xi, \alpha)$ and

$$\int_0^{\xi} \int_{\xi=0}^{\infty} R(\xi, \alpha)\, d\xi d\alpha = I(\alpha)\,. \tag{5.4}$$

The expression I/τ of Eq. (2.4) would enter into the function S. Also this function would have the dimension of length and would be expected to differ for various geometrical configurations such as baffles with various randomly distributed holes, or granulated material of various shapes and sizes, etc. In all cases, however, it appears that whatever its variations due to the correlation coefficient R, the distribution of the solid material S would have to decrease monotonically with the decrease of α. At this point one may recall Eq. (2.7) where the standard deviation due to one event was $\sigma_e = L/cu'$. When the wave reflects between the solids, $u' = 0$ [c] and $\sigma_e \approx L$ which again has the dimension of length and which in its generalised form is the deviation function S discussed above. If a new variable ψ is introduced such that $\psi = 1/\alpha$ then $\psi \to \infty$ when $\alpha \to 0$. As $S(\psi)$ would also decrease monotonically with the increase of ψ one may postulate that $dS/d\psi < 0$. One also may imply from physical consideration that the second derivative $d^2 S/d\psi^2$ always has to be positive. For such a general statement a differential relation may be ventured in the form

$$\frac{dS}{d\psi} = -C\exp(-k\psi)\,, \tag{5.5}$$

where the parameters C and k would include the integral values of the correlation coefficient R, the particular geometrical characteristics of the configuration, etc. It follows that

$$S=\frac{C}{k}\exp(-k\psi)+B=A\exp(-k\psi)+B. \tag{5.6}$$

The new constant of integration B indicates that when $\psi\to\infty$ the function may reach some limiting value. The three constants A, B and k require experimental determination and may vary from material to material and various solid configurations. The generality of one representation of S is of importance for a better understanding of the processes involved and has interesting practical aplications.

It follows from Sect. 4 that when $1.75\lesssim\psi<\infty$, i.e. for a densely packed material

$$\sigma=[A\exp(-k\psi)+B]\sqrt{\frac{x}{t}}, \tag{5.7}$$

and when $1<\psi\lesssim1.75$, i.e. for a loosely packed material like perforated baffles

$$\sigma=[A\exp(-k\psi)+B]\sqrt{\frac{x}{g}}. \tag{5.8}$$

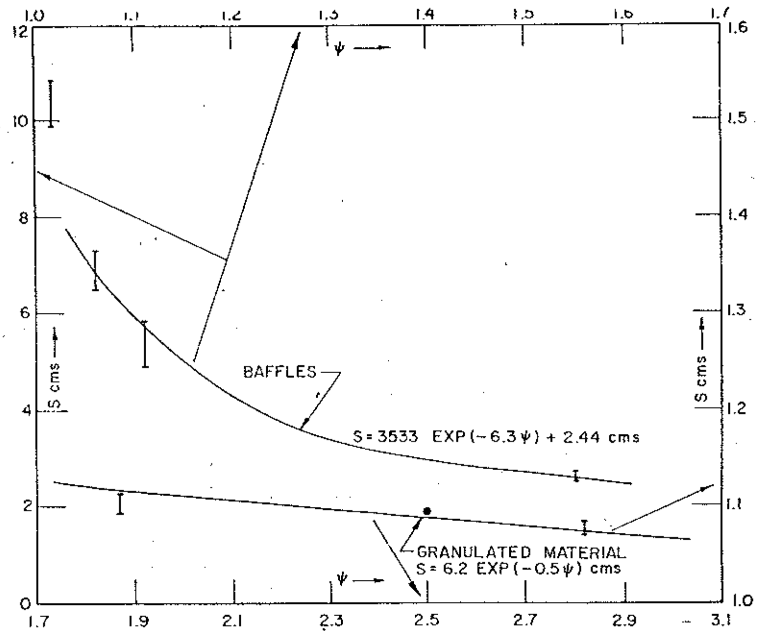

FIG. 8. Experimental and analytical values of deviation function S for baffles and granulated material.

The analysis of the available data seems to justify the above assumptions. It was found that the experimentally-calculated deviation function S was practically constant and independent of the distance x travelled by the wave. Using Eq. (5.6) for the case of baffles and granulated materials, the following expressions were derived respectively:

$$S = 3533 \exp(-6.3\psi) + 2.44 \text{ [cm]} \tag{5.9}$$

and

$$S = 6.2 \exp(-0.5\psi) \text{ [cm]}, \tag{5.10}$$

Figure 8 shows the experimental points together with the fitted curves of the assumed nature and the agreement is good. It appears that for the granulated materials the constant B can be taken as zero.

6. Optimization of the standard deviation

The argument can be pursued further. In the case of baffles Eq. (4.3) indicates that for a certain number N of those and the distance travelled by the wave h, there is a unique relation between the gap g, thikness t and void fraction α. Using $\psi = 1/\alpha$ one readily obtains that $g/t \approx 1/(\psi - 1)$. On the other hand, for a ceratin shock strength one cannot reduce the thickness t beyond certain limits and a limiting t_1 has to be adopted from stress considerations. Thus the gaps so defined, $g_1 \approx t_1 (\psi - 1)$ when this expression is inserted in Eq. (5.8), for any distance x travelled by the wave

$$\sigma_{opt} \approx [A \exp(-k\psi) + B] \sqrt{\psi - 1}, \tag{6.1}$$

which indicates that the standard deviation passes through a maximum for some value of $\psi_{optimum}$. An approximate numerical solution can be found immediately assuming $B = 0$. Then

$$\psi_{opt} = \frac{1 + 2k}{2k}.$$

Using k from Eq. (5.9), one gets $\psi_{opt} = 1.08$, $\alpha_{opt} = 0.976$. Taking $B = 2.44$, a slightly larger value for ψ_{opt} is required, i.e. $\psi_{opt} = 1.18$, $\alpha_{opt} = 0.847$. Using Eq. (4.2) one can also deduce that with a baffle thickness of 0.178 cm, the optimum gap would be $g_{opt} = 0.989$ cm for this type of baffle perforations.

