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EXPERIMENTAL INVESTIGATIONS ON SHAKEDOWN OF PORTAL FRAMES

M. H. ALI (HANNOVER), J. A. KÖNIG (WARSZAWA) and
O. MAHRENHOLTZ (HANNOVER)

The paper presents results obtained in tests consisting of cyclic loading of a two-span continuous beam as well as of some portal frames made of mild steel. The investigations were performed for various load intensities so as to determine the maximum intensity at which the deflections stabilization still takes place, i.e. a given structure shakes down. The results are in a fair agreement with the predictions of the shakedown theory and thus confirm its practical applicability in the design of steel structures.

1 INTRODUCTION

The application of plasticity methods in the design of steel engineering structures allows for a more economical use of material as well as for a more rational structural design. This is one of the reasons of the rapid development of the theory of limit analysis and of its methods during the last thirty years. However, in the case of variable repeated loads, the structural load-carrying capacity should be determined rather by the shakedown theory, cf. e.g. [1, 4, 5], which provides the criteria of stabilization of plastic deformations in the course of load variations.

The theoretical basis as well as the computational methods of structural shakedown analysis have been well developed. On the other hand, the number of experimental verifications of the theory is still rather limited. Therefore, there exists a definite need for more investigations considering the practical validity of its results.

The present paper presents some experimental data obtained on models of two-span continuous beams and for portal frames made of mild steel St. 37 and subjected to cyclic loads consisting of concentrated forces. The results seem to be in relatively good agreement with the theoretical predictions. They are of an elementary character as they were performed as an initial stage of a broader-planned program.

2 CONTENT OF INVESTIGATIONS

Our aim was to evaluate, experimentally, the shakedown loads. In all the cases the shakedown limit loads were determined by the incremental collapse. The cases in which the low cycle fatigue due to alternating plastic strains is the main factor can be investigated by means of a purely elastic analysis [5].

The experimental work consisted of the following parts.

A. Evaluation of the material properties. Here the following tests were made:

A1. Classical tension tests to evaluate the stress-strain relationship for the steel. Some of the results are shown in Fig. 1.

A2. Tests consisting of cyclic loading of simply supported beams subjected to a concentrated vertical load applied at the midspan, Fig. 2a.

B. The main shakedown tests consisting of cyclic loading of a given structure at various load amplitudes. Some characteristic displacements were measured and their stabilization or divergence indicated shakedown or incremental collapse. The following tests were made:

B1. Cyclic loading of a two-span beam, Fig. 5a. Vertical displacements at both midspans were recorded. The right-hand span deflections (at the stage II of each load cycle) are drawn against the number of loading cycles.

B2. Cyclic loading of a portal frame of clamped feet, Fig. 5b. The loading cycle consisted of four stages as indicated in the figure. The vertical displacement (at the unloading stage) at the beam midspan as well as the frame side-sway was recorded.

B3. Cyclic loading of another portal frame, Fig. 5c, subject to the constant vertical load $V=P$ and variable repeated horizontal loads H_1, H_2 varying as indicated in the figure. Tests were made for $a=0.3$ and $a=0.37$.

B4. Cyclic loading of a portal frame, Fig. 5d, with the horizontal load applied at a certain distance below the beam.

C. Tests under proportional monotone load. Frame specimens as in Figs. 5 bcd were subjected to the proportional loading $V=H=P$. The beam deflection and the side-sway of the frames were recorded; see, e.g., Fig. 6.

3. PREPARATION OF EXPERIMENTS

All the specimens were made from a mild steel St 37 cold-rolled tape of cross sections 2.0 per 0.5 cm (only some beams) or 1.4 cm per 0.4 cm (remaining beams and all frames). The frames were accomplished by plastic bending of the tape over a circular bar, the resulting radius of curvature at corners amounting to about 0.6 cm. All of the frame specimens were heat-treated in a furnace. The heat treatment consisted in full normalizing at 510°C with a soaking time of one hour, after which the specimens were cooled in the furnace for about 24 hours. This treatment was carried out to produce a complete recrystallization of the steel and to remove any residual stresses caused by bending of the material.

Specimens for the tension tests were prepared from the steel tape by reducing the cross-section to the dimensions 1.0 cm per 0.4 cm. The specimens were made both from the undeformed material as well as from the deformed one. No difference was detected between these two kinds. On the other hand, the discrepancies in the yield-point stress σ_0 as well as in the shapes of the σ, ε curves were considerable through the greater part of the material used coming from a single 5 m long tape. Some extremal cases of the σ, ε-curves are shown in Fig. 1.

