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STRESSES, TEMPERATURE AND OSCILLATIONS OF THERMOELASTIC LAYER SUDDENLY EXPOSED TO SYMMETRIC PRESSURE

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In the Principles of Thermoelasticity [1], KOVALENKO presented the limit values of temperature increase in a thermoelastic layer, exposed to a symmetric step in time pressure. This work treats the same problem. The approximate solutions are found for temperature, stresses and displacements in function of time and space variable.

1. The problem

Surfaces of the isolated thermoelastic layer, being in the homogeneous temperature field $T(x, t)=T_0=\text{const}$ $(t<0)$ at first, are pressed in the moment $t=0$ with constant pressure p_0.

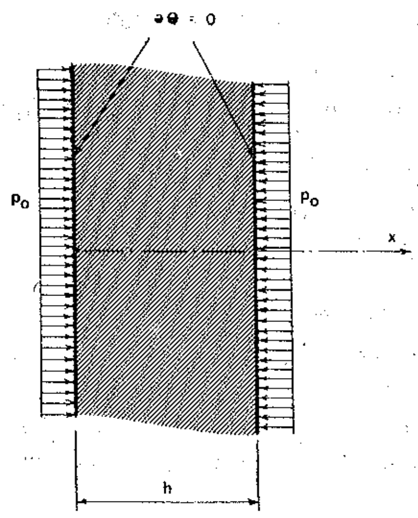

FIG. 1.

The stresses, the temperature and the oscillations of the layer in function of space and time variable are to be found.

If the unit of length, time, temperature and stress are taken as

$$u_l=\frac{a}{c_1},\quad u_t=\frac{a}{c_1^2},\quad u_T=\frac{1}{\alpha},\quad u_\sigma=\frac{E}{1-2\nu},$$

where c_1 represents the isothermic velocity of the longitudinal wave, a — thermal conductivity, α — thermal dilation coefficient, E — modul of elasticity and ν — Poisson's ratio, one gets the following set of partial differential equations, initial and boundary conditions of this coupled thermoelasticity problem in dimensionless form ([1], 275):

(1.1) $$\partial^2 \sigma - \partial_t^2 \sigma - \partial_t^2 \gamma = 0,$$

(1.2) $$\partial^2 \theta - (1+\varepsilon)\, \partial_t Q - \varepsilon \partial_t \sigma = 0,$$

(1.3) $$\sigma(x, 0) = \partial_t \sigma(x, 0) = \theta(x, 0) = \partial_t \theta(x, 0) = 0,$$

(1.4) $$\begin{aligned} &\sigma(x_b, t) = -p_0 H(t), \\ &\partial\theta(x_b, t) = 0, \quad x_b = 0 \text{ and } h. \end{aligned}$$

In the equations above, ε represents the coefficient wich couples stress and temperature fields and $H(t)$ represents the Heaviside unit function.

The system (1.1), (1.2) can be replaced by the following system of partial differential equations expressed in the matrix form:

(1.5) $$\partial_t \begin{bmatrix} \dot{\sigma} \\ \sigma' \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & (1+\varepsilon)\,\partial & -\partial^2 \\ \partial & 0 & 0 \\ 0 & -\varepsilon\partial & \partial^2 \end{bmatrix} \begin{bmatrix} \dot{\sigma} \\ \sigma' \\ \dot{\theta} \end{bmatrix},$$

where the comma beside and the dot above the symbol means partial differentiation of corresponding variable by space and time, respectively.

Briefly, Eq. (1.5) can be written in the following form:

(1.6) $$\partial_t U(x, t) = L U(x, t),$$

where the meanings of the vector function $U(x, t)$ and the matrix differential operator $L(\cdot)$ are evident.

Initial and boundary conditions for Eq. (1.5) are

(1.7) $$U(x, 0) = 0,$$

(1.8) $$\begin{aligned} &\dot{\sigma}(x_b, t) = -p_0\, \delta(t), \\ &\partial\dot{\theta}(x_b, t) = 0, \end{aligned} \qquad x_b = 0 \text{ and } h.$$

$\delta(t)$ being the Dirac symbolic function.

To simplify the procedure of solving the problem posed above, one can replace this problem, with homogeneous initial conditions (1.7) and nonhomogeneous boundary conditions (1.8), by an equivalent problem, with the nonhomogeneous initial conditions

(1.9) $$U(x, 0) = -p_0 \begin{bmatrix} 0 \\ \delta(x-h) - \delta(x) \\ 0 \end{bmatrix}$$

and the homogeneous boundary conditions

(1.10) $$\dot{\sigma}(x_b, t) = \partial\dot{\theta}(x_b, t) = 0, \quad x_b = 0 \text{ and } h.$$

It is easy to show that those two problems are equivalent by simply comparing their extended differential operators [4].

