ENGINEERING TRANSACTIONS • *Engng. Trans.* • **40**, 2, 267–282, 1992
Polish Academy of Sciences • Institute of Fundamental Technological Research

ROLE OF FLUID INERTIA IN POROUS MATERIAL SUBJECT TO DYNAMIC LOADINGS

S. J. KOWALSKI and G. MUSIELAK (POZNAŃ)

The paper is aimed at analysing the dynamical properties of fluid-filled porous media in which the inertial coupling effect (due to the complexity of the pore structure) is taken into consideration. Particular attention is given to the influence of the mass coupling coefficient $\rho_{12} = -\rho_a$ on the wave propagation velocities in fluid-filled porous medium, on the natural frequencies of fluid-filled porous cylinder, and on transmission of forces through such a cylinder, what is important for the determination of vibration-isolation properties of the medium.

1. Introduction

Any motion of fluid in a porous material is impeded even when the pore dimensions are relatively large. This is because of at least two reasons: first, the complexity of the pore structure makes the rectilinear motion of the fluid impossible and second, the fluid viscosity (adhesion) involves a resistance at the fluid-solid skeleton interface. The first reason represents some kind of inertial resistance of motion of the skeleton when it is accelerated, as for example by vibrations. The phenomenon is similar to that of a tank filled with fluid (we should consider it to be a large pore) and accelerated by a truck transporting it. During acceleration the fluid presses on the back wall of the tank and causes inertial effect which counteracts the motion of the truck. If the back wall of the tank could open during its accelerated motion, then the inertial resistance would fall down to zero.

A permeable porous medium with a complex pore structure, in which a free motion of the pore fluid with respect to the skeleton is impossible, is considered in the paper. In such a medium a part of the pore fluid participates in the inertial effect, and the rest of it is free and can move with respect to the skeleton.

Division of the pore fluid into free and non-free (constrained) was proposed by DERSKI [2]. KUBIK [7] introduced a concept of "structural permeability" which made it possible to describe the fluid division mentioned above by structural parameters. KOWALSKI [3], using a two-parameter characteristic of the pore sctructure (the volume porosity and the structural permeability), derived a set of two coupled equations of motion for fluid-filled porous medium. These equations contain the so-called coupling effect through masses in the constituent motions. Their linear forms are equivalent to those given by BIOT [1].

The method of mass and momentum balance, used by KOWALSKI [3] to derive the equations mentioned above enables an evident physical interpretation of the coefficients appearing in these equations. For instance, the coefficient of mass coupling between fluid and solid ρ_{12} can be considered as responsible for the inertial resistance of pore fluid in fluid-filled porous medium during its non-stationary motion. Internal interactions like the dynamic coupling effect and interaction forces due to mass exchange in porous medium were also discussed in [8].

The aim of this paper is an analysis of dynamical properties of fluid-filled porous medium in which the inertial effect is taken into consideration. Particular attention is given to the influence of the mass coupling coefficient $\rho_{12} = -\rho^a$ on the wave propagation velocities in fluid-filled porous medium, on the natural frequencies of fluid-filled porous cylinder, and on transmission of forces through such a cylinder, what is important in case of the determination of vibration-isolation properties of the medium. All these relations are illustrated by graphs.

2. THE FUNDAMENTAL EQUATIONS

We shall illustrate the role of inertial effects of the fluid moving in a porous medium by considering one-dimensional problem. We simplify the problem and assume that the viscous resistivity existing on the porous solid-fluid interface is negligible. The Biot equations [1], with the interpretation of mass coupling coefficient given in [3], are used for description of the motion of a fluid-filled porous medium. These equations are then referred to a porous medium with elastic both skeleton and fluid.

Let x_1 denotes the space coordinate, t time, and u_1^s and u_1^f denote the skeleton and the fluid displacements in x_1 – direction, respectively. The equations of motion have then the form [1]:

$$
(2.1)\qquad
\begin{aligned}
(2N+A)\frac{\partial^2 u_1^s}{\partial x_1^2} + Q\frac{\partial^2 u_1^f}{\partial x_1^2} &= \rho^s \frac{\partial^2 u_1^s}{\partial t^2} + \rho^a \frac{\partial^2}{\partial t^2}(u_1^s - u_1^f),\\
Q\frac{\partial^2 u_1^s}{\partial x_1^2} + R\frac{\partial^2 u_1^f}{\partial x_1^2} &= \rho^f \frac{\partial^2 u_1^f}{\partial t^2} - \rho^a \frac{\partial^2}{\partial t^2}(u_1^s - u_1^f),
\end{aligned}
$$

where N, A, Q, R are the well-known material constants of the theory of porous media, ρ^s and ρ^f are partial densities of the porous skeleton and the pore fluid, respectively, and ρ^a is the mass coupling coefficient (see [1]). In paper [3] ρ^a is just interpreted as the coefficient representing inertial effects in relative motion of the porous skeleton and the fluid. In quantitative formulation this coefficient equals