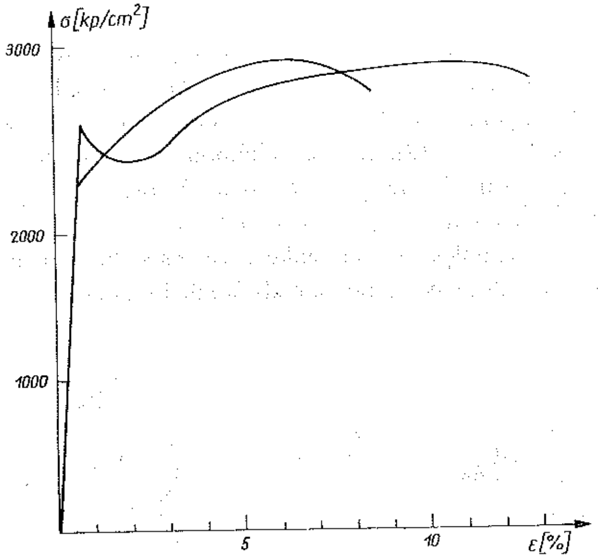

FIG. 1. Examples of the experimental σ, ε-curves.

The mean value of the yield-point stress obtained was

$$\sigma_0 = 222 \text{ N/mm}^2 = 2179 \text{ kp/cm}^2 .$$

What concerns the cyclic load tests of simply supported beams, the results (Fig. 2a) indicate a relatively rapid stabilization of the hysteresis loop. After 3 or 4 cycles the loop was becoming practically a segment of a straight line .Thus the

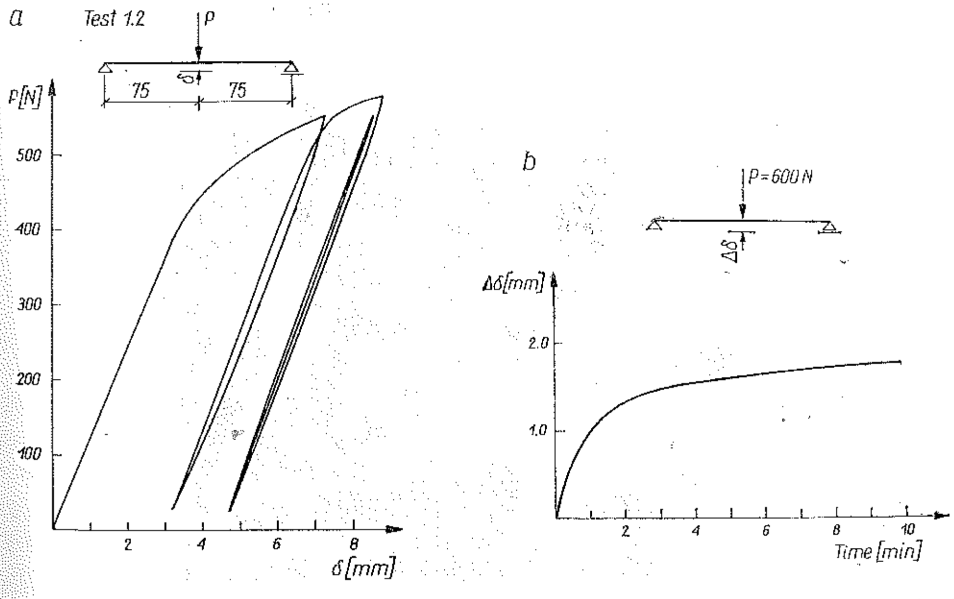

FIG. 2a. Deflections of the beam under cyclic loading. b. Creeping of the beam at elevated load.

assumption adopted in the shakedown theory, consisting of identifying the σ, ε curves in unloading and the subsequent re-loading, was satisfied nearly exactly.

The material used exhibited a considerable creep, especially at higher stresses. It is known that creeping of metals, even at room temperature, may begin also below the yield-point stress [12]. In the neighbourhood of the collapse state the creep could have the form of a steady flow. Figure 2b presents this fact for a simply supported beam subjected to a high constant load.

To overcome the troubles, in the shakedown tests the loadings were changed as fast as possible though quasi-static, namely single loading steps every 5 minutes.

