ROZPRAWY INŻYNIERSKIE • ENGINEERING TRANSACTIONS • 32, 4, 563—573, 1984
Polska Akademia Nauk • Instytut Podstawowych Problemów Techniki

ON DISTRIBUTED PARAMETER SYSTEM IDENTIFICATION WITH A PRIORI INFORMATION AS CONSTRAINTS

E. RAFAJŁOWICZ (WROCŁAW)

The identification procedure presented in the paper makes it possible to determine the unloaded estimator of parameters of a system with distributed. parameters. They are derived by minimization of the quality functional in the quadratic form with the constrains being taken into account. Various properties of the procedure proposed are discussed, and a numerical example is presented.

1. Introduction

In empirical studies we are frequently faced with the problem of extension of an existing theory. More precisely, we have a mathematical model of a physical process in which certain parameters or variables are not taken into account and our aim is to incorporate them into the model. From the methodological point of view it is desirable to find a compatible extension of the existing model. This means that if the variables incorporated into the extended model are fixed, then it behaves like the existing model. The above idea is expressed in general terms of the dimensional analysis as the correspondence principle, (see e.g. [2, 3]). In these papers a general approach called the multistage identification is proposed as a tool for model extension, retaining the correspondence principle. This approach, however, is difficult to apply when the existing model is expressed in terms of partial differential equations (PDE). We remark that an algorithm for solving this problem was proposed in [4]. This algorithm is based on the perturbation theory, what restricts its applications. Our aim is to propose an algorithm for the same problem but applicable to a wider class of linear PDE. It is based on the Ritz — Galerkin method and for this reason the correspondence principle holds in a limit sense only.

In order to fix ideas, we start from the following simple example which is further used as a testing one. Suppose that our aim is to identify an impact of a uniform tension T in the X_1 direction on the deflection of a simple supported rectangular plate subject to uniform lateral load (see Fig. 1). As it is known (see e.g. [7]), the theory in this case is well developed, but here it will be used in Sect. 5 for simulation purpose only.

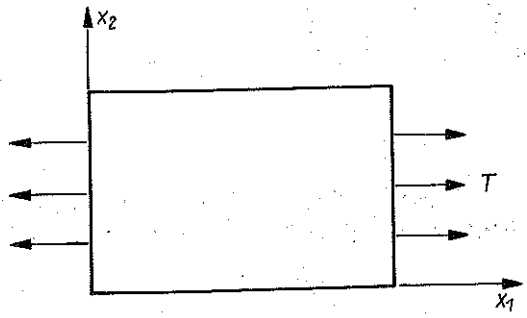

FIG. 1. A rectangular plate under uniform tension T and lateral loading.

As the existing theory, available before identification, let us take the equation for the deflection of a plate under uniform lateral load only, i.e. with $T=0$. This equation is of the form (see [7]):

$$\frac{\partial^4 F}{\partial X_1^4}+2\frac{\partial^4 F}{\partial X_1^2\,\partial X_2^2}+\frac{\partial^4 F}{\partial X_2^4}=\frac{Q}{D}\frac{16}{\pi^2}\sum_{m=1,3,5\ldots}^{\infty}\sum_{k=1,3,5\ldots}^{\infty}\frac{1}{mk}\cdot \sin(k\pi X_1/L)\sin(m\pi X_2/L), \tag{1.1}$$

with the boundary conditions

$$\begin{aligned} &F=0, \quad \partial^2 F/\partial X_1^2=0 \quad \text{for} \quad X_1=0 \quad \text{and} \quad X_1=L,\\ &F=0, \quad \partial^2 F/\partial X_2^2=0 \quad \text{for} \quad X_2=0 \quad \text{and} \quad X_2=L. \end{aligned} \tag{1.2}$$

In Eqs. (1.1) and (1.2), F denotes the deflection of the plate at the point X_1, X_2. Q means intensity of the uniformly distributed load, while D denotes the flexural rigidity of the plate, which is defined as follows:

$$D=EH^3/12\,(1-\nu^2). \tag{1.3}$$

where E is the Young modulus, ν is the Poisson coefficient and H denoted thickness of the plate.