2. Eigenvalues and eigenvector functions of matrix. Differential operator L

In order to get the spectral representation of the operator L, we seek the solution of the eigenvalue problem

$$L\Phi_i(x)=\lambda_i\,\Phi_i(x), \quad x\in(0,h), \quad i=1,2,3,\ldots, \tag{2.1}$$

where λ_i are eigenvalues and

$$\Phi_i(x)=\begin{bmatrix}\varphi_{i1}(x)\\ \varphi_{i2}(x)\\ \varphi_{i3}(x)\end{bmatrix}$$

are eigenvectors satisfying the homogeneous boundary conditions

$$\varphi_{i1}(x_b)=\partial\varphi_{i3}(x_b)=0, \quad x_b=0 \text{ and } h. \tag{2.2}$$

By eliminating the second and the third coordinate functions from the system (2.1), one finds a differential equation of the rank four, for $\varphi_{i1}(x)$. Applying the Fourier finite sine transform [3] to that equation and taking into account the boundary conditions $\varphi_{i1}(x_b)=0$, together with the approximate boundary conditions $\varphi''_{i1}(x_b)\approx 0$(1) ($x_b=0$ and h), we reach the characteristic equation

$$\lambda^3+\alpha_k^2\lambda^2+(1+\varepsilon)\,\alpha_k^2\,\lambda+\alpha_k^4=0, \tag{2.3}$$

where $\alpha_k\approx k\pi/h$ ($k=1,2,\ldots$). The sign of approximate equality is the consequence of adopted approximation in boundary conditions.

Equations (2.3) gives three eigenvalues for every k: two of them are complex conjugate $\lambda_{k1}=\bar{\lambda}_{k2}$ (with a negative real part) and one λ_{k3} is real (also negative).

Now, having found the eigenvalues λ_{km} ($k=1,2,\ldots;\ m=1,2,3$), we can easily find the eigenvectors $\Phi_{km}(x)$. The first and the second of these vectors are complex conjugate $\Phi_{k1}(x)=\bar{\Phi}_{k2}(x)$ and the third $\Phi_{k3}(x)$ is real.

We must add that the eigenvalue $\lambda_0=0$ give an untrivial solution of Eqs. (2.1) and (2.2), as well. So we include zero in the eigenvalue spectrum and a non-zero eigenvector Φ_0 in the set of eigenvectors $\Phi_{km}(x)$ ($k=1,2,\ldots;\ m=1,2,3$), which form the base of our vector space.

3. Adjoint matrix differential operator

We seek the adjoint matrix differential operator L^* by using the following equality of two complex scalar products:

$$\langle G, LF\rangle=\langle L^*G, F\rangle. \tag{3.1}$$

The domain of the operator L represents the set of three-dimensional square integrable vector functions $F(x)$ ($x\in(0,h)$), satisfying the homogeneous boundary conditions of the type (2.2) and the domain of L^* represents the set of three-dimen-

(1) These boundary conditions are equivalent to $\sigma''(x_b)\approx 0$, which are satisfied identically in an uncoupled solution.

sional vector functions $G(x)$ $(x \in (0, h))$, the boundary conditions of which can be determined from the demand that the transformation of the left side of Eq. (3.1) into the right side becomes homogeneous.

In such a way we find the adjoint operator

$$L^* = \begin{bmatrix} 0 & -\partial & 0 \\ -(1+\varepsilon)\partial & 0 & \varepsilon\partial \\ -\partial^2 & 0 & \partial^2 \end{bmatrix} \tag{3.2}$$

and its adjoint homogeneous boundary conditions

$$\begin{aligned} \bar{g}_1(x_b) &= \frac{\varepsilon}{1+\varepsilon}\,\bar{g}_3(x_b), \\ \partial\bar{g}_1(x_b) &= \partial\bar{g}_3(x_b), \end{aligned} \qquad x_b = 0 \text{ and } h. \tag{3.3}$$

The bars over the symbols mean that the adjoint boundary conditions are given by the complex conjugates of the respective functions.

4. Eigenvector functions of the adjoint operator L^*

If λ_{km} is an eigenvalue of L, then $\bar{\lambda}_{km}$ (complex conjugate of λ_{km}), is an eigenvalue of its adjoint L^* [4]. Now it is easy to find the eigenvectors Ψ_{km} $(j=1, 2, 3, \ldots,$ $m=1, 2, 3)$ in the eigenvalue problem

$$L^*\,\Psi_i(x) = \lambda_i\,\Psi_i(x), \qquad x \in (0, h), \qquad i = 1, 2, 3, \ldots \tag{4.1}$$

with boundary conditions of the type (3.3). Therefore, for every k we shall get the set of three vectors again: two of them are complex conjugates and the third one is real. We complete the set of adjoint eigenvectors with Ψ_0, obtained when we use the zero eigenvalue in Eqs. (3.3) and (4.1).

The complete set of adjoint eigenvectors form the inverse base of our vector space.

5. The solution

As usual, we replace the sets of eigenvectors Φ_0, Φ_{km} and adjoint eigenvectors Ψ_0, Ψ_{km} by two orthonormalised sets of vectors. They must satisfy the conditions

$$\langle \Psi_0, \Phi_0 \rangle = 1,$$

$$\langle \Psi_{ij}, \Phi_{km} \rangle = \delta_{ik}\,\delta_{jm} \qquad (i, k = 1, 2, \ldots; j, m = 1, 2, 3).$$

We now seek the solution of the problem (1.5), (1.9), (1.10) in the form

$$U(x, t) = d_0(t)\,\Phi_0 + \sum_{k=1}^{\infty}\sum_{m=1}^{3} d_{km}(t)\,\Phi_{km}(x), \tag{5.1}$$

where $d_0(t)$ and $d_{km}(t)$ are, for the time being, some unknown complex functions of time. Substituting Eq. (5.1) into Eq. (1.5) and using Eq. (2.1), we have

$$d_0\,\Phi_0+\sum_{k=1}^{\infty}\sum_{m=1}^{3}(\dot{d}_{km}-\lambda_{km}\,d_{km})\,\Phi_{km}=0. \tag{5.2}$$