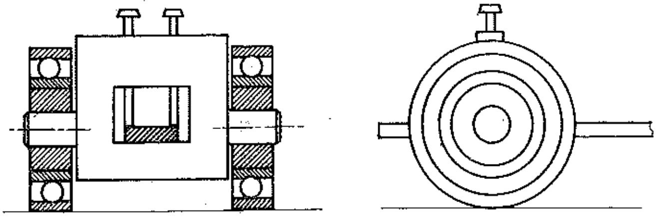

FIG. 3. Scheme of beams supports.

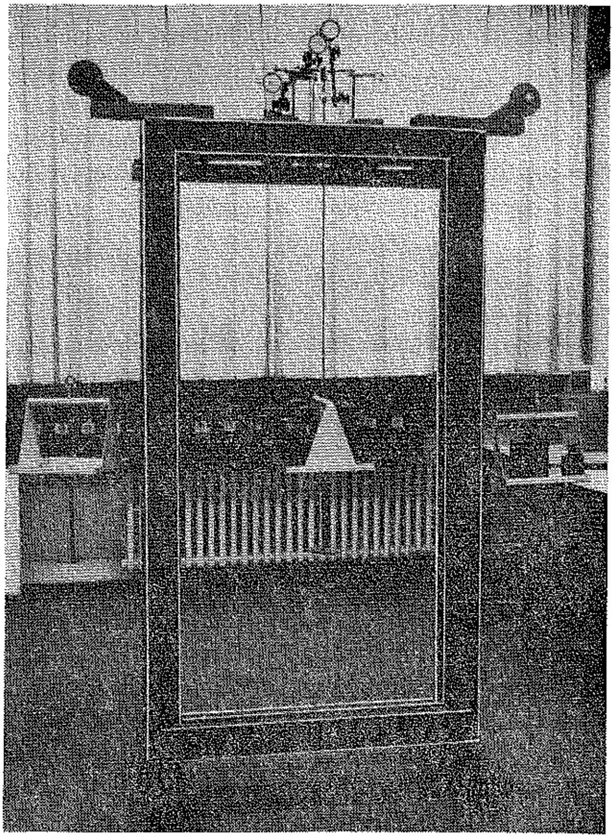

FIG. 4. A general look at the experiments arrangement.

Supports of beams were arranged by means of small steel stirrups and ball bearings, Fig. 3. Analogous stirrups were used to transfer the loads to specimens. To avoid a holding up of the left-hand beam support due to negative reaction in the loading step b) of the test B1, this support was additionally loaded with a small weight. Frame specimens were clamped by means of clamping blocks and screws. A whole specimen, together with the supports, was attached to a rigid frame, Fig. 4. The measured displacements of supports turned out to be negligibly small.

Displacements were measured by means of mechanical dial gauges with an accouracy of 0.01 mm and fabricated by Fa. Hommel, West Germany.

4. COURSE OF EXPERIMENTS AND THEIR RESULTS

Shakedown tests were made in the following way. An appropriate cyclic load program as described in Sect. 2 was applied with a certain value of the load amplitude *P*. The load cycles were repeated until stabilization of deflections was attained. Then *P* was given an increment and the whole procedure was repeated. Naturally, if the loading magnitude become sufficiently high, then there was no more deflection stabilization. The highest load intensity at which the stabilization was attained gives a lower bound to the actual shakedown load. On the other hand, the lowest load level for which deflections do not stabilize provides an upper bound. The bound obtained for given load cycles holds for any loading path containing the same external loads [15]. For the sake of comparison a few tests were made in which a relatively high load intensity was applied at the very beginning, see Fig. 5a. It has been observed that the deflection stabilization or divergence does not depend on the history of previous loading. However, the magnitudes of final deflections sta-

