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BALANCING AND UNBALANCE IDENTIFICATION OF MULTI-SUPPORT SHAFTS AND ROTORS

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The paper presents the theoretical foundations of a method to eliminate the effects of induction forces originating from the unbalancing of multi-support flexible rotors or transmission shafts. The method provides an adequate control of the rotor or shaft mass distribution and it includes the technical viability to carry out changes in this mass distribution.

Also, a proposal is given as to how unbalancing in rotor or transmission shafts can be identified.

Introduction

One of the methods used to improve the dynamic conditions of a machine structure consists in reducing induced forces acting on its components. Among the structural components of machinery often in use are rotors and shafts of various types. The specific operational character of rotors and transmission shafts (rotational movement) is the reason why induction forces, proceeding from a lack of balance in the system, make their appearance. The term of "unbalance is used here to denote the nonlinearity of geometric centres of symmetry of the rotor (shafts) cross-sections which fail to coincide with the mass centre-line of those cross-sections.

The measure of unbalance is the distance between the two lines above. The unbalance, therefore, is a function which is defined along the whole length of a rotor or a transmission shaft. The unbalance is caused by the non-homogenity of a material and a fault in the assembly or machining of the particular rotor or transmission shafts.

Each shaft or rotor has some unbalance inherent and this may be the cause of the poor dynamic condition of the machine. Hence we note a great interest in seeking a method which would eliminate the effects of rotor or transmission shaft unbalancing. The elimination of rotor unbalancing effects is called balancing.

The dynamic condition of a piece of machinery is determined from a set of physical parameters which appear during its operation. Some of those magnitudes or parameters have become the subject of standard specifications, instructions for assembly and service recomendations [8, 9, 10].

The dynamic condition of a machine is most frequently determined from the amplitudes of rotor (shaft) vibrations and those of their bearings, as well, as from the amplitudes of vibrations of certain parts of its base-plate. The so-called admissible residual unbalance value is also stated for some types of machinery.

Considering the fact that the problems of unbalancing rotors and transmission shafts are essentially similar, we shall discuss the balancing of rotors forthwith. Additionally, we shall restrict our study to the balancing of flexible rotors only, because the balancing of rigid rotors is by far easier and can be achieved, for instance, by means of the so-called method of three starts [11].

Further in this study the term "rotor balancing" will be used to denote such an elimination of unbalancing effects due to which the amplitudes of vibrations in rotor bearings will not exceed their admissible values. A natural way in which a rotor can be balanced is by an appropriate control of rotor mass distribution along the rotor length. This mode of rotor balancing will be discussed further in the course of our considerations. In theory the effect of rotor balancing can be achieved by an appropriate selection of other parameters of the rotor or the machine as a whole, for example, by matching appropriately the values of shifts and skews of the geometric axis of symmetry of the rotor on its couplings. The problem can also be altered by changing the dynamic properties of the rotor supporting components (e.g. of oil film on the slide bearings of a rotor) so as to make the rotor run possibly far from the critical rotational velocities. The induction forces originating from unbalancing will practically affect the rotor at any of its rotational speeds. If so, the balancing of a rotor at any of its rotational speeds would be equivalent to an elimination of its unbalancing along its entire length.

However, from the point of view of application, we are generally interested in having the rotor balanced within a closed interval of its rotational speeds. The appropriate control of the rotor mass distribution can take place in a finite number of rotor cross-sections which are called the planes of balancing. The change of mass distribution in the plane of balancing most frequently consists in affixing in an eccentric way additional masses or the so-called masses of balancing. The choice of number and location of the planes of balancing are of significance.

The "modal-balancing" method [3, 4, 5, 6, 7] is one of the balancing modes widely described in technical literature; it consists in balancing an adequate number of fundamental forms of free vibrations of the rotor. The planes of balancing in this case are closely connected with the nodes of these forms. The method begins with the method of solving the problem of forced vibrations of a rotor.

