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ON THE STATICALLY EQUIVALENT LOADS IN SAINT VENANT'S PRINCIPLE

M. SOKOŁOWSKI (WARSZAWA)

In traditional Saint Venant's principle formulations the loads leading to similar stress distributions are assumed to be statically equivalent, i.e. to yield identical resultant forces and moments. It is shown that this requirement may be replaced with a more general one according to which the first $N+1$ moments of order $0, 1, ..., N$ of both loading systems should be compared. In the case of an elastic half-plane loaded at the boundary it is shown that the resulting stress differences calculated at large distances from the loaded region are of the order of kr^{-N-2} where N denotes the highest order of identical moments of two loading systems, and r — distance from the loaded region.

1. Introduction

The principle was originally formulated by B. de Saint Venant in 1855. Its obvious simplicity and generality lead to considerable simplifications of the analysis of stress and strain distributions in elastic bodies and engineering structures loaded at regions which may be considered as small compared with the dimensions of the whole body. This is the reason why Saint Venant's principle represents a frequently applied tool in the analysis of such structures and in elasticity: in addition, numerous attempts have been made to adapt its formulation to various particular forms of structures and materials considered. It follows from the fact that the original St Venant's formulation of the principle [9] is rather vague and applies to the particular case of torsion of slender bars.

In the present paper we are not going to discuss the extensive literature of the subject or to present a "correct" formulation of the principle; the paper is aimed at demonstrating a certain inconsequence connected with the notion of statically equivalent loads, and at showing how this inconsequence could be avoided. The considerations will be based on an earlier paper [11] published recently (in Polish) in the Professor R. Kazimierczak Anniversary Volume by the Gdańsk Technological University.

Let us first recall two of the many possible formulations of the principle, called sometimes the "principle of elastic equivalence of statically equipollent loads" (cf. A. I. LURIE [5]).

1. "If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of surface, this redistribution of loading produces substantial changes in the stresses locally but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed" (S. TIMOSHENKO [13], p. 31).

2. "The strains which are produced in a body by the application, to a small part of its surface, of a system of forces equivalent to zero force and zero couple, are of negligible magnitude at distances which are large compared with the linear dimensions of the part" (cf. Y. C. FUNG [1], p. 300). Owing to linear stress-strain relations in elasticity, both formulations are nearly equivalent.

Analysis of both the formulations 1) and 2) makes it easy to understand the reason for repeated attempts aimed at their refinement and correction: they do not specify the form and material of the body considered; their conclusions are inaccurate and vague, expressions of the type "substantial", "negligible", "small regions", "large distances" etc. being highly ambiguous. Moreover, in the mechanics of engineering structures numerous cases are encountered in which even such general statements are found to be not true. For instance, N. J. HOFF [3] considered in 1945 the problem of torsion of bars of thin-walled cross-sections. He found that in such cases the effect of actual load distribution at the end of the rod extends far beyond the limits established for solid cross-sections, and in the case of pin-jointed space frameworks loaded by self-equilibrated force systems axial forces in some bars do not decrease at all with increasing distance from the loaded region.

It is also well-known that in elastic bodies containing certain structural defects (cracks, notches), infinite stress concentration appear at very large distances from the self-equilibrated load systems, thus contradicting the formulation 2) of the principle.

In view of such effects, attempts are made to base the principle on the notion of the strain energy density and to account for the form of the body (structure) considered. Already in 1885 J. V. Boussinesq considered the case of an elastic halfspace loaded at the surface. The reader is referred to extensive bibliography of the subject published, for instance, in books [1, 5].

It was R. VON MISES in 1945 and E. STERNBERG in 1954 ([6] and [12]) who tackled the problem first in more detail and determined the order of vanishing the strains under increasing distances from the regions loaded by self-equilibrated force systems. They introduced the notion of forces remaining in "astatic" equilibrium (under rotations of forces through equal angles about their points of application).