Our aim is to identify the dimensionally homegeneous and invariant function

$$F=\Phi(X_1, X_2, D, L, Q, T) \tag{1.4}$$

basing on measurements of F for different values of X_1, X_2, D, L, Q, T. According to the correspondence principle, we require the function Φ to be the solution (or an approximate solution) of Eq. (1.1) as $T\to 0$. In dimensionless variables the above example can be rewritten as follows: define $q=F/L$, $x^{(1)}=X_1/L$, $x^{(2)}=X_2/L$, $x^{(3)}=QL^3/D$, $y=TL^2/D$. Then Eq. (1.1) has the form

$$\frac{\partial^4 q}{\partial (x^{(1)})^4}+2\frac{\partial^4 q}{\partial (x^{(1)})^2\,\partial (x^{(2)})^2}+\frac{\partial^4 q}{\partial (x^{(2)})^4}= \tag{1.5}$$

$$= \frac{16X^{(3)}}{\pi^2} \sum_{m=1,3,5\ldots}^{\infty} \sum_{k=1,3,5\ldots}^{\infty} \frac{1}{mk} \sin(k\pi x^{(1)}) \sin(m\pi x^{(2)})$$

and our aim is to identify the real-valued function

$$q = f(x, y) \tag{1.6}$$

taking into account the fact for $y \to 0$ the function (1.6) is the solution of Eq. (1.5).

Further considerations will be expressed in dimensionless form since every dimensionally homogeneous and invariant function and partial differential equations of mathematical physics can be converted into dimensionless form (see e.g. [3] for a general procedure).

2. Problem formulation

Suppose that our aim is to identify a physical process which can be described by a real valued function

$$q = f(x, y), \tag{2.1}$$

where $x \in R^k$, $y \in R^l$ are vectors of real parameters describing the process. Two groups of parameters are distingushed since we assume that we have *a priori* knowledge concerning the process behaviour. Namely, it is assumed that for a certain $y = y_0$ the process is governed by the following equation:

$$Af(x, y_0) = u(x), \tag{2.2}$$

where A is a differential operator with respect to x-variables. It is admitted that only a part of x-variables is present in differential operations in A, while the second part of x-variables is interpreted as material constants, $u(x)$ is a given excitation influencing the process.

We assume that A is defined on a subspace V of the space $L^2(\Omega)$ of square integrable functions, where $\Omega \subset R^k$ denotes an open spatial domain of those x-variables which appear in A. Linearity of A is preassumed but univocality of the solution of Eq. (2.2) is not required. It is only assumed that the range of A is in $L^2(\Omega)$, in which the scalar product of g, h is denoted by $\langle g, h \rangle$.

Identification of the process (2.1) is based on measurements of its output q in the presence of noise. In typical situations these measurements are of the form

$$s_i = f(x_i, y_i) + z_i, \quad i = 1, 2, \ldots, n, \tag{2.3}$$

where (x_i, y_i), $i = 1, 2, \ldots, n$ denotes measurement points, s_i $(i = 1, 2, \ldots, n)$ are

the results of the measurements, while $z_i\,(i=1,2,\ldots,n)$ are the measurement noises which are assumed to be independent zero mean random variables with finite variances. Without further assumptions a finite number of the measurements $s_i\,(i=1,2,\ldots,n)$ is not sufficient for exact recovery of the function f. For this reason we assume that for a certain $N>0$ the function (2.1) can be sufficiently accurately represented by the series

$$q = a_1 g_1(x,y) + a_2 g_2(x,y) + \ldots + a_N g_N(x,y) = G_a(x,y), \tag{2.4}$$

where $a=[a_1,\ldots,a_N]^T$ are unknown constants to be estimated, while $g_k(x,y)$, $k=1,2,\ldots,N$ are given linearly independent functions such that for every y, $g_k(\cdot,y)\in V$ and $Ag_k(\cdot,y)\in L^2(\Omega)$, $k=1,2,\ldots,N$.

