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APPROXIMATE ANALYTIC SOLUTION FOR THE COMPRESSION AND TORSION PROCESS IN THE SPLIT HOPKINSON PRESSURE BAR

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Simplified analysis of the process of dynamical compression and torsion in the system of the Split Hopkinson Pressure Bar (SHPB) is presented. Bilinear relation of stress to strain, $\sigma(\varepsilon)$, for the specimen material and time-independence of the incident pulse $\sigma_J(t)$ = const has been assumed in the solution. In the compression process the effect of friction between the specimen and the rods has been taken into account. As a result of the analysis, the possibility of estimation the time-dependence of the reflected and transmitted pulses in Hopkinson bars and of the mean stress $\sigma(t)$ and strain $\varepsilon(t)$ in the specimen has been obtained. The relations were used to perform calculations, the results of which have been compared with the results of experimental investigations. Small differences between the calculated and the experimental data have been found.

1. Introduction

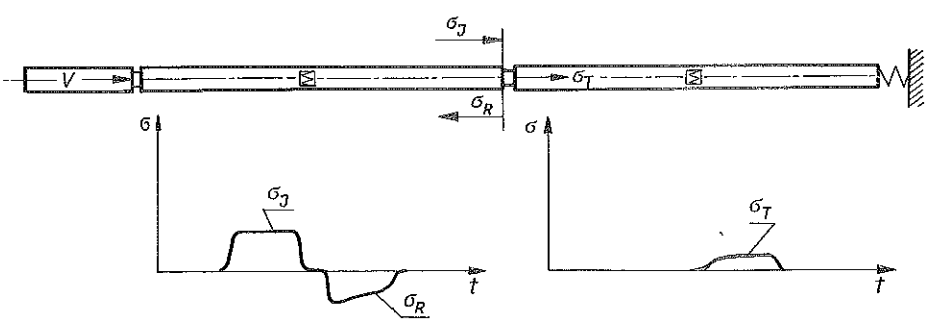

FIG. 1. Scheme of the bars-specimen system of the Split Hopkinson Pressure Bar. σ_J, σ_R – incident and reflected pulse, σ_T – transmitted pulse.

Up till now, the method of the Split Hopkinson Pressure Bar (SHPB) is the principal experimental method of investigating the plastic properties of metals at high deformation velocities, about 10^3 1/s. In Fig.1 the basic part of the SHPB apparatus is presented. It is a system consisting of two elastic bars between which the investigated specimen is placed. The specimen initially exhibits elastic and then plastic properties. The analysis of longitudinal waves propagation during dynamical compression of the specimen, based on the Taylor - Kármán - Rachmatulin theory, leads to the solution of the following set of equations:

$$\rho\frac{\partial v}{\partial t}=\frac{\partial \sigma}{\partial x},\qquad \frac{\partial v}{\partial x}=\frac{1}{\rho a^2(\sigma)}\frac{\partial \sigma}{\partial t},\qquad a^2(\sigma)=\frac{1}{\rho}\frac{\partial \sigma}{\partial \varepsilon},\tag{1.1}$$

where x and t are the Lagrange coordinate, and σ, ε, v and a denote stress, strain, particle and wave velocities.

Accuracy of the method of the SHPB has been studied in numerous works. A one-dimensional, numerical solution for the bars-specimen system was presented in the paper of RAND and JACKSON [1] and in the paper by JAHSMAN [2]. BERTHOLF and KARNES [3] obtained an exact two-dimensional solution of the problem using the numerical finite difference method. Comparison of the numerical solution of the system (1.1), in which friction on the contact surfaces between the specimen and the bars being taken into account, with the results of experiments were presented in the papers [4] and [5].

2. Solution for the case of compression

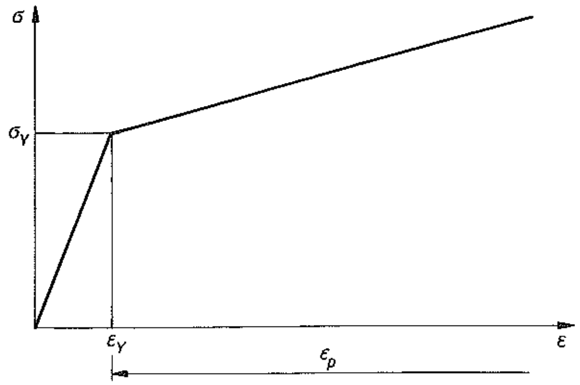

FIG. 2. Assumed bilinear relation $\sigma(\varepsilon)$ for specimen material.

Under the following two assumptions:

a) relation $\sigma(\varepsilon)$ for the specimen material is bilinear, as shown in Fig.2,

b) the value of the incident pulse is constant in time: $\sigma_J(t)$ = const,

the propagation of longitudinal stress waves in the compressed specimen can be described analytically. In the bilinear approximation of the relation $\sigma(\varepsilon)$ for the specimen material, as shown in Fig.2, two ranges are distinguished.

By convention, the one for $\sigma \leq \sigma_Y$ is denoted as "elastic", and the one for $\sigma > \sigma_Y$ – as "plastic".