There exists a very large number of more recent papers in which the authors attempt to reformulate the principle and to put it in a possibly rigorous form. Let us mention here the fundamental paper by R. A. TOUPIN [14] who considered a linear elastic cylinder loaded by a system of self--equilibrated forces applied to its end. Making the observation that the rate of decay of the strain must depend on the shape of the cross-section, he formulated and proved two fundamental theorems. The theorems are concerned with the distribution of strain energy stored within the cylinder, and with strain estimates at points located at large distances from the loaded end and at finite distances from the surface. The latter assumptions made it possible to avoid the stress concentrations mentioned before. The resulting inequality contains an exponential decay function involving several experimentally determinate parameters of definite physical meaning. Remarks concerning non-cylindrical bodies conclude the paper.

Under similar assumptions concerning the form of body considered (prismatic or not), the problem was analyzed one year later by A. ROBINSON [8] within the framework of non-standard analysis. In 1983 R. J. KNOPS and L. E. PAYNE [4] considered the nonlinear generalization of the theorem concerning a cylinder loaded at the end.

In all the cases mentioned above the loads are assumed to be "self--equilibrated", or equivalent load systems are assumed to yield the same resultant forces and moments. This problem of "statical equivalence" will now be discussed on the basis of two simple examples of an elastic half--plane and half-space loaded at the segment of the edge (surface).

2. ELASTIC HALF-PLANE LOADED AT THE EDGE

Consider the well-known solution (Flamant 1892, Michell 1900) concerning a half-plane $y>0$ loaded at its edge by a concentrated force $P=1$ (force per unit thickness of the plate) applied to the origin of the Cartesian coordinate system x, y (Fig. 1). The solution (cf. e.g. [7]) has the form

$$\sigma_{xx}^{00} = \frac{2}{\pi}\frac{x^2 y}{r^4}, \quad \sigma_{yy}^{00} = \frac{2}{\pi}\frac{y^3}{r^4}, \quad \sigma_{xy}^{00} = \frac{2}{\pi}\frac{xy^2}{r^4}, \tag{2.1}$$

where $r^2 = x^2+y^2$, and σ_{xx}^{00}, σ_{yy}^{00}, σ_{xy}^{00} are components of the plane stress state.

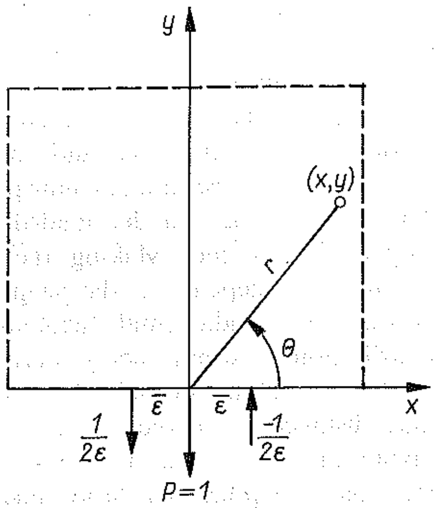

FIG. 1.

Following now the derivations given in [11] and [10], introduce the notion of a concentrated moment M^{10} which is a result of the following limiting procedure: two concentrated forces $P = 1/2\varepsilon$ and $P = -1/2\varepsilon$ are applied to the respective points $x = -\bar{\varepsilon}$, $x = \bar{\varepsilon}$ of the edge $y = 0$; with the distance $\bar{\varepsilon} \to 0$ the load transforms to the first order "concentrated moment" $M^{10} = 1$ and the corresponding stress components become

$$\sigma_{ij}^{10}(x, y) = \frac{\partial}{\partial x}[\sigma_{ij}^{00}(x, y)], \tag{2.2}$$

indices i, j assuming the values x, y. In this derivation ε is a dimensionless parameter, and $\bar{\varepsilon}$ is the corresponding multiple of length unit [1].