On the other hand the class of functions which can be generated by the series (2.4) is, in general, too "small" to contain a solution of Eq. (2.2). In order to avoid this difficulty, simultaneously retaining *a priori* information contained in Eq. (2.2), it is proposed to take Eq. (2.2) into account as follows. Let H_M be a subspace of $L^2(\Omega)$ spanned by the functions $h_1(x)$, $h_2(x),\ldots,h_M(x)$ and let Eq. (2.2) be replaced by the requirement that the orthogonal projection of $AG_a(\cdot,y_0)-u(\cdot)$ onto H_M be the zero element of H_M, i.e.

$$\langle AG_a(\cdot,y_0),h_j\rangle - \langle u,h_j\rangle = 0, \quad j=1,2,\ldots,M. \tag{2.5}$$

This means that the vector $a\in R^N$ is estimated from the measurements $s_i\,(i=1,2,\ldots,n)$ under the containts (2.5) with $M\leqslant N$.

3. Identification algorithm

Let us note that the estimation problem (2.3), (2.4) satisfies the requirements of the Gauss–Markov estimation model (see e.g. [5]) and thus the least squares method with the constraints (2.5) can be used as a tool for finding an estimate of a. This leads to the following optimization problem:

$$\min_a \sum_{i=1}^{n} (s_i - G_a(x_i,y_i))^2 \tag{3.1}$$

with the constraints

$$\sum_{k=1}^{N} a_k \langle Ag_k(\cdot,y_0),h_j\rangle = \langle u,h_j\rangle, \quad j=1,2,\ldots,M. \tag{3.2}$$

Note that for given measurements the problem (3.1), (3.2) is a quadratic optimization problem with linear equality constraints and hence it can be solved by the standard Lagrange multipliers technique (see e.g. [1]). To this end, denote by B the matrix with the elements $b_{jk}=\langle Ag_k(\cdot,y_0),h_j\rangle$, $k=1,2,\ldots,N$; $j=1,2,\ldots,M$ and let $s=[s_1,s_2,\ldots,s_n]^T$, while $g_i=[g_1(x_i,y_i),$

$g_2(x_i, y_i) \ldots, g_N(x_i, y_i)]^T$, $i = 1, 2, \ldots, n$ and $u = [\langle u, h_1\rangle, \langle u, h_2\rangle, \ldots, \langle u, h_M\rangle]^T$. Then the problem (3.1), (3.2) can be equivalently rewritten as follows:

$$\min_a [\bar{s}\bar{s}^T - 2a^T C\bar{s} + a^T CC^T a], \tag{3.1'}$$

$$Ba = \bar{u} \tag{3.2'}$$

and the Lagrange function $L(a, \lambda)$ is given by

$$L(a, \lambda) = \bar{s}\bar{s}^T - 2a^T C\bar{s} + a^T CC^T a + \lambda^T (Ba - \bar{u}), \tag{3.3}$$

where $\lambda \in R^M$ is the vector of Lagrange multipliers and C is $N \times n$ the matrix with the columns g_i $i = 1, 2, \ldots, n$. Above and further on, T denotes transposition. As it is known (e.g. [1]), the necessary condition for a^* to be a solution of the problem (3.1), (3.2) is the existence of the vector $\lambda \in R^M$ for which

$$\operatorname{grad}_a L(a^*, \lambda^*) = -2C\bar{s} + 2CC^T a^* + B^T \lambda^* = 0, \tag{3.4}$$

$$\operatorname{grad}_\lambda L(a^*, \lambda^*) = Ba^* - \bar{u} = 0. \tag{3.5}$$

Since the minimized function is convex and the constraints are linear in a, then these conditions are also sufficient for optimality of a^* (see e.g. [1]). Furthermore, if $\det CC^T > 0$, then the minimized function is strictly convex, what implies uniqueness of the optimal solution a^*.

The above results lead to the following simple identification algorithm:

Step 1. Form the matrix B and the vector $\bar{u}$ of *a priori* constraints.

Step 2. Choose the points (x_i, y_i), $i = 1, 2, \ldots, n$ in such a way that $\det CC^T > 0$ and perform the experiment in order to gain the measurements (2.3) and form the vector $\bar{s}$.