In the case of forced vibrations, the solution for non-homogeneous equations of motion is sought in a functional space in which the basis is constituted by the forms of rotor-free vibrations. Still, in practice it is not always possible to attach the balancing masses in the planes of balancing resulting from this method because the rotor can be inaccessible in these planes.

Furthermore, the "modal-balancing" method fails to take into account the free vibrations which are dependent on the rotational speed of the rotor. Such being the case, if we assume a wider range of rotational speeds, the possibility of rotor balancing by way of balancing its particular fundamental forms of free vibrations may be practically non-existent.

The above facts have coused the problem of rotor balancing to be solved in a different manner.

The number and location of the planes of balancing were assumed to be known befoιehand as, for example, they could be predicted by the rotor manufactures. It was also assumed that the magnitude of the balancing masses is limited from the above, due to the strenght of their connections to the rotor. An algorithm to slove the problem of forced rotor vibrations is the basis of any method of balancing a flexible rotor for a wide interval of its rotational speeds.

The induction forces originating from unbalancing will prediminantly result in rotor deflection, wherefore the solution is restricted to the question of forced flexional vibrations of the rotor. According to the presented method, the algorithm to solve the forced flexional vibrations of a rotor is applied to determine the so-called rotor sensitivity on the unit unbalancings, that appear successively in the particula planes in which the blancing occurs.

Since the problem is solved according to the linear theory, the magnitude of the balancing masses can be determined in a simple manner, from the known rotor sensitivity, as its susceptibility to unit unbalancings. A collection of vibration amplitudes of the rotor supports was adopted as a measure to indicate rotor sensitivity when applying unit unbalancings in the particular planes of balancing the rotor.

The above collection of amplitudes of vibrations in the rotor supports can be interpreted in terms of physics as a collection of dynamic numbers of influence.

An algorithm which matches well the above assumptions has been presented in the study [1] or, in its supplemented form, in the study [2]. The unbalancing according to the above algorithm can be an arbitrary function, continuous within intervals and defined along the whole rotor length. Such being the case, it is possible to satisfy the assumption on the definite location of the planes of balancing because one can apply, among others, unit unbalancings at any cross-section of the rotor.

It is practically almost impossible to determine by experiment the rotor sensitivity to unit unbalancings within a wide interval of rotational velocities and a high number of balancing planes. At times, in the case of two-bearing rotors, there are attempts to proceed similarly as in the mentioned method of three starts applicable to rigid rotors. Consequently, this method of balancing requires great practical experience on the part of the person conducting the balancing and, further, it is often run by trial and error.

All this testifies to wide interest in the rational methods of rotor balancing.

1. Algorithm to solve forced flexional rotor vibrations

As already mentioned in the introduction, the proposed method of rotor balancing makes use of an algorithm to solve the problem of forced rotor vibrations, as presented in the study [1], or a supplemented algorithm, as proposed in [2]. Its supplementation consists in including the effect of shearing forces acting on the rotor deflection.

Apart from this, both algorithms apply the same physical model of a rotor and its supporting structure.

The following phenomena accompanying rotor vibrations are taken into consideration in the physical model of the rotor: 1) external damping; 2) internal damping of material according to the Voight-Kelvin hypothesis; 3) the gyro effect; 4) inertia of rotor components during the flectional movement; 5) accelaration of gravity; 6) effect of shearing forces on the rotor deflections according to the Timoshenko hypothesis (in algorithm [2] only); 7) rotor unbalancing as an aribtrary function defined along the whole length of the rotor and capable of developing into a Fourier series.

The rotor supporting structure has been divided into two sub-systems: the oil film of the rotor slide bearings and rotor supports plus the machine bedplate. The oil film of the rotor slide bearings is accounted for by the hydrodynamic characteristics its flexibility and damping. The rotor supports plus the bed-plate of the machine are presented by dynamic influence numbers. Here we also have such numbers which depict the coupling of rotor supports by the machine bed plate.