The procedure of superposition of concentrated forces $1/2\varepsilon$, $-1/2\varepsilon$ outlined above (the forces may be termed zero order moments M^{00}) may now be repeated with respect to the first order moments to lead to the second order moments M^{20}. In this manner

$$\sigma_{ij}^{20}(x, y) = \frac{\partial}{\partial x}[\sigma_{ij}^{10}(x, y)], \tag{2.3}$$

and, in general,

$$\sigma_{ij}^{n0}(x, y) = \frac{\partial^n}{\partial x^n}[\sigma_{ij}^{00}(x, y)]. \tag{2.4}$$

In particular, for instance,

$$\sigma_{yy}^{10}(x, y) = -\frac{8}{\pi}\frac{xy^3}{r^6}, \quad \sigma_{yy}^{20}(x, y) = \frac{8}{\pi}\frac{(5x^2 - y^2)\,y^3}{r^8}, \tag{2.5}$$

and, in general,

(2.6) $$\sigma_{yy}^{n0} = \frac{2}{\pi} \frac{(-1)^n n!}{r^{n+1}} f_n(\theta),$$

with the notation

(2.7) $$f_n(\theta) = \frac{1}{4} [(n+3) \sin (n+1) \theta - (n+1) \sin (n+3) \theta].$$

Let us now approach the problem of determining the stress field disturbances produced by redistribution of loads acting at the edge of an elastic half-plane. Assume the segment $-a, a$ of the edge $y=0$ to be loaded by a certain distribution of vertical forces $p(x)$. The boundary conditions have the form

(2.8) $$\begin{aligned} \sigma_{yy}(x, 0) &= p(x) \quad &\text{for} \quad |x| \leqslant a, \\ \sigma_{yy}(x, 0) &= 0 \quad &\text{for} \quad |x| > a, \\ \sigma_{xy}(x, 0) &= 0 \quad &\text{for} \quad |x| < \infty. \end{aligned}$$

Function $p(x)$ satisfies the condition

$$\int_{-a}^{a} p(x)\, dx = P.$$

Treating now stresses σ_{ij}^{00} of Eq. (2.1) as a set of Green functions and denoting them by

$$\sigma_{ij}^{00} = G_{ij}(x, y),$$

solution of the boundary value problem with conditions (2.8) may be written explicitly

(2.9) $$\sigma_{ij}(x, y) = \int_{-a}^{a} p(\xi)\, G_{ij}(x-\xi, y)\, d\xi.$$

Value $-\xi$ appearing in (2.9) may now be viewed as a small increment of variable x thus enabling us to expand the Green function into a Taylor series in the neighbourhood of point x; y plays the role of a parameter.

(2.10) $$G_{ij}(x-\xi, y) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \xi^n \frac{\partial^n G_{ij}(x, y)}{\partial x^n}.$$

The region of convergence of (2.10) will be discussed on the example of stress σ_{yy} with the Green function $(2.1)_2$

$$G_{yy}(x-\xi, y) = \frac{2}{\pi} \frac{y^3}{\bar{r}^4},$$

Here

$$\bar{r}^2 = (x-\xi)^2 + y^2.$$

Application of Eq. (2.6) yields the expansion

$$\frac{2}{\pi} \frac{y^3}{[(x-\xi)^2+y^2]^2} = \frac{1}{2\pi r} \sum_{n=0}^{\infty} \left(\frac{\xi}{r}\right)^n f_n(\theta). \tag{2.11}$$

In view of the obvious inequality

$$|f_n(\theta)| \leqslant \frac{n}{2}+1,$$

series (2.11) will be convergent for any $0 \leqslant \theta \leqslant \pi$ provided

$$\xi < r = \sqrt{x^2+y^2}. \tag{2.12}$$

Remainder R_N of the expansion (2.11)

$$G_{yy}(x-\xi, y) = \frac{1}{2\pi r} \sum_{n=0}^{\infty} \left(\frac{\xi}{r}\right)^n f_n(\theta) + R_n,$$

satisfies the inequality

$$R_N < \frac{1}{4\pi r} \frac{(N+2)(\xi/r)^{N+1} - (N+1)(\xi/r)^{N+2}}{[1-(\xi/r)^2]^2}, \tag{2.13}$$