The eigenvectors Φ_0, Φ_{km} are linearly independent, so Eq. (5.2) is equivalent to an infinite system of the ordinary differential equations of the first order with respect to time. The solutions of these equations are $d_0=a_0$ and $d_{km}=a_{km}\,e^{\lambda_{km}t}$, where a_0 and a_{km} represent complex constants of integration. Therefore, the vector function $U(x,t)$ has the following form:

$$U(x,t)=a_0\,\Phi_0+\sum_{k=1}^{\infty}\sum_{m=1}^{3}a_{km}\,e^{\lambda_{km}t}\,\Phi_{km}(x). \tag{5.3}$$

We introduce the initial condition (1.9) into Eq. (5.3) and form the left scalar product with the adjoint eigenvectors ψ_0, ψ_{km} ($k=1,2,3,\ldots$; $m=1,2,3$). Integration of the obtained equations in the space interval $(0,h)$ results in the integral constants a_0, a_{km}.

Integrating the first and the third coordinate of $U(x,t)$ with respect to time and using the initial conditions $\sigma(x,0)=\theta(x,0)=0$, we obtain

$$V(x,t)=\begin{bmatrix}\sigma(x,t)\\ \theta(x,t)\end{bmatrix}=V_w(x,t)+V_c(x,t), \tag{5.4}$$

where

$$V_w=2\sum_{k=1}^{\infty}\mathrm{Re}\left\{\frac{a_{k1}\,b_{k1}}{\lambda_{k1}}\begin{bmatrix}\varphi_1(x)\\ \varphi_3(x)\end{bmatrix}_{k1}\right\}[e^{\mathrm{Re}(\lambda_{k1})t}\cos J_m(\lambda_{k1})\,t-1], \tag{5.5}$$

$$V_c=\sum_{k=1}^{\infty}\frac{a_{k3}\,b_{k3}}{\lambda_{k3}}\begin{bmatrix}\varphi_1(x)\\ \varphi_3(x)\end{bmatrix}_{k3}(e^{\lambda_{k3}t}-1), \tag{5.6}$$

and b_{k1} and b_{k3} are complex multiplicators of corresponding (orthonormalised) eigenvectors.

The vector V_w represents the stress-thermal wave (²) travelling through the layer with the velocity $\sim c_1\sqrt{1+\varepsilon}$, c_1 being the velocity of the longitudinal "isothermal" wave (of the wave travelling through the medium with the coupling factor zero). This result is in accordance with the one obtained by CHADWICK ([5], 291). The components of this vector are multiplied by the damping factors $\exp[\mathrm{Re}(\lambda_{k1})\,t]/$ $/\mathrm{Re}(\lambda_{k1})<0$, so the form of the wave is being changed constantly.

The second vector V_c represents the stress-thermal field originated by the conduction of heat from the thermal wave during the compression phase and vice versa, during the dilatation phase. This shows that in the coupled thermoelasticity solution the wave front is not a well-defined line of discontinuity, but a diffusive zone.

(²) The stress wave and thermal wave are similar in shape during the whole process as it will be hown later in the approximate solution.

One can get the function of dimensionless displacements $u(x, t)$ from

(5.7) $$U(x, t)=\frac{1+\nu}{1-\nu}\int(\sigma+\theta)\,dx+c(t),$$

where the function $c(t)$ is to be found from the symmetry condition $u(h/2, t)=$ $=0$.

6. Approximate solution

We shall discuss the approximate form of solution (5.7) which one obtains considering the fact that for real technical materials the thickness of the layer is greater than l_{mn} and the sufficient number of summation terms $k\leqslant 10^2-10^3$ the strong inequality $\alpha_k\ll 1$ is valid.

In this case, solutions of the characteristic equation (2.3) are

(6.1) $$\lambda_{k1}=\bar{\lambda}_{k2}\approx-\frac{\varepsilon}{2(1+\varepsilon)}\alpha_k^2+i\alpha_k\sqrt{1+\varepsilon},$$

(6.2) $$\lambda_{k3}\approx-\frac{\alpha_k^2}{1+\varepsilon}.$$

Using these values and neglecting the contribution of the thermal diffusion to the shape of the stress-thermal field, we get

(6.3) $$\begin{bmatrix}\sigma(x, t)\\ \theta(x, t)\end{bmatrix}=p_0\begin{bmatrix}-1\\ \dfrac{\varepsilon}{1+\varepsilon}\end{bmatrix}\left[1-\frac{4}{\pi}\sum_{k=1,3,\dots}^{10^2-10^3}\frac{1}{k}e^{-\frac{\varepsilon}{2(1+\varepsilon)}\alpha_k^2 t}\cos\alpha_k\sqrt{1+\varepsilon}\,t\sin\alpha_k x\right].$$

If we put $\varepsilon=0$ in this solution, we obtain a vector the first coordinate of which (stresses) is represented by a rectangular wave propagating with the velocity c_1 and the second (temperature) is equal to zero. Those are the well-known results of uncoupled thermoelasticity.

Expression (6.3) shows that the stress wave and thermal wave are similar in shape to within a constant multiplicator. Opposite signs of these multiplicators mean that during the compression phase of the oscillation, the temperature of the thermal wave is increased and during the dilatation phase — decreased.