With respect to the rotor geometry, the rotor is assumed to consist of circularly symmetric cylindrical sections. According to the assumption, the rotor is a multi-support structure resting on N supports. The equations of motion, following from the above physical model of a machine, are solved by a particular integral only. This follows from the assumption that the rotor free vibrations decay with time.

From the point of view of the possibility of balancing the rotor, this is a necessary condition.

The above solution of rotor movements is true for an actual rotational speed of the rotor. Among others, the solution of rotor motion equations includes the amplitudes of vibrations of the rotor supports, which are essential in the process of balancing. The values of amplitudes of the vibrations of rotor supports are designated as follows:

$$\zeta_{i(1)};\ \zeta_{i(-1)}, \quad \text{where} \quad i=1, 2, \ldots, N.$$

The above values are complex, their real parts being the amplitudes of support vibrations in the horizontal plane and their imaginary parts, the amplitudes of support vibrations in the vertical plane. Since properties of the supporting structure are anisotropic, there are two harmonic components of machinery parts vibrations for the given rotational speed. One of a frequency corresponding to the velocity Ω, the other of a frequency corresponding to the velocity $-\Omega$. This has been shown by the index sings in brackets. Rotor unbalancing is denoted by $C(z)$, where the variable z is a coordinate of rotor cross-sections measured along its length.

The unbalancing $C(z)$ is a conjugate function of a real variable. The real and the imaginary parts of unbalancing are its coordinates connected with the rotor and with those parts that are rotating jointly with the rotor, respectively.

2. Rotor balancing

Let us assume that we are to balance a rotor and have s planes of balancing at our disposal, their location being determined by the coordinates z_k where $k=1, 2, \ldots, s$, Further, let us assume that the rotor should be balanced for rotational speeds falling within the interval $<\Omega_0;\ \Omega_{max}>$. The magnitude of the balancing masses, attached in the particular planes of balancing, is designated as $m_{\bar{k}}$ where $\bar{k}=1, 2, ..., s$.

It is assumed that the balancing masses induce a change only in the unbalancing, without affecting any other parameters of the rotor. The acceptability of this assumption follows from the fact that the magnitudes of the balancing masses, compared with the rotor mass, are negligibly small.

An additorial assumption is that the balancing masses take up sections, $\varepsilon_{\bar{k}}$ ($\bar{k}=1, 2, ..., s$) long, measured along the rotor axis and that these sections are located symmetrically with respect to the planes of balancing.

The application of a unit mass of balancing $m_{\bar{k}}=1$ in the $\bar{k}$-th balancing plane causes the following change in unbalancing of the rotor, into sections of $\varepsilon_{\bar{k}}$ length, situated symmetrically with respect to this plane of balancing:

(2.1)
$$\Delta C_{\bar{k}}(z)=\mathrm{Re}\Delta C_{\bar{k}}(z)=\frac{r_{\bar{k}}}{\pi R_{\bar{k}}^2\,\varepsilon_{\bar{k}}\,\rho},$$
$$ImC_{\bar{k}}(z)=0.$$

For the sake of simplicity it is assumed here that the unit (test) unbalancing takes place along the axes of the system of coordinates rotating together with the rotor.

In the formula (2.1), $r_{\bar{k}}$ denotes the radius of attachment of the balancing mass; $R_{\bar{k}}$ the radius of the cross-section determined by the $\bar{k}$-th plane of balancing and ρ denotes the density of the rotor material. The change in unbalancing of the entire rotor, caused by a unit unbalancing applied, is as follows:

(2.2)
$$c_{\bar{k}}(z)=\begin{cases}\Delta c_{\bar{k}} & \text{for } z\in <z_{\bar{k}}-\frac{1}{2}\varepsilon_{\bar{k}};\ z_{\bar{k}}+\frac{1}{2}\varepsilon_{\bar{k}}>,\\ 0, & \text{for the remaining rotor part.}\end{cases}$$

When the function (2.2) is applied as the rotor unbalancing, we can obtain (with the use of the algorithm [1] or [2]) the rotor sensitivity to unit unbalancing in the $\bar{k}$-th plane of balancing, in the form of amplitudes of vibrations of its supports:

(2.3)
$$\zeta_{i(1)\bar{k}} \quad \text{for the frequency } \Omega,$$
$$\zeta_{i(-1)\bar{k}} \quad \text{for the frequency } -\Omega,$$

where $i=1, 2, ..., N$.