Substitute now the expansion (2.10) into (2.9). In the formula for stresses σ_{ij}

$$\sigma_{ij}(x, y) = \sum_{n=0}^{\infty} M_n \frac{(-1)^n}{n!} \sigma_{ij}^{n0}(x, y), \tag{2.14}$$

or, in the particular case of σ_{yy},

$$\sigma_{yy}(r, \theta) = \frac{1}{2\pi} \sum_{n=0}^{\infty} M^{n0} \frac{f_n(\theta)}{r^{n+1}}. \tag{2.15}$$

Coefficient M^{n0} appearing under the summation sign

$$M^{n0} = \int_{-a}^{a} \xi^n p(\xi)\, d\xi, \tag{2.16}$$

is independent of variables x, y and represents the n-th moment of load $p(x)$ about the origin of the coordinate system.

Application of dimensionless coordinates $\varrho = r/a$ and $\bar{x} = x/a$ makes it possible to write the formula (2.15) in the form

$$\sigma_{yy}(\varrho, \bar{x}) = \frac{1}{2\pi} \sum_{n=0}^{\infty} \left[\int_{-1}^{1} \bar{x}^n p(a\bar{x})\, d\bar{x} \right] \frac{f_n(\theta)}{n+1}. \tag{2.17}$$

Convergence of the series (2.14), (2.15), (2.17) may be analyzed on the basis of condition (2.12); it depends on the interval of integration (2.16); since ξ should be less than r, and ξ varies between $-a$ and a, series (2.14)—(2.17) will converge for all $r > a$, i.e. outside the semicircle of radius a centered at the midpoint of the loaded portion of the edge (shaded area in Fig. 2). The same conclusion may be drawn directly from Eq. (2.17).

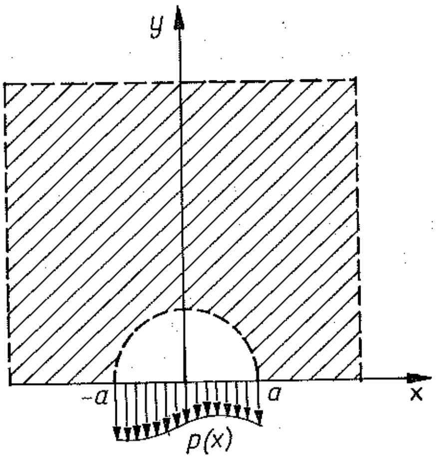

FIG. 2.

3. CONCLUSIONS

Formulae (2.14)—(2.17) are of fundamental importance for establishing the effects of load rearrangement upon the stress field at considerable distances from the loaded region.

Denote by $p(x)$ and $p'(x)$ two different load distributions over the segment $-a, a$ both of them satisfying the condition

$$\int_{-a}^{a} p(x)\,dx = \int_{-a}^{a} p'(x)\,dx = P.$$

Stresses produced by these loads in region $r > a$ of the half-plane $y > 0$ are denoted by $\sigma_{ij}(r, \theta)$ and $\sigma'_{ij}(r, \theta)$, respectively. In the case of stresses σ_{yy}, the difference of stresses σ_{ij} and σ'_{ij} is written, according to Eq. (2.15), in the form

$$\sigma_{yy} - \sigma'_{yy} = \frac{1}{2\pi} \sum_{n=0}^{\infty} (M^{n0} - M'^{n0}) \frac{f_n(\theta)}{r^{n+1}}; \tag{3.1}$$

this result enables us to draw the following simple conclusions.

1. If both loads $p(x)$ and $p'(x)$ yield the same resultant force (zero order moments) but different resultant moments (of first order), the difference of the corresponding stresses produced by the loads at distances $r > a$ will be of order $K(a/r)^2$; the value of K may be established by an analysis of coefficients M^{n0} and M'^{n0} similar to that used in the preceding section (Eq. (2.11)).