A similar discussion can be drawn for granulated materials. According to their shape, hardness, etc., an empirical relation can be assumed between the grain diameter d and the parameter ψ (In Eq. (5.7) t should be substituted by d). For the tested granulated materials an adequate expression was found in the form

$$d = a\psi^{-n}, \tag{6.2}$$

in which a could be understood as a hypothetical diameter for $\psi = 1.0$, where the range of ψ does not apply anymore. The measured data gave $d = 22\psi^{-4.1}$ cm. Substituting Eq. (6.2) in (5.7) with $d = t$ and $B = 0$, one obtains

$$\sigma_{opt} \sim A \exp(-k\psi)\, \psi^{n/2} \sqrt{\frac{x}{a}}, \tag{6.3}$$

This function passes through a maximum when

$$\psi_{opt} = \frac{n}{2k}$$

which for $n=4.1$, $a=22$ and $k=0.5$ from Eq. (5.10) gives $\psi_{opt}=4.1$, $\alpha_{opt}=0.244$ and $d_{opt}=22\times 0.0031=0.0682$ cm.

7. Conclusions

It appears that not only can a better understanding of shock wave dispersion be derived from this approach but also important practical applications are available. An area of further research should be explored like, for example, scaling the thickness of the baffles, changing the size of their perforations, etc. It appears from Fig. 8 that in many respects the perforated baffles are superior to granulated materials thinking in terms of Eqs. (6.1) and (6.2), especially when breathing is required through the passages where the shock wave is dispersed. It may also be possible to extend this reasoning to other cases like foams which present a very interesting case of shock wave dispersion, waves in detonating mixtures, etc.

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Streszczenie

DYSPERSJA FALI UDERZENIOWEJ W CIECZACH Z LUŹNO ROZŁOŻONYMI SZTYWNYMI WTRĄCENIAMI

Probabilistyczne podejście G. I. Taylora [1, 5] do fal typu wybuchowego w turbulentnym powietrzu rozszerzono na przypadek fal uderzeniowych rozprzestrzeniających się przez rozłożone w sposób przypadkowy sztywne wtrącenia w gazie. Fala uderzeniowa ulega dyspersji i zostaje zamieniona na ciąg fal słabych rozchodzących się przez dłuższy okres czasu. Jeśli straty wskutek

tarcia są małe, to impuls pozostaje w przybliżeniu stały i standardowe odchylenie fali tłumionej, jak wykazano, jest postaci

$$\sigma=[A\exp(-k\psi)+B]\sqrt{\psi-1}\sqrt{\frac{x}{t}}.$$

Wzór ten zachodzi dla przegród z przypadkowo rozłożonymi perforacjami wzdłużnymi. Podobne wyrażenie otrzymano dla materiałów ziarnistych w gazie. Stałe A, B i k zostały określone doświadczalnie. Powyższy wzór dla odchylenia standardowego został zmaksymalizowany ze względu na odwrotność części pustej, ψ, dla danej grubości przegrody i równoważnej średnicy ziaren. Zatem dla grubości przegrody 0,178 cms, $\psi=1.18$, a dla równoważnej średnicy ziaren 0,068 cms, $\psi=4.1$ przy maksimum dyspersji. Straty wskutek tarcia zaniedbane w tej pracy zostały podjęte przez autorów w innym opracowaniu.

Резюме

ДИСПЕРСИЯ УДАРНОЙ ВОЛНЫ В ЖИДКОСТЯХ С РЫХЛО РАСПРЕДЕЛЕННЫМИ ЖЕСТКИМИ ВКЛЮЧЕНИЯМИ

Пробабилистический подход Г, И. Тейлора [1, 5] к волнам взрывного типа в турбулентном воздухе расширен на случай ударных волн распространяющихся сквозь жесткие, распределенные случайным образом включения в газе. Ударная волна испытывает дисперсию и заменяется на ряд слабых волн распространяющихся через длительный период времени. Если потери вследствие трения малы, тогда импулс остается в приблимении постоянным и стандартное отклонение затухающей волны, как показано, имеет вид:

$$\sigma=[A\exp(-k\psi)+B]\sqrt{\psi-1}\sqrt{\frac{x}{t}}.$$

Эта формула имеет место для преград со случайно распределенными продольными перфорациями. Аналогичное выражение получено для зернистых материалов в газе. Постоянные A, B и k определены экспериментально. Вышеприведенное выражение для стандартного отклонения максимизировано из-за обратной величины пустой части, ψ, для данной толщины преграды и эквивалентного диаметра зерен. Итак для толщины преграды 0,178 см, $\psi=1{,}18$, а для эквивалентного диаметра зерен 0,068 см, $\psi=4{,}1$, при максимум дисперсии. Потери вследствие трения, которыми пренебрегается в данной работе, обсуждены авторами в других разработках.

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