The following vectors are constructed from the value (2.3):

(2.4)
$$Z_{\bar{k}(1)}=\mathrm{col}\,\{\zeta_{1(1)\bar{k}}, \zeta_{2(1)\bar{k}}, ..., \zeta_{N(1)\bar{k}}\},$$
$$Z_{\bar{k}(-1)}=\mathrm{col}\,\{\zeta_{1(-1)\bar{k}}, \zeta_{2(-1)\bar{k}}, ..., \zeta_{N(-1)\bar{k}}\},$$

(2.5)
$$Z_{\bar{k}}=\mathrm{col}\,\{Z_{\bar{k}(1)}, Z_{\bar{k}(-1)}\}.$$

Considering the fact that the values (2.4) were determined for just one actual rotational speed of the rotor, we propose to denote it by using the symbol $Z_{\bar{k}\Omega}$ and in this way we denote the vector (2.5).

If we apply unit unbalancing in each of the balancing planes, we obtain a whole collection of the above vectors from which the following matrix is formed:

$$\mathbf{Z}_{\Omega}=[Z_{\bar{1}\Omega}, Z_{\bar{2}\Omega}, \ldots, Z_{\bar{s}\Omega}\,. \tag{2.6}$$

Let us select a finite number of rotational speeds from that range of rotor speeds $\langle\Omega_0, \Omega_{\max}\rangle$ for which the rotor is to be balanced:

$$\Omega_0, \Omega_1, \Omega_2, \ldots, \Omega_q=\Omega_{\max}\,. \tag{2.7}$$

The selection of the above speeds should be adequate to make the rotor balancing at these revolutions tantamount with the balancing of the rotor within the entire interval. Such a procedure is dictated by the possibility of rotor balancing for a finite number of rotational speeds only. Otherwise, rotor balancing might result in the elimination of its unbalancing, which may be impossible to accomplish with the finite number of its balancing planes.

Certain indication can be given regarding the selection of the rotational speeds (2.7), viz. these should include possibly all the critical revolutions falling within the interval of $\langle\Omega_0, \Omega_{\max}\rangle$. The matrix (2.6) is to be determined for the particular velocities (2.7).

A successive matrix is constructed from those obtained in the above manner:

$$\mathbf{Z}=[\mathbf{Z}_{\Omega 0}, \mathbf{Z}_{\Omega 1}, \ldots, \mathbf{Z}_{\Omega q}]\,. \tag{2.8}$$

This matrix is the requested collection of the numbers of influence of unit unbalancings on the rotor.

The process of balancing takes place according to the measurements of amplitudes of support vibrations, while the rotor rotates at the successive speeds contained in the collection of its r.p.m. numbers (2.7). The measured magnitudes of the amplitudes of support vibrations are marked thus:

$$\zeta^e_{i(1)\Omega\mu}, \zeta^e_{i(-1)\Omega\mu}, \text{ where } i=1, 2, \ldots, N \tag{2.9}$$

and

$$\mu=0, 1, 2, \ldots, q\,.$$

The following vectors are constructed from the magnitudes (2.9):

$$\begin{aligned} Z^e_{(1)\Omega}&=\text{col}\,\{\zeta^e_{1(1)\Omega}, \zeta^e_{2(1)\Omega}, \ldots, \zeta^e_{N(1)\Omega}\},\\ Z^e_{(-1)\Omega}&=\text{col}\,\{\zeta^e_{1(-1)\Omega}, \zeta_{2(-1)\Omega}, \ldots, \zeta^e_{N(-1)\Omega}\}, \end{aligned} \tag{2.10}$$

$$Z^e_{\Omega}=\text{col}\,\{Z^e_{(1)\Omega}, Z^e_{(-1)\Omega}\}, \tag{2.11}$$

$$Z^e=\text{col}\,\{Z^e_{\Omega_0}, Z^e_{\Omega_1}, \ldots, Z^e_{\Omega_q}\}\,. \tag{2.12}$$