2. If both loads yield the same resultant force and moment, i.e. if they are "statically equivalent" according to the traditional definition, and differ by the second order moments M^{20} and M'^{20}, the stress differences will be of the order of $K(a/r)^3$.

3. If the first $N+1$ moments $M^{00}, M^{10}, \ldots, M^{n0}$ of both load systems are identical, the corresponding differences will be of the order of $K(a/r)^{N+2}$.

It is seen that the static equivalence requirement appearing in most of the Saint Venant principle formulations should be considered as a particular case of a more general equivalence requirement to be imposed on the load systems leading to similar stress distributions at large distances from the loaded region. Estimation of the resulting stress differences seems to be relatively simple. The traditional formulation of the principle results probably from the fact that the resultant forces and moments of the first order appear in the fundamental laws of motion (conservation of momentum and moment of momentum); however, such approach seems to be ungrounded in the case of formulation of the Saint Venant principle.

4. Numerical examples

To illustrate the conclusions let us consider two simple examples of an elastic half-plane $y > 0$ and a half-space $z > 0$ acted on by loads acting at the segment $-a \leqslant x \leqslant a$ (Fig. 3a, b). In the first case normal load p is constant along the segments $-a \leqslant x < 0$ and $0 \leqslant x < a$ so that the boundary conditions have the form

$$\sigma_{yy} = \begin{cases} p_1 & \text{for} \quad -a \leqslant x < 0, \\ p_2 & \text{for} \quad 0 \leqslant x < a. \end{cases}$$

The accurate solution for stresses $\sigma_{yy}(x, y)$ is easily obtained by integration of Eq. (2.9) and

$$(4.1) \quad \sigma_{yy}(x, y) = \frac{p_1}{\pi}\left[y\left(\frac{x+1}{(x+1)^2+y^2} - \frac{x}{x^2+y^2} \right) + \left(\tan^{-1}\frac{x+1}{y} - \tan^{-1}\frac{x}{y} \right) \right] + \frac{p_2}{\pi}\left[y\left(\frac{x}{x^2+y^2} - \frac{x-1}{(x+1)^2+y^2} \right) + \right.$$

$$+\left(\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-1}{y}\right)\Bigg].$$

The approximate solutions involving the resultant moments of order 0, 1 and 2 are written in the form

$$\sigma_{yyI} = M^{00}\sigma_{yy}^{00}, \tag{4.2}$$

$$\sigma_{yyII} = M^{00}\sigma_{yy}^{00} - M^{10}\sigma_{yy}^{10}, \tag{4.3}$$

$$\sigma_{yyIII} = M^{00}\sigma_{yy}^{00} - M^{10}\sigma_{yy}^{10} + \frac{1}{2}M^{20}\sigma_{yy}^{20}, \tag{4.4}$$

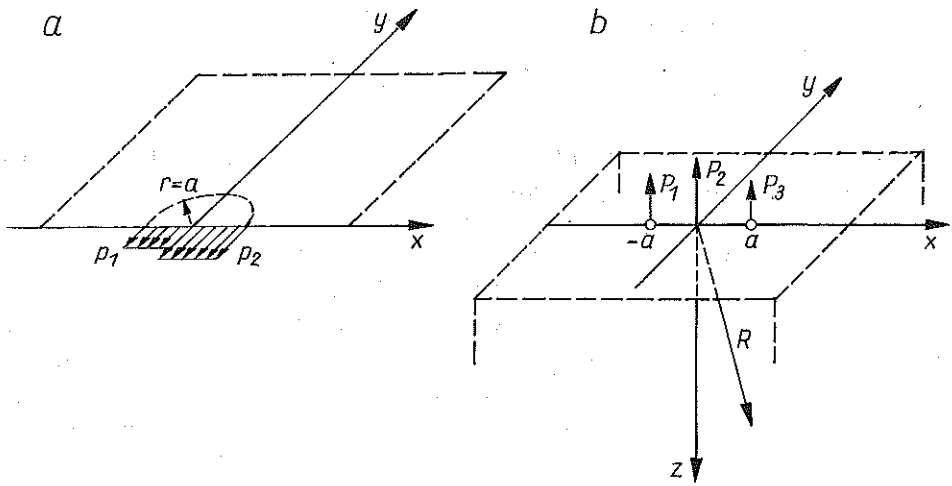

FIG. 3.