$$
(2.2)\qquad \rho^a = \rho^f \frac{\rho^c}{\rho^e},
$$

where $\rho^f = \rho_r^f f_v$ is the partial density of the fluid as a whole, $\rho^e = \rho_r^f f_e$ is the free fluid partial density, $\rho^c = \rho_r^f(f_v - f_e)$ is the partial density of the non-free (constrained by pore structure) fluid. Here ρ_r^f denotes the real fluid density, f_v – the volume porosity ratio and f_e – the structural permeability ratio (see [7]). The coefficient ρ^a characterizes both the inertial property of the pore fluid by density ρ^f and the amount of the constrained fluid expressed by the ratio

$$
\frac{\rho^c}{\rho^e} = \frac{f_v}{f_e} - 1 .
$$

The equations (2.1) with $\rho^a = 0$ were used in paper [4] for analysing the vibration-isolation properties of deformable fluid-filled porous media. Here, we take into account both the dilatational coupling characterized by coefficient Q and the mass coupling determined by coefficient ρ^a. Thus, the set of equation of motion is here doubly coupled.

We solve the problem using dimensionless coordinates x, τ

$$
(2.3)\qquad x = \frac{x_1}{l}, \qquad \tau = t\frac{\sqrt{2N+A}}{l\sqrt{\rho^s}}
$$

and the dimensionless displacements

$$
(2.4)\qquad u_s = \frac{u_1^s}{l}, \qquad u_f = \frac{u_1^f}{l},
$$

where l is a characteristic length.

The set of equations (2.1) in the dimensionless coordinates is expressed as follows:

$$
\begin{aligned}
\frac{\partial^2 u_s}{\partial x^2} + a_1 \frac{\partial^2 u_f}{\partial x^2} &= \frac{\partial^2 u_s}{\partial \tau^2} + \gamma_1 \frac{\partial^2}{\partial \tau^2}(u_s - u_f), \\
a_2 \frac{\partial^2 u_s}{\partial x^2} + \frac{\partial^2 u_f}{\partial x^2} &= \gamma \frac{\partial^2 u_f}{\partial \tau^2} - \gamma_1 \gamma_2 \frac{\partial^2}{\partial \tau^2}(u_s - u_f),
\end{aligned}
\tag{2.5}
$$

where

$$
\begin{aligned}
a_1 &= \frac{Q}{2N + A}, & a_2 &= \frac{Q}{R}, & \gamma &= \gamma_\rho \gamma_2, \\
\gamma_1 &= \gamma_\rho \frac{\rho^c}{\rho^e}, & \gamma_2 &= \frac{2N + A}{R}, & \gamma_\rho &= \frac{\rho^f}{\rho^s}.
\end{aligned}
\tag{2.6}
$$

The constitutive relations for one-dimensional problems are

$$
\begin{aligned}
\sigma_s &= \frac{\partial u_s}{\partial x} + a_1 \frac{\partial u_f}{\partial x}, \\
\sigma_f &= \left(a_2 \frac{\partial u_s}{\partial 2} + \frac{\partial u_f}{\partial x} \right) / \gamma_2 .
\end{aligned}
\tag{2.7}
$$

They will be used to formulate the boundary conditions. The dimensionless stresses in the skeleton σ_s and in the fluid σ_f are related to σ_1^s and σ_1^f by

$$
\sigma_s = \frac{\sigma_1^s}{2N + A}, \qquad \sigma_f = \frac{\sigma_1^f}{2N + A}. \tag{2.8}
$$

3. General solution of the problem

The solution of the problem is searched in a form of series expanded with respect to eigenfunctions. Applying the method of separation of variables, i.e. writing displacements in the form

$$
\begin{aligned}
u_s(x, \tau) &= U_s(x) T(\tau), \\
u_f(x, \tau) &= U_f(x) T(\tau)
\end{aligned}
\tag{3.1}
$$

and substituting them into (2.5), we obtain

$$
\begin{aligned}
U_s'' + a_1 U_f'' + \omega^2 [U_s + \gamma_1 (U_s - U_f)] &= 0, \\
a_2 U_s'' + U_f'' + \omega^2 [\gamma U_f - \gamma_1 \gamma_2 (U_s - U_f)] &= 0, \\
\ddot{T} + \omega^2 T &= 0.
\end{aligned}
\tag{3.2}
$$

In the above equations ω^2 is the constant of separation of variables, and comma and dot over a symbol denote differentiation with respect to the space and time coordinates, respectively.