Step 3. Solve, with respect to a, λ, the following equations:

$$\begin{array}{c} \\ N \text{ rows} \\ \\ M \text{ rows} \end{array} \begin{array}{c} \begin{array}{cc} N \text{ columns} & M \text{ columns} \end{array} \\ \begin{bmatrix} 2CC^T & B^T \\ B & 0 \end{bmatrix} \end{array} \begin{bmatrix} a \\ \lambda \end{bmatrix} = \begin{bmatrix} 2C\bar{s} \\ \bar{u} \end{bmatrix}$$

and substitute the estimate a^* into the series (2.4) in order to predict the process behaviour for different x, y.

If not only CC^T but also the matrix $D \triangleq B(CC^T)^{-1} B$ is nonsingular, then

$$\lambda^* = 2D^{-1} B\hat{a}, \tag{3.7}$$

$$a^* = \hat{a} - (CC^T)^{-1} B^T D^{-1} [B\hat{a} - \bar{u}], \tag{3.8}$$

where $\hat{a}$ is defined as

$$\hat{a} = (CC^T)^{-1} C\bar{s}. \tag{3.9}$$

Note that $\hat{a}$ can be interpreted as an estimate of the vector a, obtained without a priori information or ignoring it ($\hat{a}$ minimizes the problem (3.1) without any constraints).

4. Advantages of the proposed approach

From the fact that the measurements (2.3) are observations of random variables it follows that also the vector a^* and $\hat{a}$ are observations of random vectors and thus they should be compared in a statistical sense. It is easy to see that both a^* and $\hat{a}$ are the unbiased estimators for a, i.e. $Ea^* = E\hat{a} = a$, where E stands for the expectation. It can also be shown (see e.g. [5] Chapter 3) that

$$\operatorname{cov}[\hat{a}] = \sigma^2 (CC^T)^{-1}, \tag{4.1}$$

$$\operatorname{cov}[a^*) = \sigma^2 (CC^T)^{-1} [I_N - B^T D^{-1} B (CC^T)^{-1}], \tag{4.2}$$

where cov [ ] means the covariance matrix of a random vector in brackets, I_N denotes the $N \times N$ unit matrix and $\sigma^2 = Ez_i^2$ $(i = 1, 2, ..., n)$. Equations (4.1), (4.2) imply that for variances of elements of a^* and $\hat{a}$ we have

$$\operatorname{var}(a_k^*) \leqslant \operatorname{var}(\hat{a}_k), \quad k = 1, 2, ..., N. \tag{4.3}$$

Roughly speaking, this means that a^* is expected to be "closer" to a than $\hat{a}$. By the way, the values var (a_k^*), $k = 1, 2, ..., n$, which are equal to diagonal elements of the matrix (4.2) allow to evaluate estimation accuracy by constructing a confidence region for a, if the noises are Gaussian (see e.g. [5]).

In order to demonstrate further properties of the proposed approach, it is desirable to consider practically an important special case of the problem stated in Sect. 2. To this end, let us suppose that the series (2.4) is of the form

$$q = \sum_{m=1}^{M} \sum_{l=1}^{L} e_{ml}\, p_m(x)\, r_l(y), \tag{4.4}$$

for certain unknown constants $\{e_{ml}\}$ and given functions $p_m(x)$, $r_l y$; $m = 1, 2, ..., M$, $l = 1, 2, ..., L$. Note that Eq. (4.4) agrees with the series (2.4) if $N = M \cdot L$ and we take $g_k x, y) = p_m(x) r_l(y)$ and $a_k = e_{ml}$ for certain uniquely defined (m, l); $k = 1, 2, ..., N$. It is clear that ordering chosen in the above equalities is immaterial. Let us assume that the functions $p_m \in V$ are such that $Ap_m \in L^2(\Omega)$ $m = 1, 2, ..., M$ and they are chosen in such a way that

$$\langle Ap_m, h_j \rangle = \begin{cases} \lambda_m \neq 0 & \text{if} \quad m = j, \\ 0 & \text{if} \quad m = j, \end{cases} \quad m, j = 1, 2, ..., M. \tag{4.5}$$