according to formulae (2.14), (2.15) with notations (2.1) and (2.5). Resultant moments are in this case equal to

$$M^{00} = (p_1+p_2)\,a, \quad M^{10} = (-p_1+p_2)\frac{a^2}{2}, \quad M^{20} = (p_1+p_2)\frac{a^3}{3}.$$

Numerical results of approximations (4.2)—(4.4) given in Table 1 have been evaluated for the case when $a=1$, $p_1 = p = 1$, $p_2 = 2p = 2$. Accurate values of stresses $\sigma_{yy}(x, y)$ are determined from Eq. (4.1) at several locations x, y and given in column (0) of Table 1. In columns (1), (2), (3) are given the absolute values of errors (in %) following from the consecutive approximations (4.2), (4.3), (4.4) according to the formula

$$\gamma_n\% = 100\%\,\frac{\sigma_{yy0}-\sigma_{yyn}}{\sigma_{yy0}}, \qquad n = 1, 2, 3.$$

Table 1.

		stress σ_{yy}	Errors of consecutive approximations (%)		
x	y	(0)	(1)	(2)	(3)
0	2	.82472	15.8	15.8	3.5
0	3	.59373	7.2	7.2	.71
0	4	.45863	4.1	4.1	.22
0	5	.37214	2.6	2.6	.081
0	10	.18972	.67	.67	.0060
1	4	.42778	1.13	2.75	.23
2	4	.32678	6.49	.25	.36
3	4	.21696	9.85	2.65	.14
4	4	.13441	11.2	3.79	.050
5	4	.08193	11.2	4.04	.20
10	4	.00984	3.85	.65	.12

Accuracy of the consecutive approximations increases with n in most cases, and "statical equivalence" is found to be of no particular importance in this respect; in some cases the condition of equal "zero-order moments" M^{00} may already lead to satisfactory results provided $x^2+y^2 > a^2 = 1$ (outside the semicircle shown in Fig. 3a).

Another numerical example is given in Fig. 3b and Table 2. It is known that stresses produced in an elastic semispace $z > 0$ loaded at its surface $z = 0$ by a concentrated force $P = 1$ applied to the origin of coordinate system (the Boussinesq problem, cf., e.g., [5, 7, 13]) are given by the formulae

$$\sigma_{zz}(x, y, z) = \frac{3}{2\pi}\frac{z^3}{R^5}, \qquad R^2 = x^2+y^2+z^2, \tag{4.5}$$

followed by similar expressions for the remaining stress components. Denoting stress (4.5), by analogy with (2.1), by $\sigma_{zz}^{00}(x, y, z)$, solution of the problem shown in Fig. 3b (three concentrated forces P_1, P_2, P_3 acting at $x = -a, 0, a$, $y = z = 0$, respectively) assumes the form

$$\sigma_{zz}(x, y, z) = P_1\,\sigma_{zz}^{00}(x+a, y, z)+P_2\,\sigma_{zz}^{00}(x, y, z)+P_3\,\sigma_{zz}^{00}(x-a, y, z). \tag{4.6}$$

Stresses σ_{ij}^{n0} produced by unit n-th order concentrated moments (2.16) applied at $x = y = z = 0$ are calculated from the formulae analogous with (2.4), and it is easily found that

$$\begin{aligned} \sigma_{zz}^{00}(x, y, z) &= \frac{3}{2\pi}\frac{z^3}{R^5}, \\ \sigma_{zz}^{10}(x, y, z) &= \frac{3}{2\pi}\frac{-5xz^3}{R^7}, \end{aligned} \tag{4.7}$$