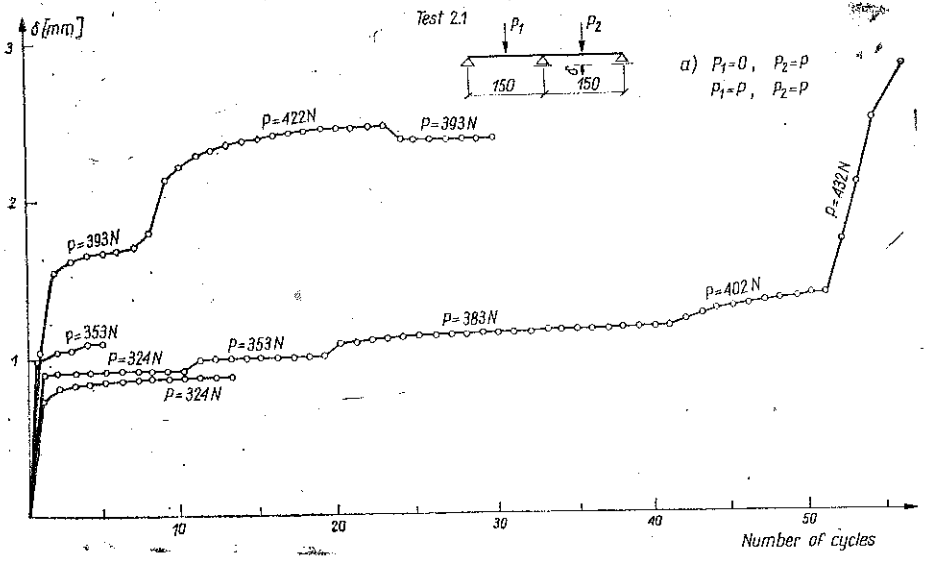

Fig. 5a.

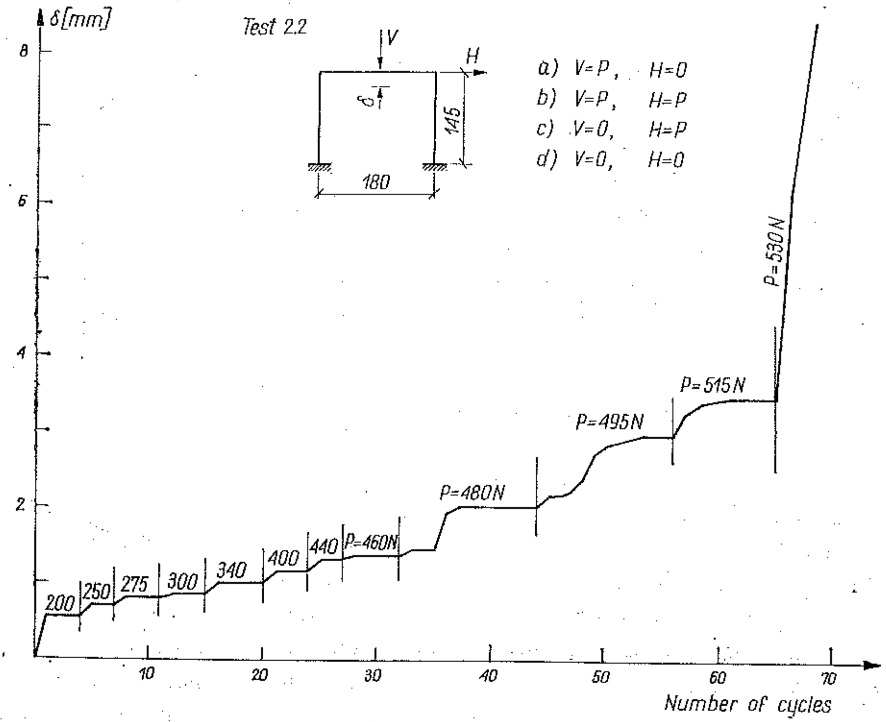
δ[mm]
Test 2.2
V
H
δ
145
180
a) V=P, H=0
b) V=P, H=P
c) V=0, H=P
d) V=0, H=0
8
6
4
2
200
250
275
300
340
400
440
P=460N
P=480N
P=495N
P=515N
P=530N
10
20
30
40
50
60
70
Number of cycles

Fig. 5b.