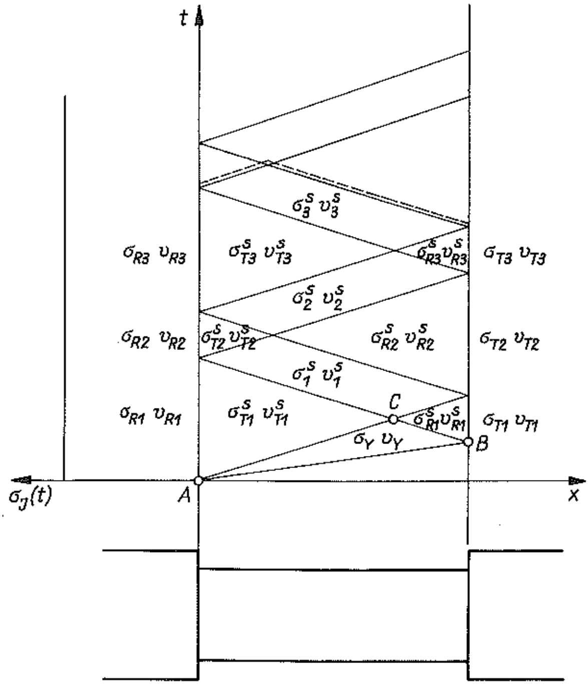

FIG. 3. Schematic diagram of plastic wave propagation for a compressed specimen in the initial state.

In Fig.3 is presented the form of characteristics in the plane (x, t) for the first three passages of the plastic wave through a specimen. The notations for stresses and particle velocities used in Fig.3 in various regions of the (x, t)-plane will be applied in further analysis. To derive the relations for stresses and particle velocities, the system of equations consisting of the equation of forces equilibrium, the equation of particle velocity continuity, and the relations between velocity and stress along the characteristics were used.

To simplify the description it can be assumed that incident pulse is produced at the left-hand side bar with respect to the specimen, as it has been shown in Fig.1. In that case, the following set of equations can be written down for the points on the left-hand side surface of the specimen:

$$A_b(\sigma_J - \sigma_R) = A_s(\sigma^s + \Delta\sigma_p),$$

$$(2.1) \qquad v_J + v_R = v^s + \Delta v_p,$$

$$v_J = \frac{\sigma_J}{\rho_b c_b}, \qquad v_R = \frac{\sigma_R}{\rho_b c_b}, \qquad \Delta v_p = \frac{\Delta\sigma_p}{\rho c},$$

where A_b, ρ_b and c_b denote cross-sectional area of the rod, density and propagation velocity of an elastic wave for the bar material, respectively; similarly, A_s, ρ and c denote the same quantities for the specimen, and Δv_p and $\Delta\sigma_p$ denote velocity and stress increments. The remaining quantities in the system (2.1) correspond to velocities and stresses in the regions of the phase plane (x,t), as it has been shown in Fig.3.

There exists also a possibility of taking into account the friction effect at the contact surfaces between the specimen and the bars. According to the method proposed in [6], if friction is included, the first equation of the system (2.1) can be represented in the form

$$f(\mu)A_b(\sigma_J - \sigma_R) = A_s(\sigma^s + \Delta\sigma_p), \qquad f(\mu) = 1 - \frac{\mu}{3s}, \tag{2.2}$$

where μ is the friction coefficient, and $s = l/d$ is the length-to-diameter ratio of the specimen.

At the boundary point A in Fig.3 the particular form of the set (2.1) is

$$\begin{aligned} \left(1 - \frac{\mu}{3s}\right) A_b(\sigma_J - \sigma_{R1}) &= A_s(\sigma_Y + \Delta\sigma_p), \\ \frac{\sigma_J}{\rho_b c_b} + \frac{\sigma_{R1}}{\rho_b c_b} &= \frac{\sigma_Y}{\rho c'} + \frac{\Delta\sigma_p}{\rho c''}, \end{aligned} \tag{2.3}$$

where c' denotes the wave propagation velocity in the specimen for stresses $0 < \sigma \leq \sigma_Y$, and c'' – for stresses $\sigma > \sigma_Y$.

By solving the system (2.3) it is possible to find the unknowns σ_{R1} and $\Delta\sigma_p$ and then to determine σ^s_{T1} and v^s_{T1}. By introducing the notations

$$F = \frac{A_b}{A_s}\left(1 - \frac{\mu}{3s}\right), \qquad N = 1 - \frac{c''}{c'}, \qquad K = F\frac{\rho_b c_b}{\rho c''}, \qquad R = \frac{A_b}{A_s},$$

the following form of equations for σ^s_{T1} and v^s_{T1} can be found:

$$\begin{aligned} \sigma^s_{T1} &= \sigma_J \frac{2F}{K+1} + \sigma_Y \frac{NK}{K+1}, \\ v^s_{T1} &= \sigma_J \frac{2F}{(K+1)\rho c''} - \sigma_Y \frac{N}{(K+1)\rho c''}. \end{aligned} \tag{2.4}$$

In a similar way one can find σ^s_{R1} and v^s_{R1} for the problem with friction, at the point B at the right-hand side of the specimen. Then, by substituting the relations for $\sigma^s_{T1}, v^s_{T1}, \sigma^s_{T1}$ and v^s_{R1} found for the points A and B into the equations for the point C lying inside the specimen, the relations for σ^s_1 and v^s_1 can be derived. By carrying out the same operations for the consecutive boundary and at the inner points of characterictics intersection, one can subsequently derive the relations for $\sigma^s_{Ti}, \sigma^s_i, \sigma^s_{Ri}, v^s_{Ti}, v^s_i, v^s_{Ri}$ (for $i =$

1, 2, 3 ...) within the entire phase plane region bounded by dotted line in Fig.3.