We assume exponential form of the eigenfunctions, i.e.

$$U_s(x) = A_s \exp(rx) , \qquad U_f(x) = A_f \exp(rx) . \tag{3.3}$$

After substituting these functions into equations $(3.2)_{1,2}$ we obtain a fourth-order equation. The solution of it gives us the general shape of the eigenfunctions

$$\begin{aligned} U_s(x) &= A_{s1} \exp\left(i\frac{\omega}{c_s^*}x\right) + A_{s2} \exp\left(-i\frac{\omega}{c_s^*}x\right) \\ &\quad + A_{s3} \exp\left(i\frac{\omega}{c_w^*}x\right) + A_{s4} \exp\left(-i\frac{\omega}{c_w^*}x\right) , \\ U_f(x) &= A_{f1} \exp\left(i\frac{\omega}{c_s^*}x\right) + A_{f2} \exp\left(-i\frac{\omega}{c_s^*}x\right) \\ &\quad + A_{f3} \exp\left(i\frac{\omega}{c_w^*}x\right) + A_{f4} \exp\left(-i\frac{\omega}{c_w^*}x\right) , \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} c_s^* &= \sqrt{\frac{\gamma_3 + \sqrt{\gamma_3^2 - 4(1 - a_1 a_2)[\gamma + \gamma_1(\gamma + \gamma_2)]}}{2[\gamma + \gamma_1(\gamma + \gamma_2)]}} , \\ c_w^* &= \sqrt{\frac{\gamma_3 - \sqrt{\gamma_3^2 - 4(1 - a_1 a_2)[\gamma + \gamma_1(\gamma + \gamma_2)]}}{2[\gamma + \gamma_1(\gamma + \gamma_2)]}} \end{aligned} \tag{3.5}$$

are the dimensionless velocities of longitudinal fast and slow waves, respectively, where the notation is introduced

$$\gamma_3 = 1 + \gamma + \gamma_1[1 + a_2 + \gamma_2(1 + a_1)] .$$

Assuming the mass coupling coefficient γ_1 to be zero in relations (3.5), we obtain dimensionless velocities of fast and slow waves used in [4,5,6]:

$$c_s^* = c_s/a_s , \qquad c_w^* = c_w/a_s$$

where

$$a_s = \sqrt{\frac{2N + A}{\rho^s}}$$

denotes the wave velocity in skeleton. Assuming additionally the dilatational coupling coefficient Q to be equal to zero, c_s^* and c_w^* will express the uncoupled dimensionless wave velocities in the porous skeleton and fluid, respectively. Generally, however, the wave velocities depend on the coupling coefficients ρ^a and Q.

Functions (3.4) have to fulfill equations (3.2) and this implies the following relations between constants:

$$A_{f1} = \delta_1 A_{s1}, \qquad A_{f2} = \delta_1 A_{s2},$$
(3.6)
$$A_{f3} = \delta_2 A_{s3}, \qquad A_{f4} = \delta_2 A_{s4},$$

where

$$\delta_1 = -\frac{1-(1+\gamma_1)c_s^{*2}}{a_1+\gamma_1 c_s^{*2}} = -\frac{a_1+\gamma_1\gamma_2 c_s^{*2}}{1-(\gamma+\gamma_1\gamma_2)c_s^{*2}},$$
(3.7)
$$\delta_2 = -\frac{1-(1+\gamma_1)c_w^{*2}}{a_1+\gamma_1 c_w^{*2}} = -\frac{a_1+\gamma_1\gamma_2 c_w^{*2}}{1-(\gamma+\gamma_1\gamma_2)c_w^{*2}}.$$

The generalized coordinate, which results from the solution of differential equation $(3.2)_3$, takes the form

(3.8)
$$T(\tau) = C\sin\omega\tau + D\cos\omega\tau.$$

Existence of two longitudinal waves in fluid-saturated porous solid involves two sets of real-valued natural frequencies in the case of standing waves

(3.9)
$$\left\{\omega_n^{(1)}\right\} \quad \text{and} \quad \left\{\omega_n^{(2)}\right\}; \qquad n = 1, 2, 3, \ldots$$

The first set is related to the fast wave velocity and the second one to the slow wave velocity. Consequently, there are two sets of eigenfunctions and generalized coordinates and the general solution of initial-boundary value problems takes the form

$$u_s(x,\tau) = \sum_{n=1}^{\infty}\left[U_{sn}^{(1)}(x)T_n^{(1)}(\tau) + U_{sn}^{(2)}(x)T_n^{(2)}(\tau)\right],$$
(3.10)
$$u_f(x,\tau) = \sum_{n=1}^{\infty}\left[U_{fn}^{(1)}(x)T_n^{(1)}(\tau) + U_{fn}^{(2)}(x)T_n^{(2)}(\tau)\right].$$