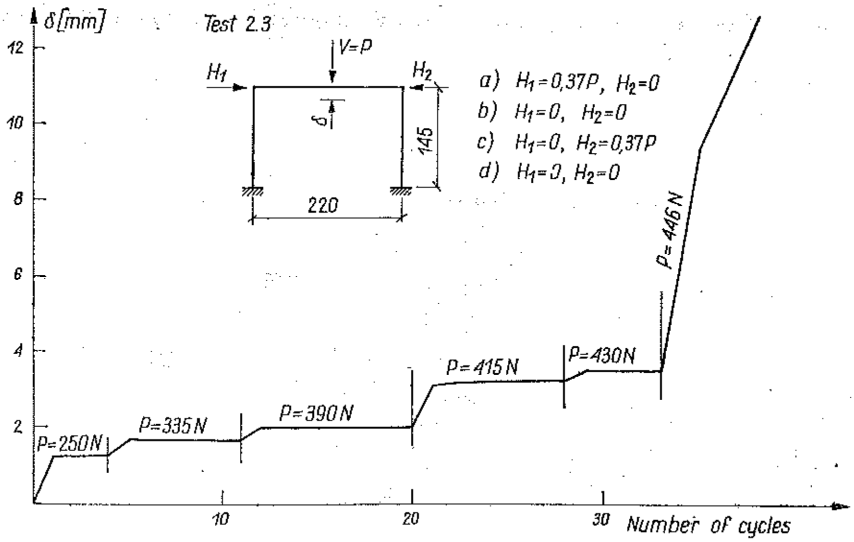
δ[mm]
Test 2.3
V=P
H1
H2
δ
145
220
a) H1=0,37P, H2=0
b) H1=0, H2=0
c) H1=0, H2=0,37P
d) H1=0, H2=0
12
10
8
6
4
2
P=250N
P=335N
P=390N
P=415N
P=430N
P=446N
10
20
30
Number of cycles

Fig. 5c.

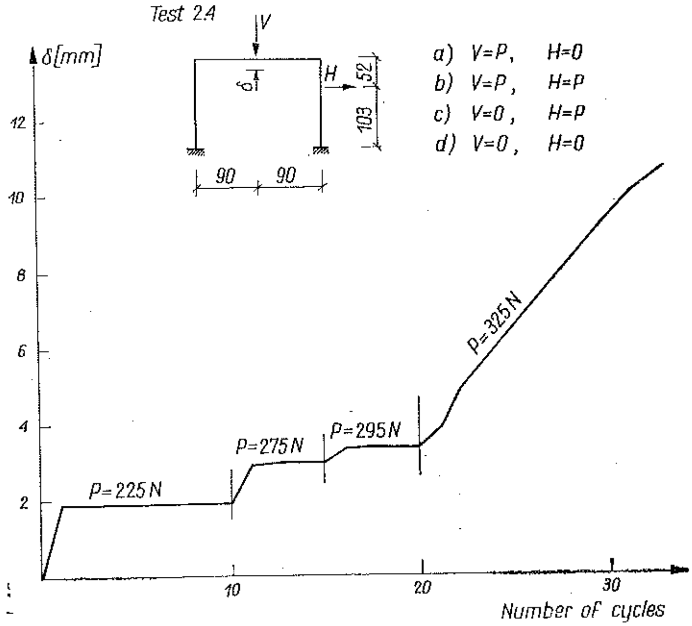

Fig. 5d.

FIG. 5a. Deflection of the beam when subject to cyclic loadings in the test B1. b. Vertical deflection of the upper beam of the frame in the test B2. c. Vertical deflection of the upper beam of the frame in the test B3. d. Frame side-sway of the frame in the test B4.

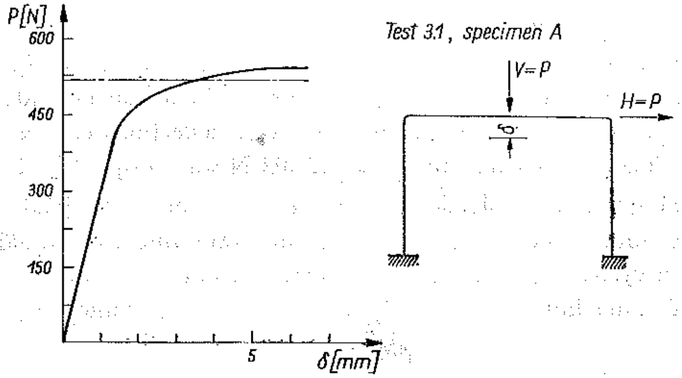

FIG. 6. Vertical deflection — load curve in the test C (proportional loading).

bilized are different. In principle, if a certain level of cyclic loading was proceeded by some lower-amplitude load cycles, then the resulting deflections were smaller.

Examples of the results obtained are drawn out in Figs. 5 abcd. The load carrying capacity tests show also a fairly good agreement with theoretical values of the collapse loads, cf. Fig. 6.

Table 1 summarizes all the results obtained.

