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DESCRIPTION OF CREEP OF THE LIGNOSTONE IN BIAXIAL STRESS STATES

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The results of creep investigations of the beech-wood lignostone specimens cut along the grains are presented. In the two-dimensional stress state the shearing stress is kept constant while the tensile stress varies. The description of creep of the lignostone as a nonlinear and orthotropic material is proposed by means of the equations

$$\varepsilon_{ij}(t) = \sigma_{kl}\left[a_{ijkl} + \int_0^t k_{ijkl}(\tau)\,d\tau\right] + \sigma_{kl}\sigma_{mn}\left[a_{ijklmn} + \int_0^t k_{ijklmn}(\tau)\,d\tau\right] + \ldots .$$

This description has been found to be satisfactory.

1. Introduction

The aim of the paper is to formulate a description of creep of a nonlinear orthotropic body in the state of plane stress in which $\sigma_{12} = \text{const}$ and σ_{11} varies.

2. Experiment and its results

2.1. *Results of creep tests on tension and torsion*

Specimens to test the creep were made of beech-wood lignostone with the density $\varrho = 990\,\text{kg/m}^3$ and the compression ratio 1.45. They were prepared in the Plywood Industries Factory in Białystok. The manufacturing conditions are given in [1]. The hollow cylinders had the outer diameter $d_0 = 19.2\,\text{mm}$, the inner diameter $d_z = 16.0\,\text{mm}$ and the measuring length $l_0 = 70\,\text{mm}$. The specimen axis coincided with the grain directions. The creep tests were made under the relative air humidity

$(65 \pm 2.5)\%$ and the temperature (293 ± 3) K, applying the shearing stress $\sigma_{12} = 0.6R_{12}$ ($R_{12} = 15.56$ MPa is a technical shearing strength) and the shearing-to-tensile stress ratios $\lambda = \sigma_{12}/\sigma_{11} = \infty$; 1; 0.25; 0.167.

The results of measurements of the shearing strains $2\bar{\varepsilon}_{12}$, obtained via the angle of twist, where $\tilde{\varepsilon}_{12}$ is the average value of the non-dilatational strains ε_{12} and ε_{13} in an orthotropic material, are shown in Fig. 1 while the elongations ε_{11} are shown in Fig. 2.

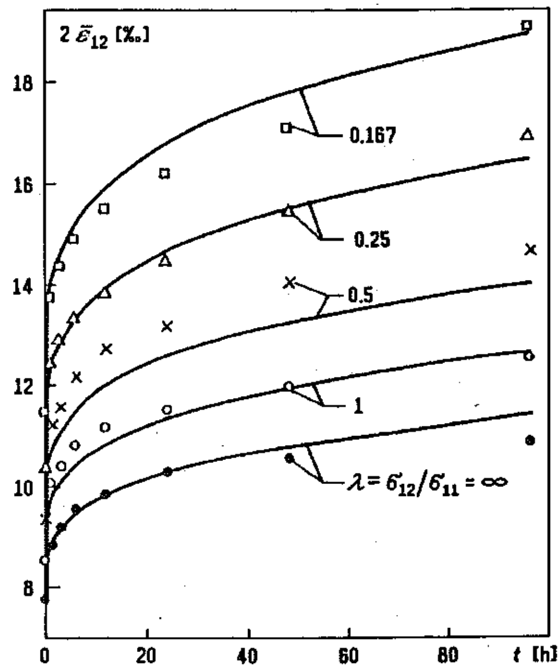

FIG. 1. Creep curves for $2\bar{\varepsilon}_{12}$: marked points represent the test results acc. to Eq. (3.7).

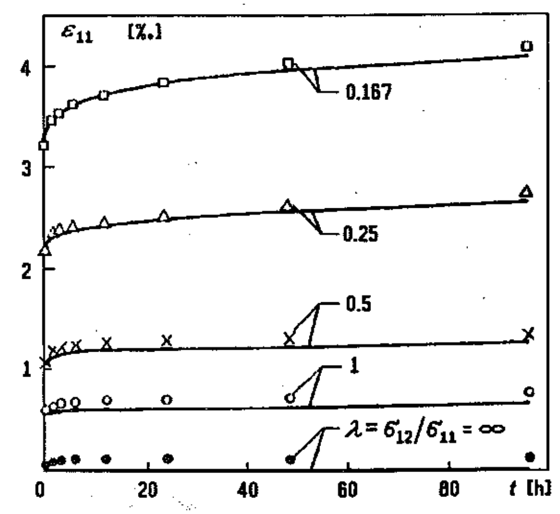

FIG. 2. Creep curves for ε_{11}: marked points represent the test results acc. to Eq. (3.17).