Next, the permissible value of the amplitude of rotor support vibrations is to be determined for each of the rotor supports and every velocity from the collection

(2.7). The vectors to be constructed from the permissible amplitudes of support vibrations are

$$A_{\Omega}=\text{col}\,\{A_{1(1)\Omega}, A_{2(1)\Omega}, \ldots, A_{N(1)\Omega}, \quad A_{1(-1)\Omega}, A_{2(-1)\Omega}, \ldots, A_{N(-1)\Omega}\}, \tag{2.13}$$

and

$$A_{\max}=\text{col}\,\{A_{\Omega_0}, A_{\Omega_1}, \ldots, A_{\Omega_q}\}.$$

Each balancing mass can be treated as a certain additional unbalancing of the rotor which, on the principle of superposition with the unknown balancing of the rotor, will eventually yield its balancing. The following vector is to be constructed from the above additional unbalancing of the rotor, due to the attachment of the balancing masses on the rotor:

$$C=\text{col}\,\{\Delta C_1^*, \Delta C_2^*, \ldots, \Delta C_s^*\}. \tag{2.14}$$

The purpose of the presented methods of rotor balancing is to find the expression (2.14).

From these considerations it follows that the expression (2.14), which solves the problem of rotor balancing, can assume the form of any vector C, satisfying the relationships

$$|ZC+Z^e|\leqslant A_{\max}, \tag{2.15}$$

$$|C|\leqslant C_{\max}. \tag{2.16}$$

The condition (2.16) appears when the magnitudes of the balancing masses are limited, due to a limited strength of the connection between these masses and the rotor.

Let us introduce an additional designation:

$$ZC+Z^e=F(C). \tag{2.17}$$

The magnitude $F(C)$ is a complex linear vectorial form. The solution of the problem (2.15)–(2.16) can be formulated as follows:

Find a minimum value of the form $F(C)$ on a hypercubicoid $0\leqslant|C\leqslant C_{\max}$ from the point of view of a certain norm (to be designated as $\|\cdot\|$) of this form, in other words find such a value $0\leqslant C_0\leqslant C_{\max}$, for which $\|F(C_0)\|=\min$.

When $F(C_0)\leqslant A_{\max}$, then the problem of rotor balancing has been solved satisfactorily.

Such a formulation enables us to apply linear or nonlinear programing methods, depending on whether the mentioned norm, $\|\cdot\|$ links the coordinates or the form $F(C)$ into a linear or into nonlinear combination. Furthermore, with such a formulation we obtain an equivocal solution, whether or not it is possible to balance the rotor in the case when $F(C_0)\geqslant A_{\max}$. One of the reasons why rotor balancing might be impossible to be effectuated is an insufficient number of the available planes of balancing, or else their inconvenient location on the rotor.

3. Identification of unbalancing

A complete recognition of rotoı unbalancing may be useful to eliminate systematic errors in machining and assembly of the ıotor in a large-bath manufacturing process. Similarly, as in the case of rotor balancing, we apply here the same algorithms that were described earlier in the studies [1] and [2].