Table 1. Results of the tests

No	Type of experiment	Loads		$\frac{Pr}{Pc}$	
		Calculated Pc N	Recorded Pr N		
1	B1	417.6	402	0.963	Cyclic loading
2	B2	294.3	315	1.070	
3	B2	294.3	310	1.053	
4	B3	512.7	530	1.034	
5	B3	512.7	535	1.043	
6	B3	410	446	1.088	
7	B3	410	435	1.061	
8	B4	328	325	0,991	
9	B4	325	335	1.021	
10	B4	328	325	0.991	
11	B4	308	320	1.039	
12	C	342.2	345	1.008	Monotone loading
13	C	491.4	510	1.038	
14	C	368.5	350	0.950	
15	C	491.4	510	1.038	

5. DISCUSSION OF THE RESULTS

1. The results show a good agreement of the theory of shakedown and of the experimental data. The discrepancies would be probably smaller in the case of a more homogeneous material.

2. The shakedown loads obtained from the diagrams are not quite clear, for example, the curve of Fig. 5a, appropriate for the load intensity 402 N seems to be a limit case as more pronouced increments of deflections can be noticed only at the 432 N load. In this case the value of 402 N was adopted in Table 1.

The effect of smaller deflections to occur if a given cyclic load intensity was reached after some proceeding load cycles of lower intensity would suggest an influence of a cyclic hardening which in fact has not been detected in tests A2. An effect of some initial self-equilibrated stress states cannot be, in principle, excluded.

3. The order of creep deformations and their possible influence should be emphasized. In initial trials creeping was observed even after 48 hours of waiting-time. Figure 7 shows the result of test A1 — a deflection-load curve obtained by waiting-time for 15 minutes on each new load level. The solid curve is related to deflections after the waiting-time, the dashed line to their immediate magnitudes. Such a strongly pronounced creep has not been reported in the hitherto papers on shakedown experiments.

4. It is worthwhile to stress that not only the magnitudes of shakedown and collapse loads seem to be in agreement with the theory. Also the modes of the in-

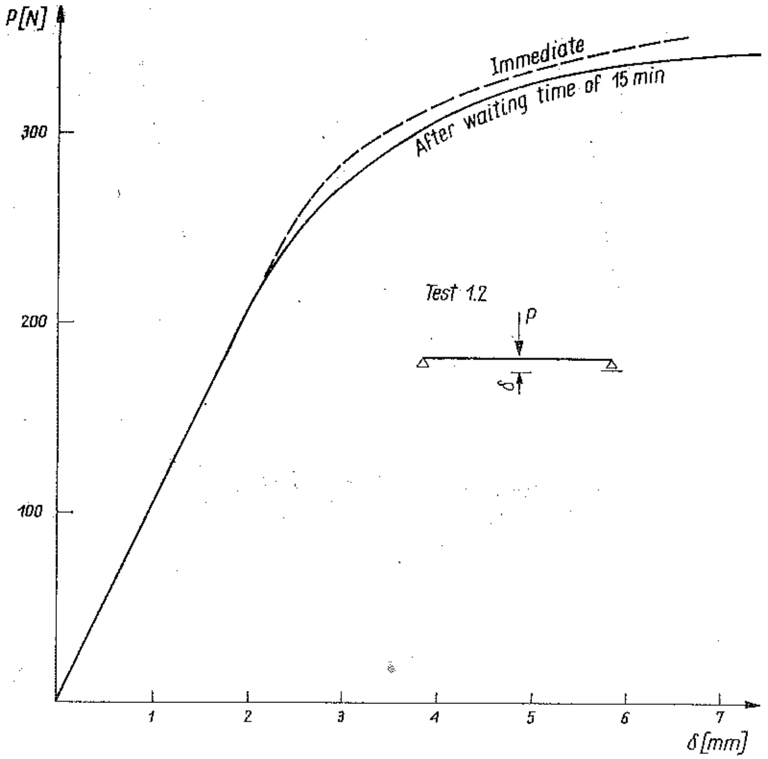

FIG. 7. Deflections of the beam of Fig. 2, solid line — recorded after 15 minutes; dashed line — record immediately after the load increase.

cremental collapse (or of the instantaneous collapse, respectively) observed followed the theoretical predictions even in tests B4 and C in which at least two of the seven possible mechanisms give similar values of shakedown (or collapse) loads. Figure 8a shows the theoretical incremental collapse mechanism for test B4 and Fig. 8b shows the respective specimen deformed after several loading cycles.