It turns out that those relations can be put down in six groups in which their properties are those of geometrical progressions. Further terms of these sequences, for $i > 3$, determine stresses and particle velocities; hence, they constitute extension of the solution in the phase plane (x, t). Using this method, the expressions for all the stress and particle velocity components, introduced in this analysis, can be derived. Putting $\sigma_i \equiv \sigma(i)$ these expressions can be written down as follows:

$$
\begin{aligned}
\sigma^s_T(m) &= \sigma_J p \frac{q^m - 1}{q - 1} + \sigma_Y \frac{KN}{K+1} q^{m-1}, \\
\sigma^s_R(n) &= \sigma_J p \frac{q^n - 1}{q - 1} + \sigma_Y \frac{KN}{K+1} q^{n-1}, \\
v^s_T(m) &= \sigma_J \frac{p}{\rho c''} \frac{q^m + 1}{q + 1} - \sigma_Y \frac{N}{\rho c''(K+1)} q^{m-1}, \\
v^s_R(n) &= \sigma_J \frac{p}{\rho c''} \frac{1 - q^n}{q + 1} + \sigma_Y \frac{N}{\rho c''(K+1)} q^{n-1}, \\
\sigma^s_T(n) &= \sigma_J p \frac{q^{n-1} - 1}{q - 1} + \sigma_Y \frac{K(2-N)}{K+1} q^{n-1}, \\
\sigma^s_R(m) &= \sigma_J p \frac{q^{m-1} - 1}{q - 1} + \sigma_Y \frac{K(2-N)}{K+1} q^{m-1}, \\
& \qquad m = 1, 3, 5, \ldots, \qquad n = 2, 4, 6, \ldots; \\
v^s_T(n) &= \sigma_J \frac{p}{\rho c''} \frac{q^{n-1} - 1}{q + 1} - \sigma_Y \frac{2-N}{\rho c''(K+1)} q^{n-1}, \\
v^s_R(m) &= \sigma_J \frac{p}{\rho c''} \frac{1 - q^{m-1}}{q + 1} + \sigma_Y \frac{2-N}{\rho c''(K+1)} q^{m-1}, \\
\sigma^s(l) &= \sigma_J p \frac{q^l - 1}{q - 1} + \sigma_Y q^l, \\
v^s(l) &= \sigma_J \frac{p}{\rho c''} \frac{1 - (-q)^l}{q + 1} + \sigma_Y \frac{1-N}{\rho c''} (-q)^l, \\
& \qquad l = 1, 2, 3, 4, 5, \ldots .
\end{aligned}
\tag{2.5}
$$

In the above expressions, p and q denote

$$p = \frac{2F}{K+1}, \qquad q = \frac{K-1}{K+1}.$$

After having derived the relations (2.5), our object will be to obtain the relations of stress and deformation to time: $\sigma[t, (k)], \varepsilon[t(k)]$, where k is the number of passages of an elastic and plastic wave through the specimen.

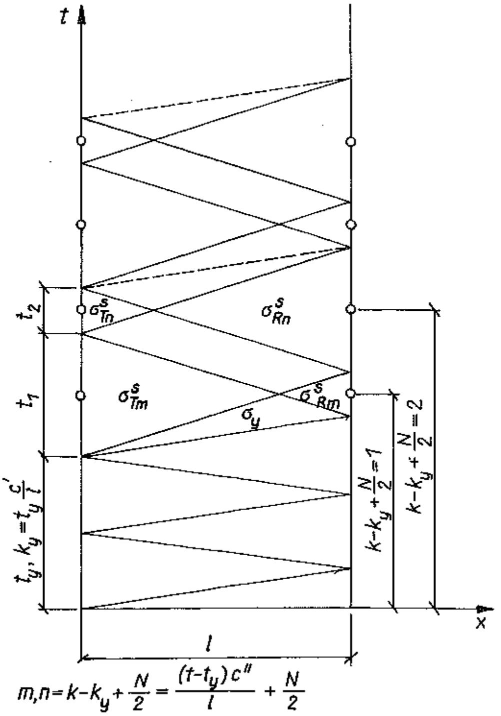

FIG. 4. Schematic diagram of elastic and plastic wave propagation in a specimen.

The values of σ and ε in these relations will correspond to a uniform stress state, averaged along the specimen.

From Figs.3 and 4 it results that in the deformation process cycles appear, the duration times of which equal the time of two passages of a plastic wave through the specimen: $t = 2l/c$. Duration time of each cycle can be split into two parts t_1 and t_2, marked in Fig.4. At these times stresses and particle velocities at the front surfaces assume values described by Eq.(2.5). According to Fig.4, times t_1 and t_2 can be expressed as

(2.6)
$$t_1 = \frac{l}{c'} + \frac{l}{c''} = \frac{l(c'+c'')}{c'c''},$$
$$t_2 = \frac{l}{c''} - \frac{l}{c'} = \frac{l(c'-c'')}{c'c''}.$$

Hence,

(2.7)
$$\frac{t_1}{t_1+t_2} = 1 - \frac{N}{2}, \qquad \frac{t_2}{t_1+t_2} = \frac{N}{2}.$$

In every subsequent time cycle $t = t_1 + t_2$ the averaged stress values σ^s_T and

σ^s_R can be found from the formula

$$(2.8)\qquad \begin{aligned} \sigma^s_T(m, n = m+1) &= \sigma^s_T(m)\frac{t_1}{t_1+t_2} + \sigma^s_T(n)\frac{t_2}{t_1+t_2}, \\ \sigma^s_R(m, n = m+1) &= \sigma^s_R(m)\frac{t_2}{t_1+t_2} + \sigma^s_R(n)\frac{t_1}{t_1+t_2}. \end{aligned}$$