The orthogonality relations for these eigenfunctions are

$$
\begin{aligned}
(3.11)\qquad \left[\left(\omega_n^{(i)}\right)^2-\left(\omega_m)^{(k)}\right)^2\right]\int_0^1 \Big\{ & \gamma_2(1+\gamma_1)U_{sn}^{(i)}(x)U_{sm}^{(k)}(x) \\
& +(\gamma+\gamma_1\gamma_2)U_{fn}^{(i)}(x)U_{fm}^{(k)}(x) \\
& -\gamma_1\gamma_2\left[U_{fn}^{(i)}(x)U_{sm}^{(k)}(x)+U_{sn}^{(i)}(x)U_{fm}^{(k)}(x)\right]\Big\}\,dx \\
& +\gamma_2\Big\{\left[U_{sn}^{(i)\prime}(x)+a_1U_{fn}^{(i)\prime}(x)\right]U_{sm}^{(k)}(x) \\
& -\left[U_{sm}^{(k)\prime}(x)+a_1U_{fm}^{(k)\prime}(x)\right]U_{sn}^{(i)}(x)\Big\}\Big|_0^1 \\
& +\Big\{\left[a_2U_{sn}^{(i)\prime}(x)+U_{fn}^{(k)\prime}(x)\right]U_{fm}^{(k)}(x) \\
& -\left[a_2U_{sm}^{(k)\prime}(x)+U_{fm}^{(k)\prime}(x)\right]U_{fn}^{(i)}(x)\Big\}\Big|_0^1=0
\end{aligned}
$$

for $i,k=1,2; n,m=1,2,3,\ldots$. Prime over the symbol denotes the ordinary derivative.

On the basis of the general solution quoted above it is seen that the mass coupling coefficient, taken into account in the equations of motion, influences the wave propagation velocities, the eigenfunctions, the generalized coordinates (in a quantitative sense) and the orthogonality relations for the eigenfunctions. The mathematical form of eigenfunctions and of generalized coordinates is the same as that for which $\rho^a=0$ (see [4]). For this reason we do not report here the detailed solution procedure of the initial-boundary value problem at hand. This procedure was presented, for instance, in paper [4].

4. Influence of fluid inertia on vibrations of fluid-filled porous cylinder

Consider now free and forced vibrations of a fluid-filled porous cylinder and analyse the influence of fluid inertia on the amplitude of free vibrations of the cylinder and on the force transmission coefficient in the case of forced vibrations.

Free vibrations

Let us assume that mass M_s (e.g. mass of a technical device) is resting on the porous cylinder (skeleton), and mass M_f is resting on the pore fluid for stimulation of the pore pressure (see Fig.1).

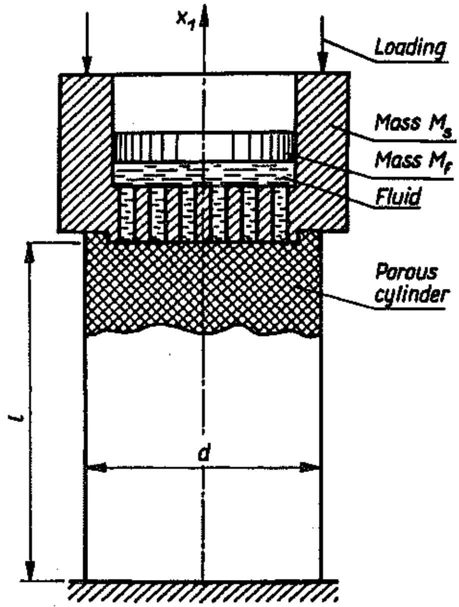

FIG. 1. Model of vibroisolator.

The boundary conditions for such a loaded cylinder, expressed in dimensionless coordinates, are following:

$$
\begin{aligned}
\sigma_s(1,\tau) &= \frac{\partial u_s(1,\tau)}{\partial x} + a_1 \frac{\partial u_f(1,\tau)}{\partial x} = -m_s \frac{\partial^2 u_s(1,\tau)}{\partial t^2}\,, \\
\sigma_f(1,\tau) &= \frac{1}{\gamma_2}\left(a_2 \frac{\partial u_s(1,\tau)}{\partial x} + \frac{\partial u_f(1,\tau)}{\partial x}\right) = -m_f \frac{\partial^2 u_f(1,\tau)}{\partial t^2}\,, \qquad (4.1)\\
u_s(0,\tau) &= 0, \qquad u_f(0,\tau) = 0
\end{aligned}
$$

where

$$
m_s = \frac{M_s}{A_0 l \rho_s}\,, \qquad m_f = \frac{M_f}{A_0 l \rho_s}
$$

are the ratios of masses M_s and M_f related to the mass of the skeleton $A_0 l \rho_s$, where A_0 is the cylinder cross-sectional area and l is the cylinder height.

Separation of variables in boundary conditions (4.1) and substitution of the functions (3.3), including (3.6) and (3.7), yields the characteristic equation of the form

$$(4.2)\qquad (\delta_2-\delta_1)\sqrt{\frac{1-a_1a_2}{\gamma_4}}(\gamma_4\cos\alpha\cos\beta+\gamma_2 m_s m_f\alpha\beta\sin\alpha\sin\beta) \\ +[(a_2+\delta_1)m_s-(1+a_1\delta_1)\gamma_2\delta_2 m_f]\alpha\cos\alpha\sin\beta \\ +[(1+a_1\delta_2)m_f\gamma_2\delta_1-(a_2+\delta_2)m_s]\beta\sin\alpha\cos\beta=0$$

with $\alpha=\omega/c_s^*$ and $\beta=\omega/c_w^*$. This equation allows us to determine the natural frequencies ω.