Table 2.

			stress σ_{zz}	errors of consecutive approximations (%)		
x	y	z	(0)	(1)	(2)	(3)
0	0	4	.16226	10.3	10.3	1.15
0	0	6	.076065	4.62	4.62	.226
0	0	8	.043628	2.60	2.60	.0714
0	0	10	.028179	1.67	1.67	.0292
2	0	4	.12088	15.2	1.07	1.75
4	0	4	.042725	25.9	10.5	.837
6	0	4	.012548	25.1	10.7	1.42
8	0	4	.0040887	21.7	8.61	1.10
10	0	4	.0015501	18.4	6.66	.755
−2	0	4	.089145	15.0	4.19	.355
−4	0	4	.028846	9.73	13.1	1.15
−6	0	4	.0086646	8.52	12.3	1.03
−8	0	4	.0029540	8.43	9.64	.747
−10	0	4	.0011688	8.24	7.31	.521
4	4	4	.013591	15.5	3.75	.160
6	6	6	.0056808	10.1	1.82	.0323
10	10	10	.0019584	5.92	.692	.000425

$$\sigma_{zz}^{20}(x, y, z) = \frac{3}{2\pi} \frac{-5(R^2-7x^2)z^3}{R^9}.$$

In the numerical example it was assumed that $a=1$, $P_1=1$, $P_2=2$, $P_3=3$, so that

$$M^{00}=6, \quad M^{10}=2, \quad M^{20}=4;$$

moments M^{ij} for $j \neq 0$ vanish, loads P being distributed along the x-axis.

Accurate values of stress σ_{zz}, Eq. (4.6), at several points x, y, z (satisfying the condition $R>1$) are given in Table 2, column (0). Columns (1), (2), (3) present, as before, absolute values of errors resulting from the consecutive approximations (4.7) analogous with (4.2)—(4.4).

Also here, in a three-dimensional case, it is seen that the statical equivalence does not decide on the accuracy of approximation, what suggests the possibility of reformulation of de Saint Venant's principle.

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Streszczenie

O STATYCZNIE RÓWNOWAŻNYCH OBCIĄŻENIACH W ZASADZIE SAINT VENANTA

W tradycyjnych sformułowaniach zasady de Saint Venanta zakłada się, że obciążenia prowadzące do zbliżonych rozkładów naprężeń powinny być statycznie równoważne, a więc prowadzić do identycznych sił i momentów wypadkowych. W pracy wykazano, że założenie to zastąpić można ogólniejszym, zgodnie z którym porównywać należy pierwsze $N+1$ momentów rzędu $0, 1, \ldots, N$ obu układów sił obciążających. W przypadku półpłaszczyzny sprężystej obciążonej na krawędzi okazuje się, że różnice naprężeń wywołane obydwoma układami i obliczone dla odległości dużych w porównaniu z rozmiarami obszaru obciążonego są rzędu kr^{-N-2}, gdzie N oznacza najwyższy rząd identycznych momentów obu układów, a r — odległość od obszaru obciążenia.

Резюме

О СТАТИЧЕСКИ ЭКВИВАЛЕНТНЫХ НАГРУЖЕНИЯХ В ПРИНЦИПЕ СЕН-ВЕНАНА

В традиционных формулировках принципа Сен-Венана предполагается, что нагружения, приводящие к сближенным распределениям напряжений, должны быть статически эквивалентными, значит должны приводит к идентичным результирующим силам и моментам. В работе показано, что это предположение можно заменить более общим, согласно с которым следует сравнивать первые $N+1$ моменты порядка $0, 1 \ldots, N$ обоих

систем нагружающих сил. В случае упругого полупространства, нагруженного на крае, оказывается, что разницы напряжений, вызванные обоими системами и расчитанные для расстояний больших по сравнению с размерами нагруженной области, порядка kr^{-N-2}, где N обозначает самый высокий порядок идентичных моментов обоих систем, а r — расстояние от области нагружения.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH

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