From the kinematics of wave propagation in the specimen it results that the power exponents m and n in Eqs. (2.5) can be expressed by the parameters k_y and k which denote respectively, the number of elastic wave passages and the total number of wave passages through the specimen, from the beginning of the deformation process. As it can be seen in Fig.4, m and n can be expressed as functions of k and k_y in the following way:

$$(2.9)\qquad m = k - k_y + \frac{N}{2}, \qquad n = k - k_y + \frac{N}{2}.$$

For such defined m and n the relations (2.5) are accurate in the middle points of time intervals t_1 and t_2 . Between the time increment Δt and parameter increment Δk there is the relation $\Delta t = \Delta kl/c$. Making use of relations (2.8) and (2.5), (2.7), (2.9), one can express the stress values σ^s_T and σ^s_R at the contact surfaces between the specimen and the bars, averaged in the given cycle, as follows:

$$(2.10)\qquad \begin{aligned} \sigma^s_T(k) = \sigma_J F\left[\left(1-\frac{N}{2}\right)\left(1-q^{k-k_y+N/2}\right)\right. \\ \left. +\frac{N}{2}\left(1-q^{k-k_y+N/2-1}\right)\right] + \sigma_Y\frac{KN(2-N)}{K+1}q^{k-k_y+N/2-1}, \\ \sigma^s_R(k) = \sigma_J F\left[\left(1-\frac{N}{2}\right)\left(1-q^{k-k_y+N/2}\right)\right. \\ \left. +\frac{N}{2}\left(1-q^{k-k_y+N/2-1}\right)\right] + \sigma_Y\frac{KN(2-N)}{K+1}q^{k-k_y+N/2-1}. \end{aligned}$$

At a given time $t(k)$, stress distribution in the specimen is assumed to be uniform, equal to the mean values of $\sigma^s_T(k)$ and $\sigma^s_R(k)$

$$(2.11)\qquad \begin{aligned} \sigma(k) &= \frac{1}{2}[\sigma^s_T(k) + \sigma^s_R(k)], \\ \sigma(k) &= \sigma_J F\left[1 - q^{k-k_y+N/2}\left(1+\frac{N}{K-1}\right)\right] \\ &\qquad + \sigma_Y\frac{KN(2-N)}{K-1}q^{k-k_y+N/2}. \end{aligned}$$

For the elastic range it holds: $c'' = c'$, $N = 0$ and $k_y = 0$; therefore, relation (2.11) yields

$$\sigma(k) = \sigma_J F(1 - q^k). \tag{2.12}$$

From the condition $\sigma(k_y) = \sigma_Y$ we obtain

$$k_y = \frac{\ln\left(1 - \dfrac{\sigma_Y}{\sigma_J F}\right)}{\ln q}. \tag{2.13}$$

With the transition from the elastic to plastic range, at point $\sigma(k_y)$, the values of c'', N, σ_Y and k_y change in a stepwise manner. This yields that the values $\sigma(k = k_y)$ calculated from Eqs.(2.11) and (2.12) differ from each other. There is, however, a possibility of reducing this difference to zero and to obtain continuity in the limiting point. To this end it is sufficient to substitute k_1 for k_y in the relation (2.11) and to find k_1 from the condition that the values of $\sigma(k = k_y)$ obtained from (2.11) and (2.12) in the limiting point should be equal. From this condition it follows that

$$k_1 = k_y + \frac{N}{2} + \frac{\ln\left(\dfrac{C\sigma_J F - D\sigma_Y}{\sigma_J F - \sigma_Y}\right)}{\ln q}, \tag{2.14}$$

where

$$C = 1 + \frac{N}{K-1}, \qquad D = \frac{KN(2-N)}{K-1}.$$

Between the mean stress $\sigma(k)$ in the specimen and the plastic strain ε_p a relation exists

$$\sigma(k) = \sigma_Y + \rho c''^2 \varepsilon_p. \tag{2.15}$$

Using this relation and the relations (2.11) and (2.14), one can find the value of plastic strain ε_p as

$$\varepsilon_p = \frac{\sigma_J F}{\rho c''^2}\left[1 - q^{k-k_1+N/2}\left(1 + \frac{N}{K-1}\right)\right] + \frac{\sigma_Y}{\rho c''^2}\left(\frac{KN(2-N)}{K-1} q^{k-k_1+N/2} - 1\right). \tag{2.16}$$

On the basis of Eqs. (2.15) and (2.16) it is possible to estimate the stress $\sigma(k)$ and the strain $\varepsilon(k)$ in the specimen for the given value of parameter k which denotes the number of wave passages through the specimen.

To obtain a proper description of the problem studied, the relation between time t and parameter k should be derived. For time increment dt and increment dk, the following relation holds:

(2.17) $$dt = \frac{1}{c}\, dk.$$

Taking the logarithmic strain measure $\varepsilon = \ln(l_0/l)$, where l_0 and l denote the initial and the current specimen length, for the strain range $\varepsilon > \varepsilon_Y$ one obtains

(2.18) $$dt = \frac{l_0}{c'' e^{\varepsilon_Y} e^{\varepsilon_p}} dk.$$

Therefore, after integrating, for the plastic deformation range is follows.