We specify the orthogonality relations (3.11) for the boundary conditions (4.1) and obtain

$$(4.3)\qquad \int_0^1\Big\{(1+\gamma_1)U_{sn}^{(i)}(x)U_{sm}^{(k)}(x)+(\gamma_\rho+\gamma_1)U_{fn}^{(i)}(x)U_{fm}^{(k)}(x) \\ -\gamma_1\left[U_{fn}^{(i)}(x)U_{sm}^{(k)}(x)+U_{sn}^{(i)}(x)U_{fm}^{(k)}(x)\right]\Big\}\,dx \\ +m_sU_{sn}^{(i)}(1)U_{sm}^{(k)}(1)+m_fU_{fn}^{(i)}(1)U_{fm}^{(k)}(1) \\ =\begin{cases}0 & \text{for}\quad m\neq n\quad\text{or}\quad i\neq k,\\ M_n^{(i)} & \text{for}\quad m=n\quad\text{and}\quad i=k\end{cases}$$

where

$$(4.4)\qquad M_n^{(i)}=\int_0^1\Big\{(1+\gamma_1)\left[U_{sn}^{(i)}(x)\right]^2+(\gamma_\rho+\gamma_1)\left[U_{fn}^{(i)}(x)\right]^2 \\ -2\gamma_1U_{sn}^{(i)}(x)U_{fn}^{(i)}(x)\Big\}\,dx+m_s\left[U_{sn}^{(i)}(1)\right]^2+m_f\left[U_{fn}^{(i)}(1)\right]^2.$$

The eigenfunctions (3.4) for boundary conditions (4.1) are described by trigonometric functions,

$$(4.5)\qquad \begin{aligned} U_{sn}^{(i)} &= A_n^{(i)}\left[a_n^{(i)}\sin\frac{\omega_n^{(i)}}{c_s^*}x-b_n^{(i)}\sin\frac{\omega_n^{(i)}}{c_w^*}x\right],\\ U_{fn}^{(i)} &= A_n^{(i)}\left[\delta_1a_n^{(i)}\sin\frac{\omega_n^{(i)}}{c_s^*}x-\delta_2b_n^{(i)}\sin\frac{\omega_n^{(i)}}{c_w^*}x\right]\end{aligned}$$

where

$$
(4.6)\qquad
\begin{aligned}
a_n^{(i)} &= (1+a_1\delta_2)\beta_n^{(i)}\cos\beta_n^{(i)} - m_s\omega_n^{(i)}\sin\beta_n^{(i)}\,,\\
b_n^{(i)} &= (1+a_1\delta_1)\alpha_n^{(i)}\cos\alpha_n^{(i)} - m_s\omega_n^{(i)}\sin\alpha_n^{(i)}\,.
\end{aligned}
$$

Constants $A_n^{(i)}$ can be assumed to be equal to unity since, finally, they will appear in products with generalized coordinate constants (Eq.(3.8)) leading to other constants $C_n^{(i)}$ and $D_n^{(i)}$, i.e.

$$
(4.7)\qquad T_n^{(i)}(\tau) = C_n^{(i)}\sin\omega_n^{(i)}\tau + D_n^{(i)}\cos\omega_n^{(i)}\tau\,.
$$

The constants $C_n^{(i)}$ and $D_n^{(i)}$ are to be determined by means of the initial conditions. For the initial conditions written below

$$
(4.8)\qquad
\begin{aligned}
u_s(x,0) &= u_{s0}(x), & u_f(x,0) &= u_{f0}(x),\\
\frac{\partial u_s(x,0)}{\partial\tau} &= v_{s0}(x), & \frac{\partial u_f(x,0)}{\partial\tau} &= v_{f0}(x),
\end{aligned}
$$

these constants are

$$
(4.9)\qquad C_n^{(i)} = \frac{G_n^{(i)}}{\omega_n^{(i)}M_n^{(i)}}\,, \qquad D_n^{(i)} = \frac{H_n^{(i)}}{M_n^{(i)}}\,,
$$

where

$$
(4.10)\qquad
\begin{aligned}
G_n^{(i)} &= \int\limits_0^1 \Big\{(1+\gamma_1)v_{s0}(x)U_{sn}^{(i)}(x) + (\gamma_\rho+\gamma_1)v_{f0}(x)U_{fn}^{(i)}(x)\\
&\qquad -\gamma_1\Big[v_{f0}(x)U_{sn}^{(i)}(x) + v_{s0}(x)U_{fn}^{(i)}(x)\Big]\Big\}\,dx\\
&\qquad +m_s v_{s0}(1)U_{sn}^{(i)}(1) + m_f v_{f0}(1)U_{fn}^{(i)}(1)\,,\\
H_n^{(i)} &= \int\limits_0^1 \Big\{(1+\gamma_1)u_{s0}(x)U_{sn}^{(i)}(x) + (\gamma_\rho+\gamma_1)u_{f0}(x)U_{fn}^{(i)}(x)\\
&\qquad -\gamma_1\Big[u_{f0}(x)U_{sn}^{(i)}(x) + u_{s0}(x)U_{fn}^{(i)}(x)\Big]\Big\}\,dx\\
&\qquad +m_s u_{s0}(1)U_{sn}^{(i)}(1) + m_f u_{f0}(1)U_{fn}^{(i)}(1)\,.
\end{aligned}
$$