When $t\to\infty$, the existence of damping multiplicatiors makes all the terms of the functional series in Eq. (6.3) tend to zero and in a limit we obtain a constant vector the coordinates of which represent dimensionless expressions for the limit values of the stress and of the temperature increase, obtained by Kovalenko [1].

The displacement, in the approximate form, is given by

(6.4) $$U(x, t)=$$
$$=-\frac{1+\nu}{1-\nu}\frac{p_0}{1+\varepsilon}\left[x-\frac{h}{2}+\frac{4h}{\pi^2}\sum_{k=1,3,\dots}^{10^2-10^3}\frac{1}{k^2}e^{-\frac{\varepsilon}{2(1+\varepsilon)}\alpha_k^2 t}\cos\alpha_k\sqrt{1+\varepsilon}\,t\cos\alpha_k x\right].$$

If we put $\varepsilon=0$ in this equation again, we shall obtain the well-known displacement function of the classical theory of elasticity.

The term tending to zero exists as well in Eq. (6.4) when time tends to infinity. Therefore, the displacements are going to stabilise in one position — the position of the static displacements for $t=\infty$. The term $1+\varepsilon$ by which the static displacement is divided represents the influence of heating (i.e. coupling) on the displacement.

7. Example

We shall expose the isolated steel layer to the pressure $p_0=100$ MPa. The thermomechanical characteristics of the material are: $\varepsilon=0.0114$, $c_1=5910$ m/s, $a=0.13$ cm²/s, $\nu=0.3$, $E=2.06\cdot10^5$ MPa, $\alpha=1.2\cdot10^{-5}$ grad^{-1}. The temperature of the natural state is taken to be $T_0=293$°K.

The limit temperature increase, i.e. $\lim T-T_0$ (see Eq. (6.3) and introduce the dimensions):

$$(T-T_0)_{\lim}=\frac{1}{\alpha}\,\frac{\varepsilon}{1+\varepsilon}\,\frac{1-2\nu}{E}\,p_0=0.18 \text{ grad}.$$

Figures 2–6 represent the stress-thermal waves in different phases of oscillations. The velocity of wave propagation is $c_1\sqrt{1+\varepsilon}$ and the period of oscillations is $2h/c_1\sqrt{1+\varepsilon}$.

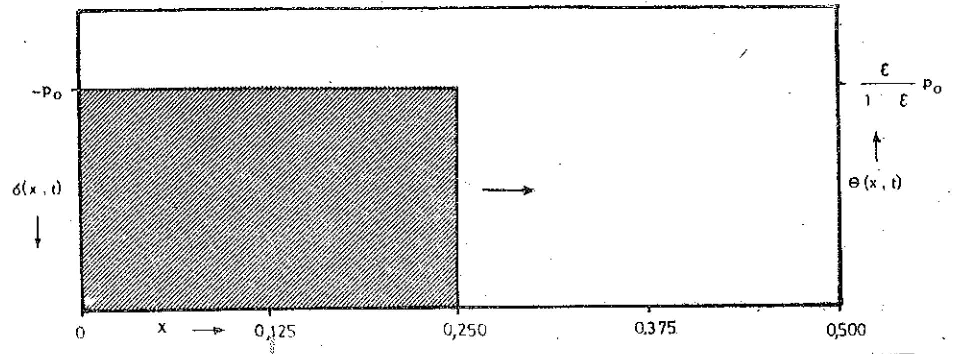

Fig. 2. Beginning of the oscillations. The stress-thermal wave for $t=T/8$ $(T=2h/\sqrt{1+\varepsilon})$.

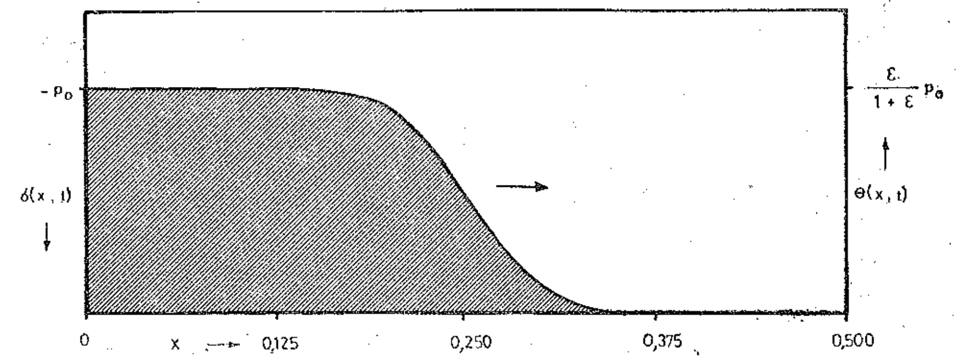

Fig. 3. The stress-thermal wave for $t\approx0{,}005n_t\,T+T/8$ (n_t is the total number of oscillations).