(2.19) $$t = \frac{l_0}{c'' e^{\varepsilon_Y}} \int e^{-\varepsilon_p} dk + R_2,$$

where ε_p is described by the relation (2.16). Introducing the notations

(2.20) $$A_1 = \frac{\sigma_J F}{\rho c''^2}, \qquad B_1 = 1 + \frac{N}{k-1},$$
$$A_2 = \frac{\sigma_Y}{\rho c''^2}, \qquad B_2 = \frac{KN(2-N)}{k-1}, \qquad a = \frac{N}{2} - k_1;$$

on the basis of (2.16) and (2.19) one obtains

(2.21) $$t = \frac{l_0}{c''} e^{A_2 - A_1 - \varepsilon_Y} \int e^{Gq^k} dk + R_2,$$

where

$$G \equiv q^a (A_1 B_1 - A_2 B_2).$$

After integrating and finding the constant R_2 from the condition that $t(k = k_y) = t_y$, the formula for time $t(k)$ is found in the form

(2.22) $$t = t_y + \frac{l_0}{c'' \ln q} e^{A - \varepsilon_Y} \left[\ln q^{k-k_y} + \sum_{i=1,2,3\ldots} (Gq^{k_y})^i \frac{q^{i(k-k_y)} - 1}{i \cdot i!} \right],$$

where

$$A = A_2 - A_1 = (\sigma_Y - \sigma_J F)/\rho c''^2.$$

In the relation (2.22), the time of duration of the elastic deformation process of the specimen t_y is still the last unknown. To find t_y it is necessary to derive the equation for $t(k)$ for the elastic range, i.e. for $0 < t < t_y$. The relation (2.19) has in this case the following form:

(2.23) $$t = \frac{l_0}{c'} \int e^{-\varepsilon}\, dk + R_1.$$

Finding ε from (2.12) and substituting in (2.23) one gets

$$t = \frac{l_0}{c'} e^{-B} \int e^{Bq^k}\, dk + R_1, \qquad \text{where} \qquad B = \frac{\sigma_J F}{\rho c'^2}. \tag{2.24}$$

After integration and determination of the constant R_1 from the condition that $t(k=0)=0$, the expression for time $t(k)$ in the elastic range will be

$$t = \frac{l_0}{c' \ln q} e^{-B} \left(\ln q^k + \sum_{i=1,2,3\ldots} \frac{B^i(q^{ki}-1)}{i \cdot i!} \right). \tag{2.25}$$

The same equation can be obtained by substituting in Eq. (2.22) the values

$$t_y = 0, \qquad c'' = c', \qquad k_y = 0, \qquad \sigma_Y = 0, \tag{2.26}$$

and, in connection with this, $N = 0$, $k_1 = 0$, $A = -B$, $G = B$. These values are suitable for the elastic range of deformation, and by taking them into account Eq. (2.22) assumes the same form as Eq. (2.25).

The relations (2.15), (2.16) and (2.22), (2.25) derived above allow for finding $\sigma(k), \varepsilon(k)$ and $t(k)$ and this concludes the solution of the considered problem.

3. Solution for the case of torsion

The system of the torsional SHPB consist of two elastic rods between which a specimen in the form of a thin-wall tube is located. For the description of the dynamically twisted specimen, similarly to the case of compression, according to the Taylor - Kármán - Rachmatulin theory of elastic-plastic waves propagation in rods, the bilinear relations between the torsional stress and deformation $\tau(\gamma)$ and the rectangular shape of the impulse $\tau_J(t)$ have been assumed. As it was done above, in order to derive the relations for stresses and particle velocities, the system of equations is used, consisting of the torsional moments equilibrium equation, particle velocity continuity equation and relations between velocity and stress along characteristics. The difference consists in replacing the force equilibrium equation with the torsional moment equilibrium equation. The method of solution is then nearly identical to that for compression; for example, to adopt the sets of Eqs. (2.1) and (2.3) to the description of torsion, the quantities σ, A_b, A_s should be replaced by $\tau, J_b/r_b, A_s r$, respectively, where J_b, r_b denote the polar moment of inertia and the outer radius of the bar cross-section, and A_s and r denote the area and the mean radius of the specimen cross-section.

Furthermore, in the torsion case c_b and c denote the propagation velocities of the transverse wave in the rod and in the specimen; since the friction effect does not appear, $\mu = 0$, $f(\mu) = 1$ is assumed. The general form of

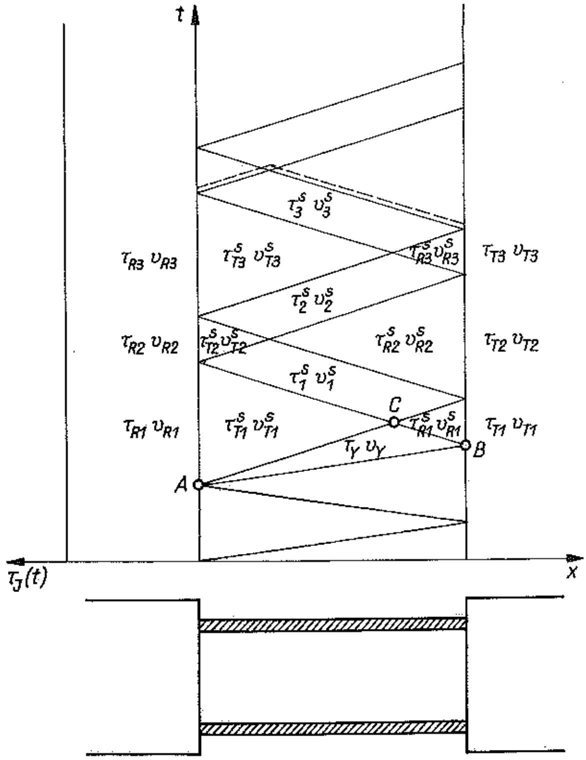

FIG. 5. Schematic diagram of plastic wave propagation for a twisted specimen in the initial state.