The eigenfunctions (4.5) and the generalized coordinates (4.7) with the constants defined by (4.6), (4.9) and (4.10) give the solution of the initial-boundary value problem.

Forced vibrations

The forced vibrations of fluid-filled porous cylinder subject to a harmonic loading (Fig.1) are analysed here to study the influence of fluid inertia on the vibration-isolation properties of the porous materials. The force is applied to the mass M_s.

The boundary conditions differ slightly from those of (4.1). The difference appears in the first of them, which is now

$$\sigma_s(1,\tau) = \frac{\partial u_s(1,\tau)}{\partial x} + a_1 \frac{\partial u_f(1,\tau)}{\partial x} = -m_s \frac{\partial^2 u_s(1,\tau)}{\partial \tau^2} - f_0 \sin p\tau \,, \tag{4.11}$$

where f_0 denotes the amplitude of the loading force and p is the forced frequency. The initial conditions for this problem are assumed to be homogeneous, i.e.

$$u_s(x,0) = u_f(x,0) = 0, \qquad \frac{\partial u_s(x,0)}{\partial \tau} = \frac{\partial u_f(x,0)}{\partial \tau} = 0. \tag{4.12}$$

The procedure of solving such a problem has been given in details in previous authors' papers, for instance in [4]. This is the reason why we shall present here only the final solution.

Our task here is to determine how the mass coupling coefficient ρ^a influences the vibration-isolation properties of fluid-filled porous media. We will answer this question by analysing the force transmission coefficient μ. This coefficient expresses the ratio of the force acting on the cylinder to the loading force (Fig.1):

$$\mu = \frac{P(0,\tau)}{f_0 \sin p\tau} \,. \tag{4.13}$$

The dimensionless axial force in the cylinder $P(x,\tau)$ is equal to

$$P(x,\tau) = \sigma_s(x,\tau) + \sigma_f(x,\tau) \,. \tag{4.14}$$

The final form of the force transmission coefficient μ is then as follows:

$$\begin{aligned} \mu = {} & (1+a_1)\left[\kappa_s + p^2 \sum_{i=1}^{2}\sum_{n=1}^{\infty} \frac{\omega_n^{(i)} L_n^{(i)}}{(\omega_n^{(i)})^2 - p^2}\left(\frac{a_n^{(i)}}{c_s^*} - \frac{b_n^{(i)}}{c_w^*}\right)\right] \\ & + \frac{(1+a_2)}{\gamma_2}\left[\kappa_f + p^2 \sum_{i=1}^{2}\sum_{n=1}^{\infty} \frac{\omega_n^{(i)} L_n^{(i)}}{(\omega_n^{(i)})^2 - p^2}\left(\delta_1 \frac{a_n^{(i)}}{c_s^*} - \delta_2 \frac{b_n^{(i)}}{c_w^*}\right)\right] \end{aligned} \tag{4.15}$$

where

$$
\begin{aligned}
\kappa_s &= \frac{(1-m_f p^2)}{(1-m_s p^2)(1-m_f p^2)-a_1 a_2}\,, \\
\kappa_f &= \frac{a_2}{(1-m_s p^2)(1-m_f p^2)-a_1 a_2}\,, \qquad (4.16)\\
L_n^{(i)} &= \frac{1}{M_n(i)}\left\{\int_0^1 [G_s x U_{sn}^{(i)}(x) + G_f x U_{fn}^{(i)}(x)]dx \right.\\
&\qquad \left. + m_s\kappa_s U_{sn}^{(i)}(1 + m_f\kappa_f U_{fn}^{(i)}(1)\right\}\,.
\end{aligned}
$$

and

$$
G_s = (1+\gamma_1)\kappa_s - \gamma_1\kappa_f\,, \qquad G_f = (\gamma_\rho + \gamma_1)\kappa_f - \gamma_1\kappa_s \qquad (4.17)
$$

are the notations used.

5. Analysis of results

The main aim of this work is the analysis of influence of the mass coupling coefficient ρ^a on dynamical properties of the fluid-filled porous medium, i.e. on the wave velocities, on the natural frequencies of a fluid-filled porous cylinder, and on the force transmission coefficient (in the case of forced vibrations).