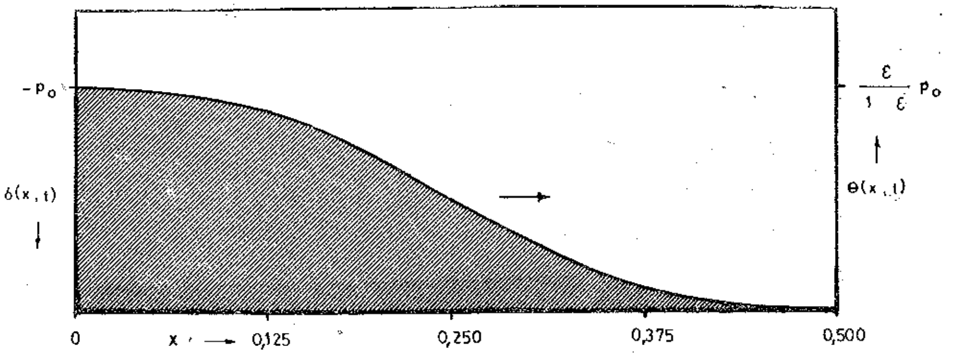

FIG. 4. The stress-thermal wave for $t \approx 0.05 n_t T + T/8$.

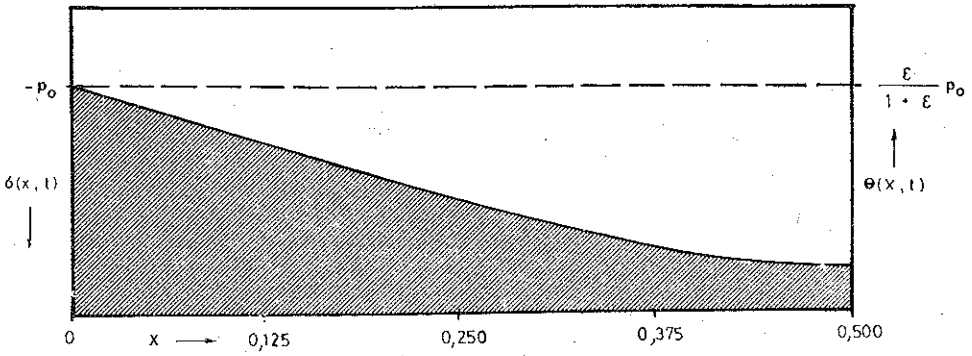

FIG. 5. The stress-thermal wave for $t \approx 0.2 n_t T + T/8$.

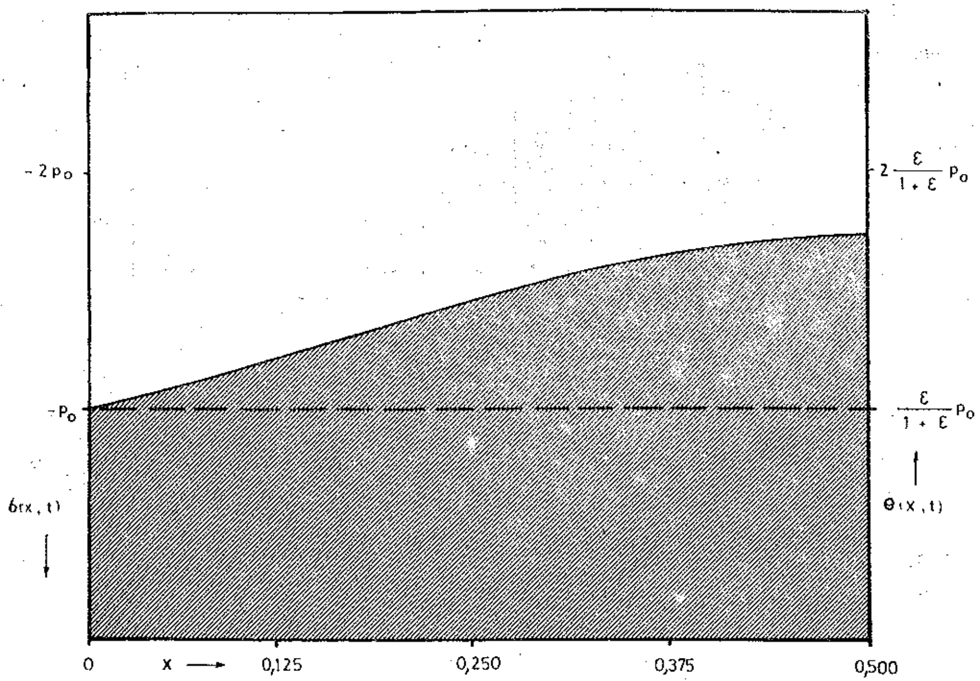

FIG. 6. The stress-thermal wave for $t \approx 0.2 n_t T + 3T/8$. The broken line denotes the limit values of stresses and temperature.