characteristics in the plane (x,t) displayed in Fig.5, is the same as that for compression. For point A the set of equations, analogous to the set (2.3), is

$$\frac{J_b}{r_b}(\tau_J - \tau_R) = A_s r(\tau_Y + \Delta\tau_p),$$
(3.1)
$$\frac{\tau_J}{\rho_b c_b} + \frac{\tau_R}{\rho_b c_b} = \frac{\tau_Y}{\rho c'} + \frac{\Delta\tau_p}{\rho c''}.$$

By introducing the similar notations as those for the case of compression,

$$F = \frac{J_b}{r_b A_s r}, \qquad N = 1 - \frac{c''}{c'}, \qquad K = \frac{J_b \rho_b c_b}{r_b A_s r \rho c''},$$

one can represent the expressions for τ_{T1}^s and v_{T1}^s in an identical form as expressions (2.4) for σ_{T1}^s and v_{T1}^s in the case of compression,

$$\tau_{T1}^s = \tau_J \frac{2F}{K+1} + \tau_y \frac{NK}{K+1},$$
(3.2)
$$v_{T1}^s = \tau_J \frac{2F}{(K+1)\rho c''} - \tau_Y \frac{N}{(K+1)\rho c''}.$$

Further relations have also the same form as the relations derived for compression. Final equations for stress $\tau(k)$ and plastic deformation $\gamma_p(k)$ have the form

$$\tau(k) = \tau_Y + \rho c''^2 \gamma_p, \tag{3.3}$$

$$\gamma_p = \frac{\tau_J F}{\rho c''^2}\left[1 - q^{k-k_1+N/2}\left(1 + \frac{N}{K-1}\right)\right] + \frac{\tau_Y}{\rho c''^2}\left(\frac{KN(2-N)}{K-1} q^{k-k_1+N/2} - 1\right). \tag{3.4}$$

In paper [7] it has been found experimentally that the length changes during the process of torsion are very small and can be neglected. Assuming constant specimen length l_0, the relation for time $t(k)$ can be expressed in the form

$$t(k) = \frac{l_0}{c'} k_y + \frac{l_0}{c''}(k - k_y). \tag{3.5}$$

4. Comparison of the results of solution with the experiment

To check the correctness of the solution, the compression process in the system of the SHPB was analyzed on the basis of the derived relations (2.10), (2.15), (2.16), (2.22) and (2.25). Calculations were made for aluminium specimens with initial length-to-diameter ratios $s_0 = 0.1$, 0.2, 0.33, 0.5, 0.67 and 1.0. Such specimens have been tested in earlier experiment, which has been described in [5]. In the calculations the data corresponding exactly to the conditions of experiments were assumed, what made it possible to compare the calculation results to those obtained experimentally. For friction the previously estimated values of the dynamic friction coefficient μ were assumed. In Fig.6 the exeperimentally found dynamic hardening curve $\sigma(\varepsilon)$ for aluminium and the bilinear characteristics assumed in its basis are presented. Depending on the value of total specimen deformation, which was approximately 0.08, 0.11 or 0.14, three different bilinear characteristics were taken, each of them being optimized for the particular deformation range. Calculations were made on a computer by means of a simple program taking into account the changes of cross-sectional area of the specimen A_s and increase of the deformation.

In Fig. 7 the incident pulse σ_J, the transmitted pulse σ_T and the reflected pulse σ_R are shown. These pulses were recorded in experiment and calculated from the relations (2.10), (2.15), (2.16) and (2.22) for an aluminium specimen with dimensions ratio $s_0 = 0.1$. As it is shown in Fig.7, in the calculations it has been assumed that the incident pulse has constant value

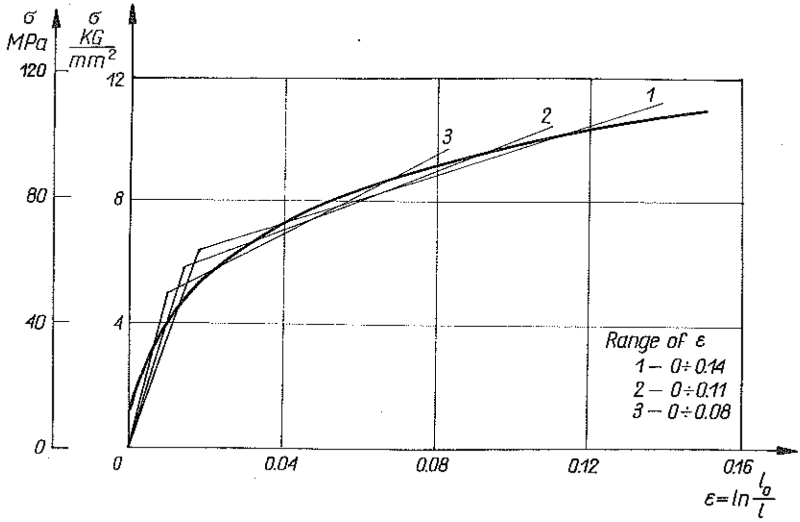

FIG. 6. Bilinear characteristics $\sigma(\varepsilon)$ assumed in calculations and the real hardening curve.