The condition (4.5) is fulfilled if the operator A is symmetric and $p_m = h_m$

$(m = 1, 2, ..., M)$ are eigenfunctions of A, while λ_m $(m = 1, 2, ..., M)$ are the corresponding eigenvalues. If, furthermore, A is a positive definite operator and for a certain $C > 0$, $\langle Av, v\rangle \geq C \|v\|^2$ for every $v \in V$, then the equation

$$Aw = u \tag{4.6}$$

has the unique solution $w \in V$, which is of the form

$$w(x) = \sum_{m=1}^{\infty} \langle u, p_m\rangle p_m(x)/\lambda_m \tag{4.7}$$

provided that the eigenfunctions p_m $(m = 1, 2, ...)$ are complete in $L^2(\Omega)$ (see [6] for definitions and derivation of Eq. (4.7)).

Let us suppose that the functions $r_l(y)$, $l = 1, 2, ..., L$ fulfill the following conditions:

$$r_1(y_0) = 1, \quad r_l(y_0) = 0, \quad l = 2, 3, ..., L \tag{4.8}$$

which hold if, for example these functions are polynomials and $y_0 = 0$.

The above assumptions allow to solve the constraints (2.5). Indeed, substitution of Eq. (4.4) into Eq. (2.5) with the subsequent use of Eqs. (4.5), (4.8) yields

$$e_{m1} = \langle u, h_m\rangle/\lambda_m, \quad m = 1, 2, ..., M. \tag{4.9}$$

Thus the function

$$q = r_1(y) \sum_{m=1}^{M} \langle u, h_m\rangle p_m(x)/\lambda_m + \sum_{l=2}^{L} \sum_{m=1}^{\infty} e_{ml} p_m(x) r_l(y) \tag{4.10}$$

fulfills the constraints (2.5) for every e_{ml} $(l = 2, 3, ..., L, m = 1, 2, ..., M)$. This implies that minimization of the sum of squares with respect to these parameters can be carried out using standard library subroutines for the regression function fitting.

It is to be noted that for $y = y_0$ we have from the function (4.10)

$$q = \sum_{m=1}^{M} \langle u, p_m\rangle p_m(x)/\lambda_m. \tag{4.11}$$

Comparison of Eq. (4.11) with Eq. (4.7) shows that for $M \to \infty$ the function (4.10) fulfills Eq. (2.2). This means that for large M *a priori* information contained in Eq. (2.2) is fully incorporated into the proposed identification algorithm.

If all the above assumptions hold and it is known that the function (4.4) with $L = 2$ describes the process sufficiently accurate in a domain of interest, then a striking effect of using *a priori* knowledge can be demonstrated. To this end, let us note that in this case we have from the function (4.10)

$$q = r_1(y) \sum_{m=1}^{M} \langle u, p_m \rangle p_m(x)/\lambda_m + r_2(y) \sum_{m=1}^{M} e_{m2}\, p_m(x) \tag{4.12}$$

and only the parameters e_{m2} $(m = 1, 2, \ldots, M)$ are to be identified. Suppose that x_i $(i = 1, 2, \ldots, n)$ are chosen in such a way that the $M \times n$ matrix with the elements $p_m(x_i)$, $(i = 1, 2, \ldots, n,\ m = 1, 2, \ldots, M)$ is of the rank M. If for a certain y, $r_2(y) \neq 0$, then e_{m2} $(m = 1, 2, \ldots, M)$ are estimable by the least squares method using the measurements (2.3) with $y_i = y$ $(i = 1, 2, \ldots, n)$ (see [5] for definition of estimability). That is, *a priori* information allows to identify an influence of the y-variable on the process by taking measurments at one y point only. It is clear that approximation with $L = 2$ can be valid in a limited domain of y-variables. However, possible reduction of a number of measurements, demonstrated above for $L = 2$, takes place also for $L > 2$.