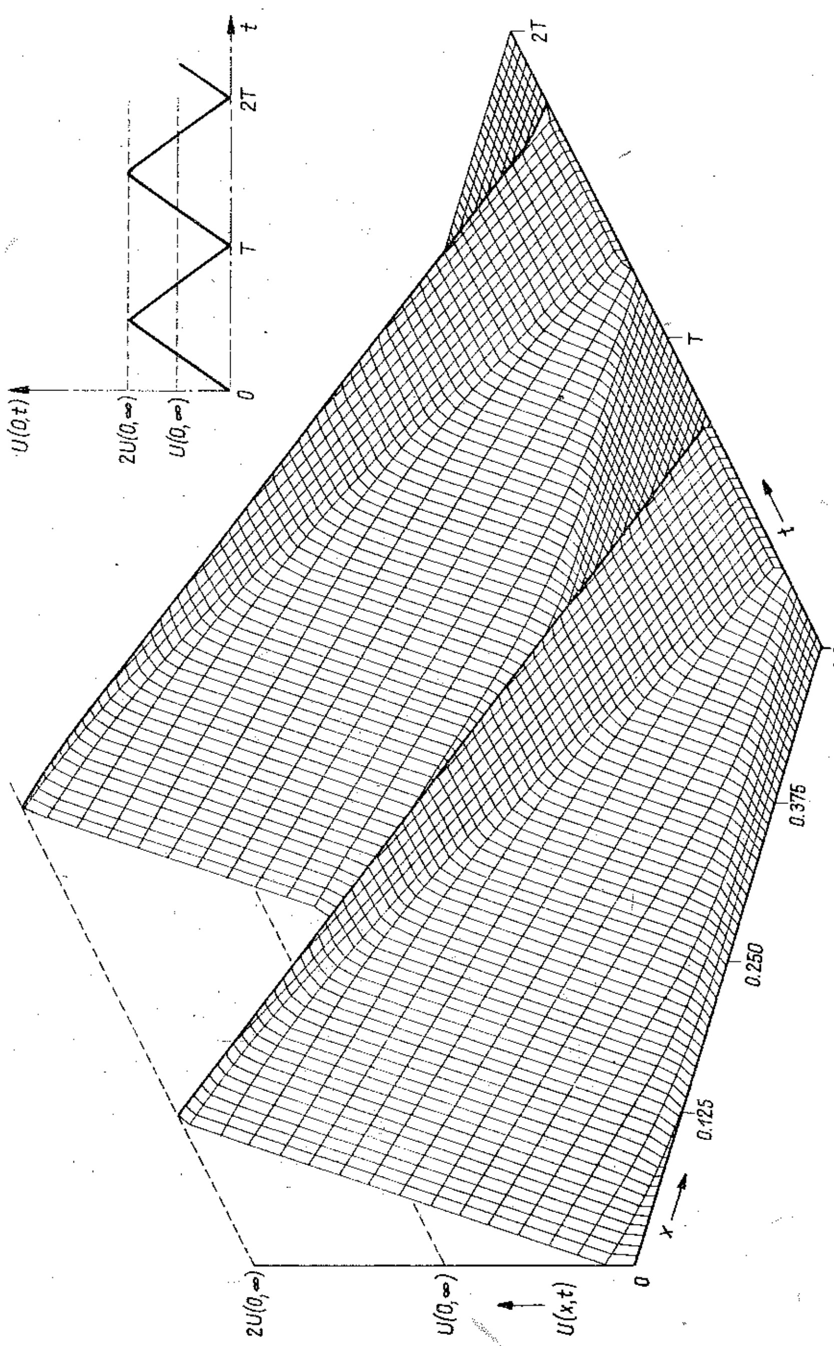

FIG. 7. The displacement field. Beginning of the oscillations.