The information on rotor unbalancing due to the vibrations of its supports will be restricted to an analysis of their synchronous harmonic components. The basis for the identification of balancing will thus be the measured values of amplitudes of the synchronous harmonic component vibrations of rotor supports. As in the previous chapter, the following vector can be constructed with the use of these magnitudes:

(3.1) $$Z^e_{(1)}=\text{col}\,\{Z^e_{(1)\Omega_0},\, Z^e_{(1)\Omega_1},\, \ldots,\, Z^e_{(1)\Omega_q}\}.$$

Hence we seek the unbalancing of the rotor in the form of the finite expansion into a functional series as follows:

(3.2) $$C(z)=\sum_{i=1}^{k} b_i\,\varphi_i(z).$$

The complex functions $\varphi_i(z)$ $(i=1, 2, \ldots, k)$ are the pre-set, lineary independent functions defined along the whole rotor length and developed into a Fourier series. This is how they satisfy the assumption on the unbalancing, set out in the algorithm [1] and [2]. Among others, a polynomial or trigonometric functions can be adopted as functions:

$$\varphi_i(z)/i=1, 2, \ldots, k/.$$

The problem of identification of rotor unbalancing can be solved after the coefficients b_i $(i=1, 2, \ldots, k)$ are determined.

The subsequent vector to be construed from these coefficients is

(3.3) $$B=\text{col}\,\{b_1, b_2, \ldots, b_k\}.$$

Similaıly, as in the problem of rotor balancing, we proceed here with the determination of rotor sensitivity to the action of induction forces following from the unbalancing which is described by the successive functions $\varphi_i(z)$ $(i=1, 2, \ldots, k)$.

To do this, one has to bring successively to zero all the expansion factors in the expansion (3.2), except the factor next to the function for which the rotor sensibility is to be determined. This factor shall be taken as equal to one.

The results of calculations for the above unbalancing can be recorded in the form of amplitudes of the synchronous harmonic components of the supports vibrations as follows:

(3.4) $$U_{i\Omega\mu}=\text{col}\,\{\zeta_{1(1)\Omega\mu},\, \zeta_{2(1)\Omega\mu},\, \ldots,\, \zeta_{N(1)\Omega\mu}\}.$$

where

$$i=1, 2, \ldots, k, \qquad \mu=1, 2, \ldots, N.$$

The subsequent vector is construed from the value (3.4) in the form of

(3.5) $$U_i = \text{col}\,\{U_{i\Omega 0}, U_{i\Omega 1}, \ldots, U_{i\Omega_q}\},$$

where

$$i = 1, 2, \ldots, k.$$

The vectors (3.5) will serve to prepare a matrix containing all the numbers of the effects of unbalancing $\varphi_i(z)$ $(i=1, 2, \ldots, k)$ on the rotor:

(3.6) $$\mathbb{U} = [U_1, U_2, \ldots, U_k].$$

The expansion factors (3.2) can be found by solving the linear system of equations:

(3.7) $$\mathbb{U}B = Z^e.$$

In order to solve the indentification problem completely, one should develop a method for error assessment in the determination of rotor unbalancing.

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Streszczenie

WYWAŻANIE I IDENTYFIKACJA NIEWYWAŻENIA WIELOPODPOROWYCH WAŁÓW I WIRNIKÓW

W pracy podano teoretyczne podstawy metody eliminacji skutków działania sił wzbudzających, pochodzących od niewyważenia wielopodporowych giętkich wirników lub wałów napędowych. Metoda polega na odpowiednim sterowaniu rozkładem masy wirników lub wałów — z uwzględnieniem możliwości technicznej realizacji zmiany tego rozkładu.

Podano również koncepcję identyfikacji niewyważenia wirników lub wałów napędowych.

Резюме

УРАВНОВЕШИВАНИЕ И ИДЕНТИФИКАЦИЯ НЕУРАВНОВЕШЕННОСТИ ВАЛОВ И РОТОРОВ С МНОГИМИ ОПОРАМИ

В работе даются теоретические основы метода исключения следствий действия возбуждающих сил[происходящих от неуравновешенности гибких роторов или приводных валов с многими опорами. Метод заключается в соответствующем управлении распределением массы роторов или валов[с учетом технической возможности реализации изменения этого распределения. Дается тоже концепция идентификации неуравновешенности роторов или приводных валов.

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