5. Simulation example-rectangular plate with simple supported edges

In order to verify the theory presented above we continue the example of Sect. 1. To identify the function (1.6) with *a priori* information given by Eq. (1.5) we need measurements of q for different values x, y. These measurements will be simulated by using the equation for the deflection of the plate under uniform tension T in the X_1 direction subjected to uniform lateral load (see [7]).

In the dimensionless variables introduced in Sect. 1 this equation has the form

$$\frac{\partial^4 f}{\partial (x^{(1)})^4} + 2\frac{\partial^4 f}{\partial (x^{(1)})^2\, \partial (x^{(2)})^2} + \frac{\partial^4 f}{\partial (x^{(2)})^4} - y\frac{\partial^4 f}{\partial (x^{(1)})^4} = \tag{5.1}$$

$$= \frac{16x^{(3)}}{\pi^2} \sum_{m=1,3,5}^{\infty} \sum_{k=1,3,5}^{\infty} \frac{1}{mk} \sin(k\pi x^{(1)}) \sin(m\pi x^{(2)})$$

with the boundary conditions

$$f = 0, \quad \partial^2 f/\partial (x^{(1)})^2 = 0 \quad \text{for} \quad x^{(1)} = 0 \quad \text{and} \quad x^{(1)} = 1, \tag{5.2}$$

$$f = 0, \quad \partial^2 f/\partial (x^{(2)})^2 = 0 \quad \text{for} \quad x^{(2)} = 0 \quad \text{and} \quad x^{(2)} = 1. \tag{5.3}$$

In [7] §91 it was shown that the solution of Eqs. (5.1)–(5.3) is of the form

$$f(x, y) = \frac{16x^{(3)}}{\pi^2} \sum_{m=1,3,5}^{\infty} \sum_{k=1,3,5}^{\infty} \sin(k\pi x^{(1)}) \sin(m\pi x^{(2)}) / \tag{5.4}$$

$$/\left(km\left[(k^2+m^2)^2 + k^2\, y/\pi^2\right]\right).$$

This solution, after truncation of both of the series at the 9-th element, was used for measurements simulation according to the relations, with random $\{Z_i\}$ uniformly distributed over the interval $[-\delta, \delta]$.

Three simulation runs were performed for $x^{(3)} = 10/343.4$ and the following values of other variables were obtained:

a) 25 measurements for $x^{(1)}$, $x^{(2)}$ uniformly distributed over the plate and y changing from 5/343.3 to 25/343.3 with $\delta = 0$ i.e. without the measurement noise;

b) as above but with the measurement noise;

c) 100 measurements for $x^{(1)}$, $x^{(2)}$ uniformly distributed over the plate and y changing from 0 to 25/343.3 with the measurement noise.

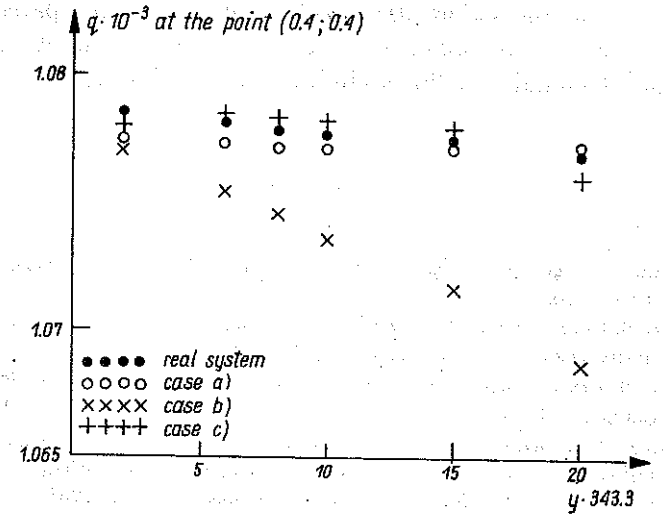

FIG. 2. Simulation results.