The solution of the problem was given in the form of a series of eigenfunctions. The first thirty terms of the series were taken into account in numerical calculations (greater number of those terms have not influenced the final values).

The values of material constants used in numerical calculations are similar to those used in [4,5,6], i.e.

$$
\begin{aligned}
a_1 &= 0.02, & a_2 &= 1.0, & \gamma_\rho &= 0.1,\\
m_s &= 100.0, & m_f &= 0.5
\end{aligned} \qquad (5.1)
$$

and the coefficient γ_1 responsible for the inertia of the pore fluid was arbitrarily selected from the range

$$
\gamma_1 = 0 \div 1\,, \qquad (5.2)
$$

where $\gamma_1 = 0$ corresponds to the case of uncoupled constituents motions via mass cross-couplings, and $\gamma_1 = 1$ is a theoretically admissible value of this coefficient.

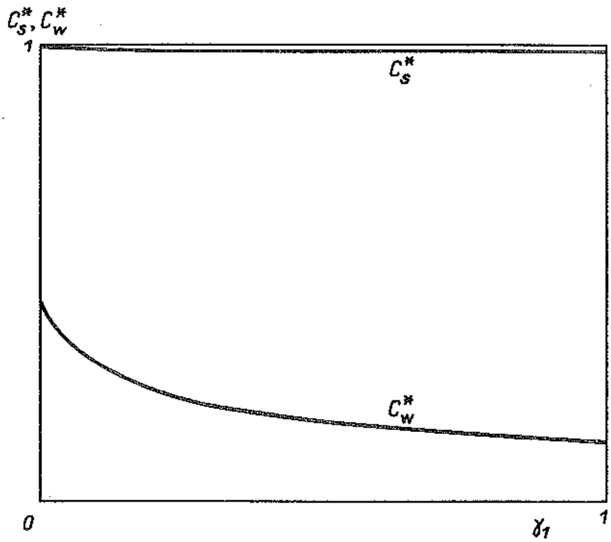

FIG. 2. Velocities of dilatational waves versus parameter γ_1.

The dimensionless wave velocities (fast c_s^* and slow c_w^*) versus parameter γ_1 are shown in Figure 2 for the material constants (5.1).

On the basis of this figure it may be observed that both the fast c_s^* and the slow c_w^* wave velocities decrease with the increase of the mass coupling

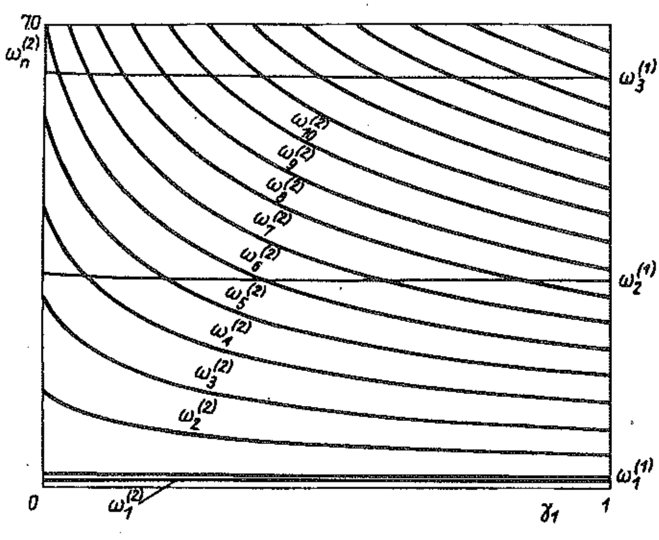

FIG. 3. Natural frequencies of vibroisolator model versus parameter γ_1.

coefficient. The influence of this coefficient is rather small in the case of the fast wave velocity and significant in the case of the slow one. This influence is clearly reflected in the natural frequencies for the porous cylinder analysed above (Fig.3).

An increase of coefficient γ_1 causes a decrease of the natural frequencies. The decrease is insignificant for the natural frequencies corresponding to the fast wave, but is considerable for those corresponding to the slow wave.

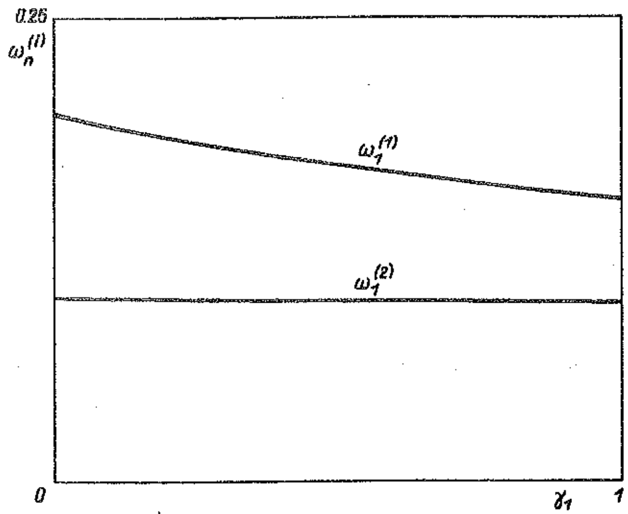

FIG. 4. The first two natural frequencies of vibroisolator model versus parameter γ_1.