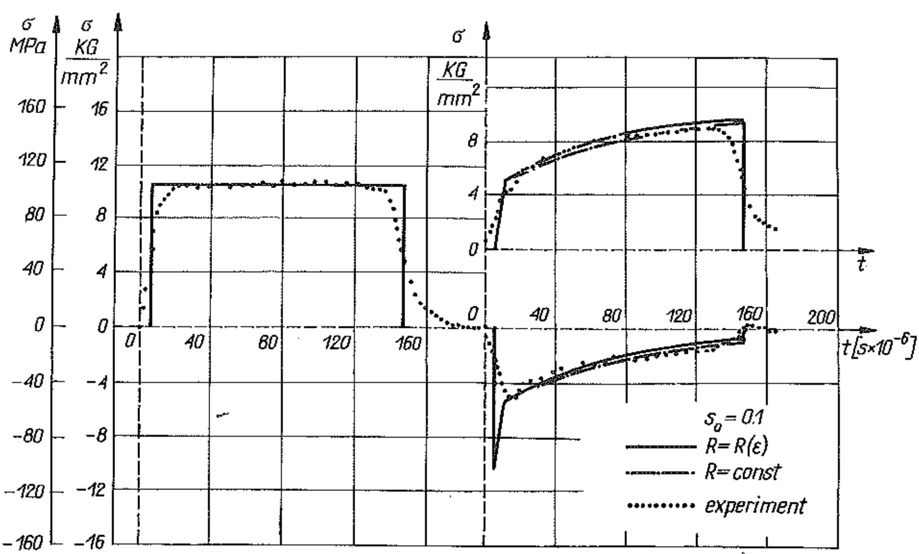

FIG. 7. Pulses $\sigma_J, \sigma_R, \sigma_T$ recorded in the experiment and calculated for a short specimen.

in the whole time range. The pulses σ_R and σ_T have been calculated for a constant value of $R = A_b/A_s$ and for R changing according to the change of the cross-sectional area of the specimen A_s. Difference of results for these two cases is not large. It is easy to notice that the pulses σ_R and σ_T obtained experimentally and calculated in the manner described above exhibit quite good agreement, except for the initial time period. The difference within this period results from the fact that real pulse σ_J of trapezoidal shape with a finite time of growth is replaced in the calculations by an averaged rectangular impulse with zero growth time.

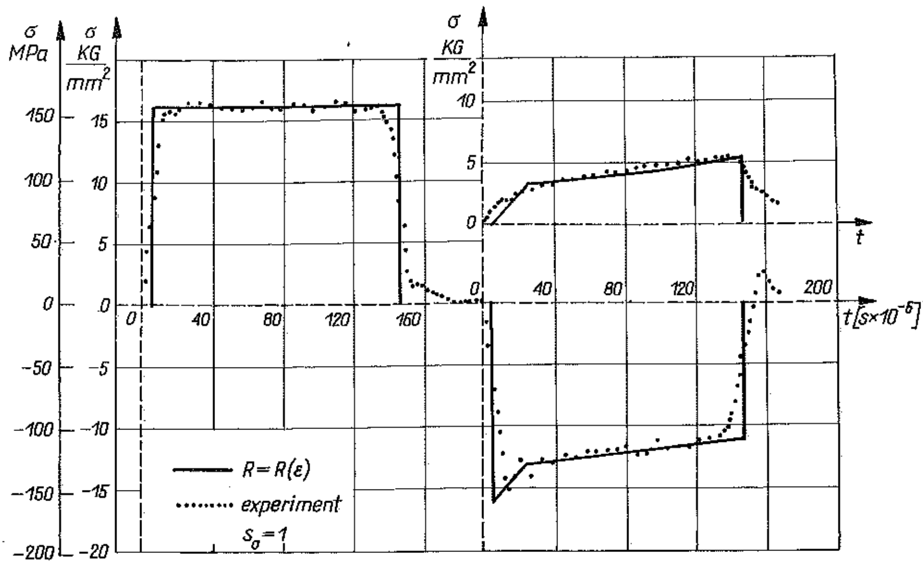

FIG. 8. Pulses $\sigma_J, \sigma_R, \sigma_T$ recorded in the experiment and calculated for a long specimen.

Comparison of results of calculatons with those of an experiment for a specimen of dimensions ratio $s_0 = 1$ is displayed in Fig.8. Also in this case we observe a good agreement of graph shapes for the reflected pulse σ_R and the transmitted one σ_T.

In Fig. 9 is shown the comparison of maximum deformations ε_{m0} calculated from Eq. (2.16) and found on the basis of a numerical solution by the method of characteristics, with the maximum deformation ε_{md} estimated experimentally using SHPB technique for specimens with various dimensions ratio s_0.