The largest values of q, which will serve as a base for accuracy evaluation, was obtained for $x^{(1)} = x^{(2)} = 0.5$ and $y = 0$ and equals $q_{\max} = 0.00118$. In the cases b), c) the noises were uniformly distributed over the interval $[-0.005, 0.005]$, i.e. the largest relative measurement error was $\pm 0.4\%$.

After simulation the data were stored and the model (5.1)–(5.3) was "forgotten".

As an *a priori* information for identification of f the model (1.5) was assumed. It was decided to look for the unknown dependence of q on x, y in the form (4.10) with $r_1(y) = 1$, $r_2(y) = y$, $r_3(y) = y^2$ and $p_1(x) = 2/\pi \sin(\pi x^{(1)}) \sin(\pi x^{(2)})$, $p_2 x = 2/\pi \sin(\pi x^{(1)}) \sin(3\pi x^{(2)})$, $p_3 x = 2\sin(3\pi x^{(1)}) \cdot \sin(\pi x^{(2)}/\pi)$.

According to the theory of Sects. 3 and 4 it remains to identify six parameters e_{ml} for $m = 1, 2, 3$ and $l = 2, 3$. These values were estimated using the least squares method and then substituted into the function (4.10) in order to test the model accuracy. In each case a), b), c) the identified model was tested by comparing 20 values of the model output with the "real" values calculated from the solution (5.4). The largest differences related to $q_{\max}$ were the following: 0.06% in the case a), 0.7% in the case b) and 0.1% in the case c). These results show that the largest relative model errors are comparable with the relative measurement errors. Comparing cases b) and c) one can notice that, according to the theoretical results, the model error decreases the number of measurements increases.

The results of model testing are shown in Fig. 2 where the "real" behaviour of q at the point (0.4; 0.4) versus y is compared with the behaviour of the models obtained in cases a), b), c). As it can be seen the behaviour of model c) follows closely the "real" system.

REFERENCES

1. E. J. BELTRAMI, *An algorithmic approach to nonlinear analysis and optimization*, Academic Press, New York 1970.
2. W. KASPRZAK, B. LYSIK, *Construction and interpretability of a mathematical model for process description*, Studia Geotechnica et Mechanica, **2**, 1, 59–73.
3. W. KASPRZAK, B. LYSIK, *Analiza wymiarowa w projektowaniu eksperymentu*, PAN — Oddział Wrocław, Ossolineum, 1978.
4. E. RAFAJŁOWICZ, *Identification of partly unknown systems using dimensional analysis and perturbation methods*, Studia Geotechnica et Mechanica [to be published].
5. C. R. RAO, *Linear statistical inference and its applications*, J. Wiley, 1965.
6. I. STACKOLD, *Boundary value problems of mathematical physics*, Collier–Mac Millan, London 1968.
7. S. P. TIMOSHENKO, S. WOINOWSKY-KRIEGER, *Theory of plates and shells*, MacGraw-Hill Book Company, New York 1959.

STRESZCZENIE

IDENTIFIKACJA UKŁADU O PARAMETRACH ROZŁOŻONYCH Z OGRANICZENIEM INFORMACJĄ *A PRIORI*

Przedstawiona w pracy procedura identyfikacji pozwala na wyznaczanie nieobciążonego estymatora parametrów układu z parametrami rozłożonymi. Parametry te otrzymuje się w wyniku minimalizacji funkcjonału jakości w postaci formy kwadratowej z uwzględnieniem ograniczeń. Przeanalizowano różne właściwości zaproponowanej procedury. Zamieszczono przykład liczbowy.

Резюме

ИДЕНТИФИКАЦИЯ СИСТЕМЫ С РАСПРЕДЕЛЕННЫМИ ПАРАМЕТРАМИ ОГРАНИЧЕННЫМИ ИНФОРМАЦИЕЙ *A PRIORI*

В работе представлена процедура идентификации, позволяющая определить несмещенную оценку параметров системы с распределенными параметрами. Эти параметры получают в результате минимализации функционала качества в квадратичной форме с учетом ограничений. Проанализированы различные свойства предложенной процедуры. Приведены численные примеры.

TECHNICAL UNIVERSITY OF WROCŁAW.

Received April 20, 1984.