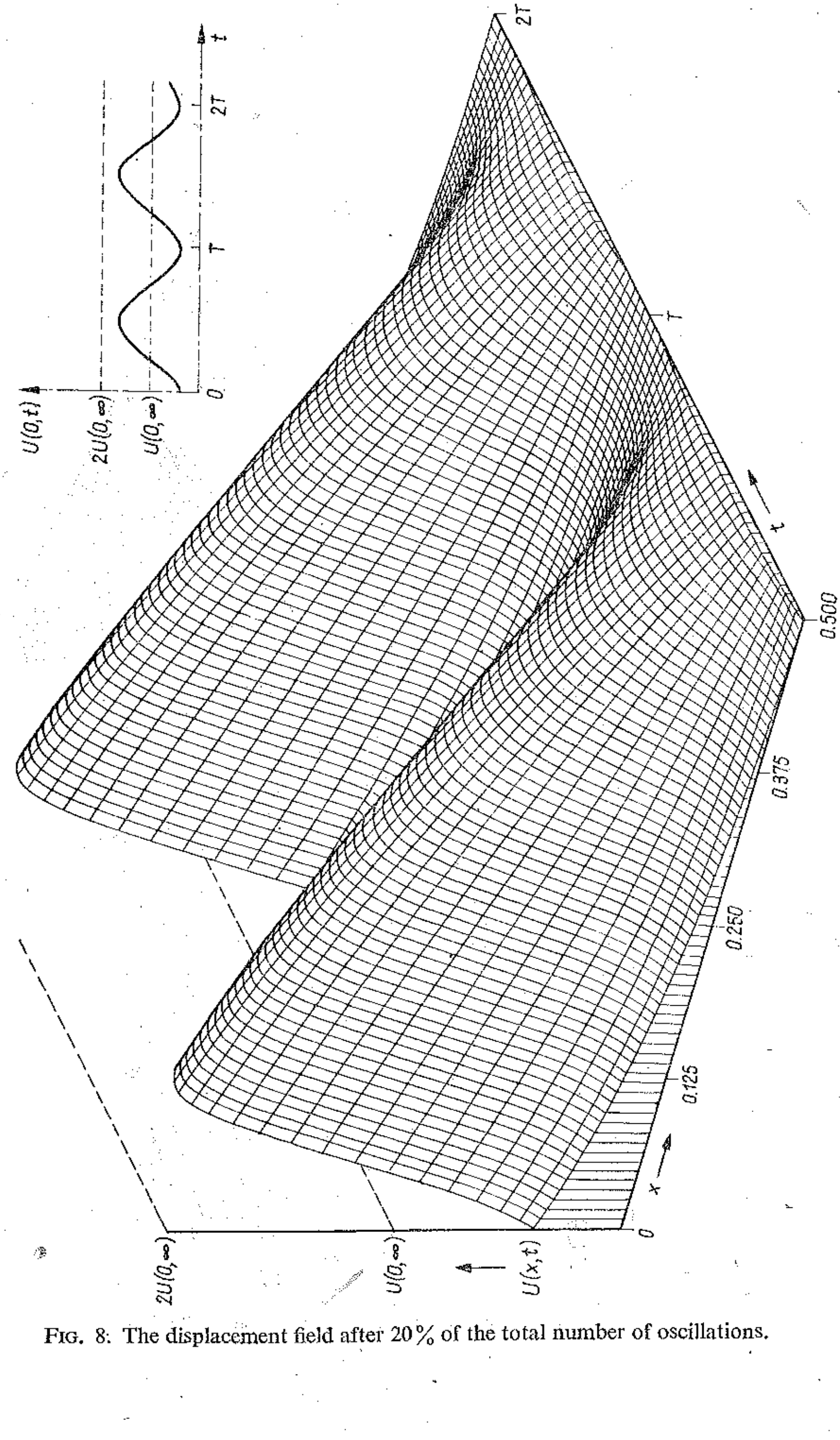

FIG. 8: The displacement field after 20% of the total number of oscillations.

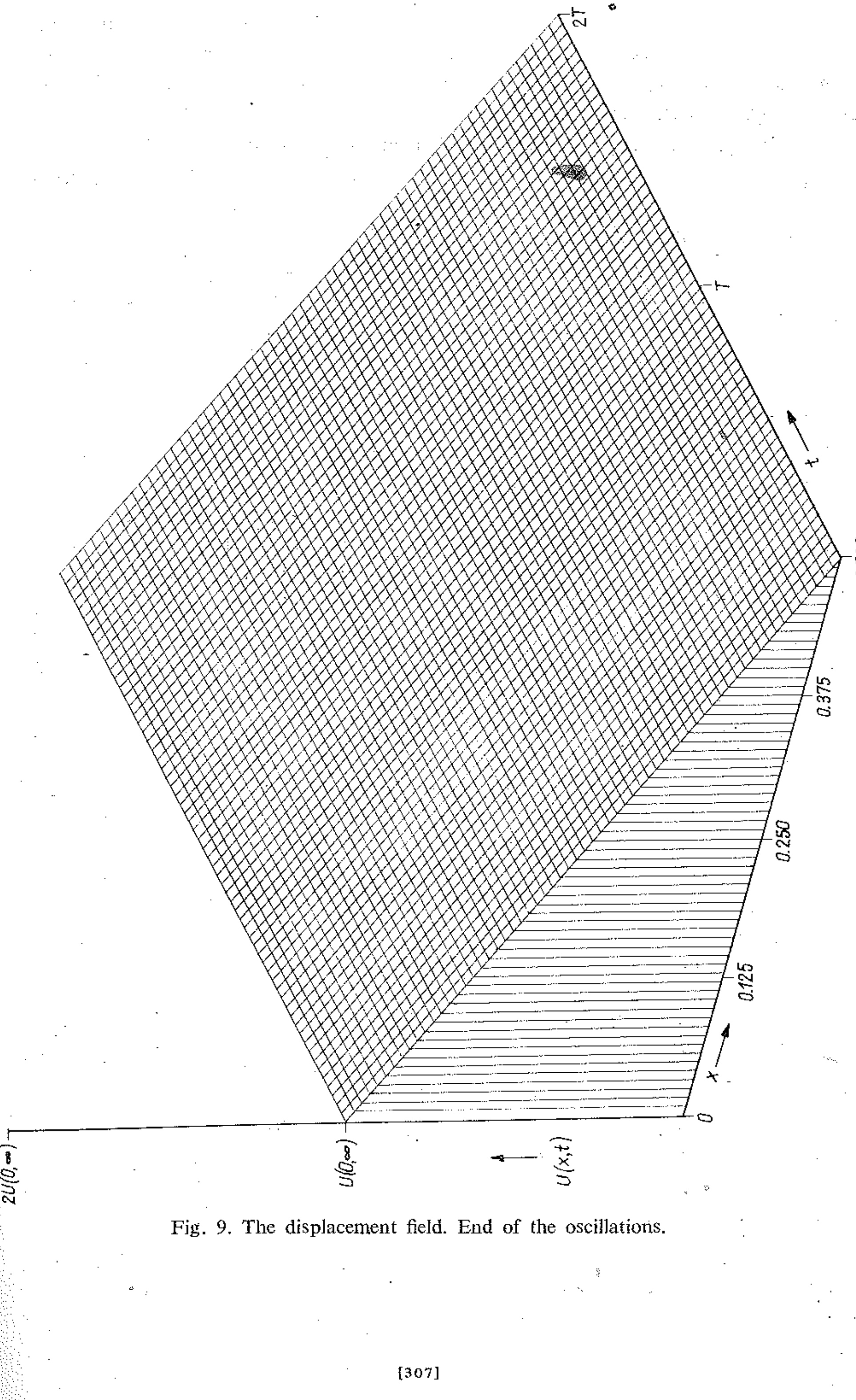

Fig. 9. The displacement field. End of the oscillations.

At the beginning of oscillations the magnitude of the greatest (the first one) damping factor is close to one, so the shape of the wave is rectangular. Figure 2 represents such a wave for $t=T/8$. At the beginning there is no difference between the stress wave of this solution and the stress wave in the elastic solution but one must not forget that here we have the thermal wave as well.