An exception of this rule is represented by the first frequencies from both sets of the natural frequencies (Fig.4), where the first natural frequency of fast vibrations decreases more than the first one of the slow vibrations. We can observe this effect in the force transmission coefficient μ versus $p/\omega_1^{(z)}$ which is shown for some values of γ_1 in Fig.5.

On the basis of this figure and Fig.4 it can be observed that an increase of inertia of the pore fluid, i.e. an increase of γ_1, reduces the practically very important distance between the first and the second resonance of the fluid-filled porous cylinder. It means that the vibration-isolation properties of the cylinder are reduced in the range between two resonances and, in some cases, for material constants other than (5.1), these properties vanish.

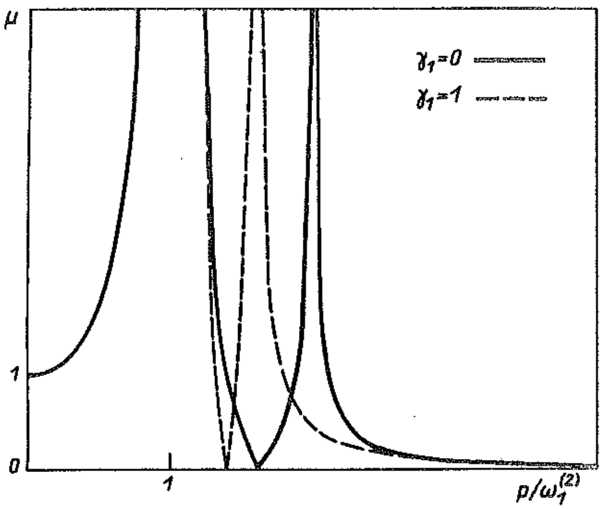

FIG. 5. The force transmission coefficient μ for different values of parameter γ_f.

Summing up we can say that the inertia of pore fluid coupled mechanically (through complex pore structure) with the skeleton influences the velocities of waves propagating through fluid-filled porous medium. In the one-dimensional problem analysed here, there are two longitudinal waves. Both these waves decrease with an increase of γ_1, however, the slow wave velocity decreases more than the fast one. In the extreme case of closed pores, what means that the whole fluid is moving together with the skeleton, the coefficient γ_1 tends to infinity and the slow wave disappears ($c_w^* \to 0$). The only one longitudinal wave propagates then through the medium and has the velocity $c_s = \sqrt{(2N + A + R + 2Q)/(\rho^s + \rho^f)}/a_s$. The fluid-filled porous medium becomes then a composite with one kinematics, and therefore more "rigid". It explains thus the reduction of vibration-isolation effect between the first and the second natural frequencies of the cylinder with the growth of γ_1. If the difference between the natural frequencies $\omega_1^{(1)}$ and $\omega_1^{(2)}$ is maximum, then the phase distance between impulses (loadings) transported by the fast and slow waves also reaches a maximum. The large phase distance between the impulses is the necessary condition of a good vibration-isolation effect in fluid-filled porous medium.

As the mass coupling phenomena reduce the vibration-isolation effect, we should choose for construction of vibroisolators a porous material with minimal value of parameter γ_1.

References

1. M.A.Biot, *Theory of propagation of elastic waves in a fluid-saturated porous solid, I. Low-frequency range*, J. Acoust. Soc. Amer., **28**, 2, 1956.
2. W.Derski, *Equation of motion for a fluid-saturated porous solid*, Bull. Acad. Pol. Sci., serie Sci. Tech., **26**, 1, 11-16, 1978.
3. S.J.Kowalski, *On motion of fluid-saturated porous solid*, Transport in Porous Media 1991 (in press).
4. S.J.Kowalski, R.Kuc, G.Musielak, *Analysis of vibration-isolation properties of fluid-filled perforated media*, Engng. Trans., **36**, 3, 681-706, 1988 [in Polish].
5. S.J.Kowalski, R.Kuc, G.Musielak, *Analysis of damping properties of fluid-filled porous materials* [in Polish], Engng. Trans., **37**, 4, 651-673, 1989.
6. S.J.Kowalski. T.Świderski, *An influence of boundary conditions on damping properties of fluid-saturated porous materials*, Engng. Trans., **39**, 2, 1989.
7. J.Kubik, *A macroscopic description of geometrical pore structure of porous solids*, Int. J. Engng. Sci., **24**, 6, 971-980, 1986.
8. J.Kubik, M. Cieszko, *On internal forces in a porous, liquid-saturated medium* [in Polish], Engng. Trans., **35**, 1, 55-70, 1987.

DEPARTMENT OF MECHANICS OF POROUS MEDIA
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received December 12, 1991.