In order to verify the accuracy of the obtained solution, comparison of the relation $\sigma(\varepsilon)$ calculated from Eqs. (2.15) and (2.16) with the bilinear approximation of the hardening curve of the specimen material has been made. Such comparison is shown in Fig.10, where it is easy to see that the relation $\sigma(\varepsilon)$ calculated with the assumption of variability of the parameter $R[A_s(\varepsilon)]$ nearly ideally conform with the bilinear material characteristics assumed for calculations. However, under the condition of constant R, the

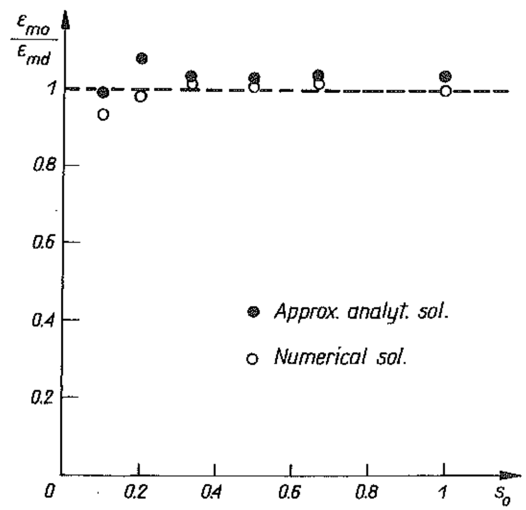

FIG. 9. Ratio of maximum deformations obtained from calculations and measured in experiment.

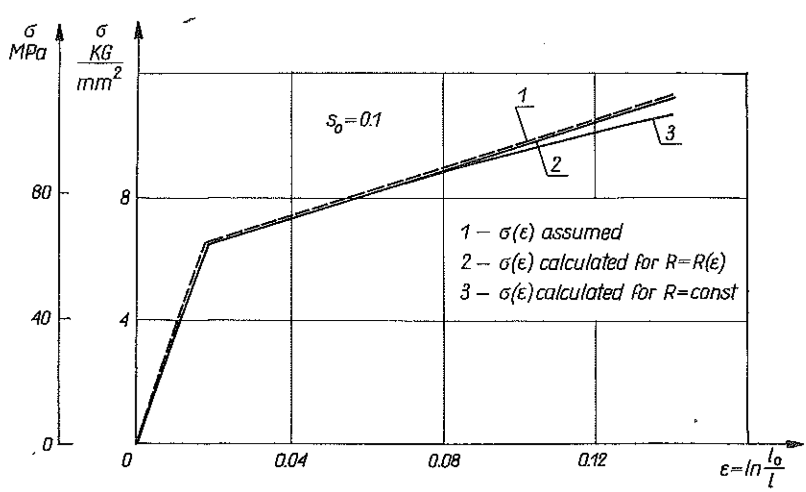

FIG. 10. Comparison of the assumed and calculated ration $\sigma(\varepsilon)$.

calculated relation $\sigma(\varepsilon)$ differs slightly for strain $\varepsilon > 0.1$ from the previously assumed bilinear approximation of the hardening curve. This indicates that more accurate results are obtained if variability of the parameter F together with deformation is taken into consideration.

5. Conclusions

Comparison of the results of calculations with the experimental data indicates that the solution presented in this paper with a good approximation describes the compression process of a specimen in the system of the SHPB. No experimental tests for torsion were made here, but it seems that the proposed approximate method of the solution should not be less accurate for the case of torsion than it is for compression. The simplifying assumptions yield the largest error in the description of the initial and the final stage of specimen deformation.

The simplified analysis presented in this paper can be useful in designing test stands with the SHPB as well as in adequate experiment preparation and proper interpretation of the obtained experimental results.

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Streszczenie

PRZYBLIŻONE, ANALITYCZNE ROZWIĄZANIE DLA PROCESU ŚCISKANIA I SKRĘCANIA W UKLADZIE ZMODYFIKOWANEGO PRĘTA HOPKINSONA

W pracy przedstawiono uproszczoną analizę procesu dynamicznego ściskania i skręcania w układzie zmodyfikowanego pręta Hopkinsoana. W rozwiązaniu przyjęto biliniową zależność naprężenia od odkształcenia $\sigma(\varepsilon)$ dla materialu próbki oraz stalą wartość w czasie impulsu inicjującego $\sigma(t)$ = const. W procesie ściskania uwzględniono efekt tarcia pomiędzy próbką i prętami. W wyniku analizy uzyskano możliwość określenia zależności od czasu impulsów odbitego i przenoszonego w prętach Hopkinsona oraz średniego naprężenia $\sigma(t)$ i odkształcenia $\varepsilon(t)$ w próbce. Korzystając z tych zależności przeprowadzono obliczenia, których wyniki porównano z wynikami badań doświadczalnych. Stwierdzono niewielkie różnice mi edzy wynikami obliczeń i uzyskanymi doświadczalnie.

Резюме

ПРИБЛИЖЕННОЕ, АНАЛИТИЧЕСКОЕ РЕШЕНИЕ ДЛЯ ПРОЦЕССА СЖАТИЯ И СКРУЧИВАНИЯ В СИСТЕМЕ МОДИФИЦИРОВАННОГО СТЕРЖНЯ ГОПКИНСОНА

В работе представлен упрощенный анализ процесса динамического сжатия и скручивания в системе модифицированного стержня Гопкинсона. В решении приняты билинейная зависимость напряжения от деформации $\sigma(\varepsilon)$ для материала образца и постоянное значение во времени инициирующего импульса $\sigma_J(t)$ = const. В процессе сжатия учтен эффект трения между образцом и стержнями. В результате анализа получена возможность определения зависимости от времени отраженного и проходящего импульсов в стержнях Гопкинсона, а также среднего напряжения $\sigma(t)$ и деформации $\varepsilon(t)$ в образце. Используя эти зависимости, проведены расчеты, результаты которых сравнены с результатами экспериментальных исследований. Констатировано небольшие разницы между результатами расчетов и полученными экспериментально.

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