5. Though the material used exhibits strain-hardening, see Figs. 1 and 2, and different samples gave different σ, ε-curves, no influence of those effects on shakedown loads was visible. Only the slightly curvilinear form of the δ, N-lines (Figs. 5 a, b, c, d) in the case of incremental collapse may be recognized as a result of strain-hardening.

6. Similarly to previously reported investigations, the stabilization of plastic deflections usually took place only after several loading cycles even if the theory foresees that in a single one (what is usually the case for simple structures) cf. [5]. However, some curves appropriate for lower loads agree with the theory also with this respect. The discrepancy of the theoretical and actual structural behaviour can be explained by the effect of the variations of the σ, ε curve over the initial cyclic loading, Fig. 2.

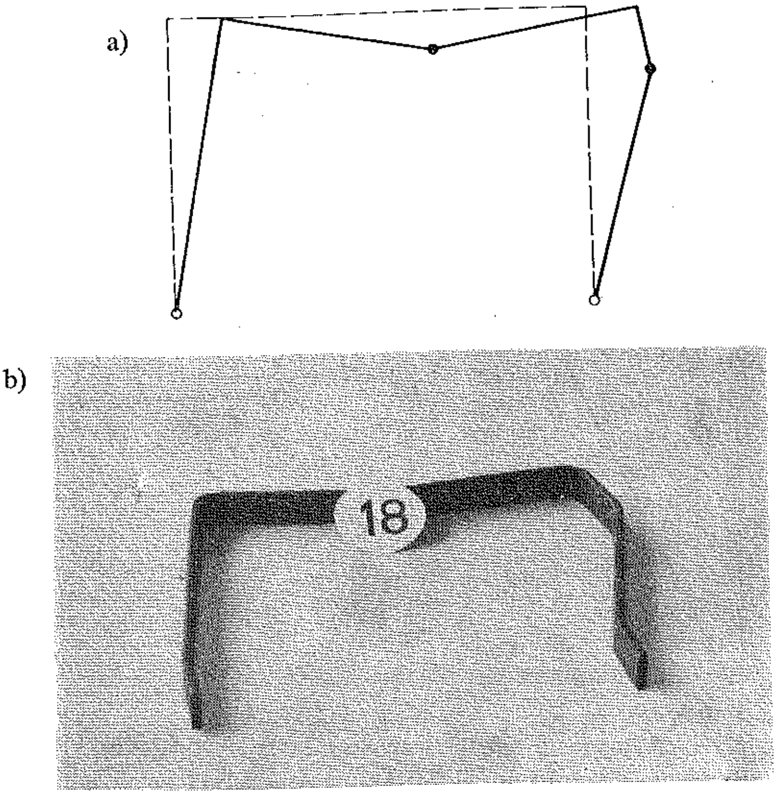

FIG. 8a. Possible incremental collapse mechanisms in the test B4. b. A frame deformed actually in the test B4.

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STRESZCZENIE

DOŚWIADCZALNE BADANIE PRZYSTOSOWYWANIA SIĘ RAM PORTALOWYCH

Praca przedstawia rezultaty otrzymane w badaniach modeli dwuprzęsłowej belki ciągłej i kilku przypadków ram portalowych, poddanych obciążeniom cyklicznym (quasi-statycznym). Badania przeprowadzano przy różnych intensywnościach obciążeń, wyznaczając maksymalne ich wartości, pozwalające jeszcze na stabilizację ugięć, tzn. na przystosowanie. Wyniki te pozostają w dobrej zgodzie z obliczeniami przeprowadzonymi według teorii przystosowania i potwierdzają jej praktyczną stosowalność w projektowaniu konstrukcji stalowych.

Резюме

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ПРИСПОСОБЛЕНИЯ ПОРТАЛЬНЫХ РАМ

Работа представляет результаты полученные в исследованиях моделей двупролетной непрерывной балки и нескольких случаев портальных рам, подвергнутых циклическим (квазистатическим) нагрузкам. Исследования проведены при разных интенсивностях нагрузок, определяя максимальные их значения, позволяющие еще стабилизировать прогибы, т. зн. приспособлять. Эти результаты хорошо совпадают с расчетами, проведенными согласно теории приспособления и подтверждают ее практическую применяемость в проектировании стальных конструкций.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH
and
TECHNICAL UNIVERSITY OF HANNOVER, FRG.

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