Figure 3 represents the stress-thermal wave after approximately 0.5% of the total number of oscillations ([3]): $t \approx 0.005\ n_t + T/8$. The wave in the moment $t \approx 0.05\, n_t\, T + T/8$ is represented in Fig. 4.

After approximately 20% of the total number of oscillations, because of damping, there remains the first term of the functional series in Eq. (6.3) only and instead of the wave travelling along the x axis; we now have a standing wave (Figs. 5 and 6).

Finally, when the greatest term of the functional series becomes practically zero ($t=t_d$) ([4]), the stresses and temperature of the layer stabilise to their limit values — broken lines in Figs 5 and 6.

Figures 7, 8 and 9 are stereometric representations of the displacement function $u(x, t)$ in different phases of oscillations.

At the beginning (Fig. 7), the displacement function corresponds to the one obtained in the elastic solution, taking into account the heating of the layer due to coupling. The figure in the upper right side represents the oscillations of the point $x=0$.

The same function after approximately 20% of the total number of oscillations appears in Fig. 8. Its shape is perceptibly modified, due to the thermoelastic energy dissipation. The triangular wave describing the motion of the point $x=0$ at the beginning becomes now a dislocated sinusoidal wave with considerably smaller amplitude.

The last figure represents the displacement function after re-establishment of the stability.

8. Conclusion

The analysis of the limit value of the temperature increase in the layer indicates that it is a function of the pressure the layer is exposed to, of the coupling coefficient, of the thermal dilatation coefficient, of the Poisson's ratio and of the elasticity modulus. The limit does not depend on the thickness of the layer.

If we analyse the exponent of the damping function of the greatest term in the functional series of Eq. (6.3) or Eq. (6.4) — i.e. $\mathrm{Re}(\lambda_{11})\, t$, we see that the number of oscillations until total damping is accomplished is proportional to the thickness of the layer, the velocity of the wave propagation, the coupling coefficient and that it is in inverse proportion to the coefficient of the temperature conductivity.

([3]); ([4]) Reestablishment of the mechanical and thermal stability in this solution requires of course $t=\infty$ but for the practical purpose one can take that there is no more motion when the greatest damping factor $\exp[\mathrm{Re}(\lambda_{11})\, t]$ becomes smaller than a given small and positive number δ. In this work we took $\delta=10^{-4}$ and so we could compute the "total number of oscillations" n_t and the "decay time" t_d.

Proportionality to the thickness indicates that a ten times thickeer layer of one material gives a ten times greater number of oscillations.

The decay time is also in inverse proportion to the coefficient of the temperature conductivity and it is proportional to the coupling coefficient and to the square of the thickness of the layer. Consequently, a ten times thicker layer of one material oscillates a hundred times longer.

Now, if we want to see the influence of the coupling factor in the problem, we can take, as Kovalenko did, polyvinyl-butiral and suppose that this material is thermoelastic. The thermomechanical characteristics of this material are: $\varepsilon=0.432$, $c_1=2344$ m/s, $a=0.0016$ cm^2/s, $\nu=0.4$, $E=2747$ MPa, $\alpha=2.3\cdot10^{-4}$ grad^{-1}. The pressure is $p_0=100$ MPa again and the temperature of the natural state $T_0=293°$K.

The limit increase of the temperature is now

$$(T-T_0)_{\lim}=9.4 \text{ grad}.$$

The coupling coefficient is an important factor as regards the temperature increase in the layer. On the other hand, the coupling coefficient does not influence the stress-thermal wave or the displacement to a great extent. After the same number of oscillations, a wave (primary rectangular) in the polyvinyl-butiral layer will not be more deformed than a wave in the steel layer. Moreover, due to their dependence on other thermomechanical factors, the total number of oscillations and the decay time in a „thermoelastic" polyvinyl butiral are greater than corresponding quantities in the steel layer of the same thickness.

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Streszczenie

NAPRĘŻENIA, TEMPERATURA I DRGANIA WARSTWY TERMOSPRĘŻYSTEJ PODDANEJ NAGŁEMU DZIAŁANIU SYMETRYCZNIE ROZŁOŻONEGO CIŚNIENIA

W książce „Zasady Termosprężystości" [1] Kowalenko przedstawił graniczne wartości wzrostu temperatury w warstwie termosprężystej pod działaniem symetrycznego ciśnienia przyłożonego w sposób nagły według funkcji Heaviside'a. W pracy niniejszej rozważono podobny problem; wyznaczono przybliżone rozwiązania dla temperatury, naprężeń i przemieszczeń jako funkcje czasu i zmiennych przestrzennych.

Резюме

НАПРЯЖЕНИЕ, ТЕМПЕРАТУРА И КОЛЕБАНИЯ ТЕРМОУПРУГОГО СЛОЯ ПОДВЕРГНИТОГО ВНЕЗАПНОМУ ДЕЙСТВИЮ СИММЕТРИЧЕСКИ РАСПРЕДЕЛЕННОГО ДАВЛЕНИЯ

В книге „Принципы термоупругости" [1] Коваленко представил предельные значения роста температуры в термоупругом слое под действием симметричного давления, приложенного внезапным образом согласно функции Хевисайда. В настоящей работе рассмотрена аналогичная проблема; определены приближенные решения для температуры, напряжений и перемещений как функций времени и пространственных переменных.

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