2.2. *Description of creep in simple stress states (torsion, tension)*

1. TORSION

The non-dilational strains $2\bar{\varepsilon}_{12}$ were measured during torsion of a thin-walled tubular specimen. Suitable results are reported in [2], where the description of these strains is as follows:

$$(2.1)\qquad 2\bar{\varepsilon}_{12}(t,\sigma_{12}) = \tilde{A}_{1212}\sigma_{12}/R_{12} + \tilde{A}_{12121212}(\sigma_{12}/R_{12})^3 + \left[\tilde{E}_{1212}\sigma_{12}/R_{12} + \tilde{E}_{12121212}(\sigma_{12}/R_{12})^3\right] B_{1212}\left[1 - e^{-C_{1212}(t/t_0)^{D_{1212}}}\right].$$

for

$$\tilde{A}_{1212} = 13.18\cdot10^{-3},\qquad \tilde{A}_{12121212} = 0,\qquad B_{1212} = 11.31\cdot10^{-3},$$

$$C_{1212} = 0.0806,\qquad D_{1212} = 0.340,\qquad \tilde{E}_{1212} = 0.771,$$

$$\tilde{E}_{12121212} = 2.557,\qquad t_0 = 1\,\mathrm{h}.$$

2. TENSION

The thin-walled tube was stretched to measure the strains $\varepsilon_{11}(t)$. The results are also given in [2], where the description of these strains is

$$(2.2)\qquad \varepsilon_{11}(t,\sigma_{11}) = A_{1111}\sigma_{11}/R_{11} + A_{11111111}(\sigma_{11}/R_{11})^3 + \left[E_{1111}\sigma_{11}/R_{11} + E_{11111111}(\sigma_{11}/R_{11})^3\right] B_{1111}\left[1 - e^{-C_{1111}(t/t_0)^{D_{1111}}}\right],$$

with

$$A_{1111} = 11.19\cdot10^{-3},\qquad A_{11111111} = 0,\qquad B_{1111} = 3.612\cdot10^{-3},$$

$$C_{1111} = 0.118,\qquad D_{1111} = 0.283,\qquad E_{1111} = 1.531,\qquad E_{11111111} = 10.602.$$

3. DESCRIPTION OF GLOBAL CREEP

3.1. *Description of creep strain $2\bar{\varepsilon}_{12}$*

Describing the short-term deformations of anisotropic materials a non-linear expression is often used in the form

$$(3.1)\qquad \varepsilon_{ij} = a_{ijkl}\sigma_{kl} + a_{ijklmn}\sigma_{kl}\sigma_{mn} + a_{ijklmnop}\sigma_{kl}\sigma_{mn}\sigma_{op} + \ldots\,.$$

Similar form is used for the creep under $\sigma = \text{const}$, i.e.

$$(3.2)\qquad \varepsilon_{ij}(t) = \sigma_{kl}\left(a_{ijkl} + \int\limits_0^t k_{ijkl}(\tau)\,d\tau\right) + \sigma_{kl}\sigma_{mn}\left(a_{ijklmn} + \int\limits_0^t k_{ijklmn}(\tau)\,d\tau\right) + \sigma_{kl}\sigma_{mn}\sigma_{op}\left(a_{ijklmnop} + \int\limits_0^t k_{ijklmnop}(\tau)\,d\tau\right) + \ldots .$$

For an orthotropic material subjected to tension and torsion the first three terms can be taken and a notation introduced in [2] leads to

$$(3.3)\qquad 2\bar{\varepsilon}_{12}(t) = (\sigma_{12}/R_{12})\left(\tilde{A}_{1212} + \int\limits_0^t K_{1212}(\tau)\,d\tau\right) + (\sigma_{12}/R_{12})^3\left(\tilde{A}_{12121212} + \int\limits_0^t K_{12121212}(\tau)\,d\tau\right) + (\sigma_{11}\sigma_{12}/R_{12}^2)\left(\tilde{A}_{121112} + \int\limits_0^t K_{121112}(\tau)\,d\tau\right) + (\sigma_{11}^2\sigma_{12}/R_{12}^3)\left(\tilde{A}_{12111112} + \int\limits_0^t K_{12111112}(\tau)\,d\tau\right).$$

Using notation as in [3], we obtain

$$(3.4)\qquad 2\bar{\varepsilon}_{12}(t) = G_1(t)\sigma_{12} + G_2(t)\sigma_{12}^3 + G_3(t)\sigma_{11}\sigma_{12} + G_4(t)\sigma_{11}^2\sigma_{12}.$$

On comparing the formulae (3.4) and (2.1), we get

$$(3.5)\qquad \begin{aligned} G_1(t) &= A_{1212}/R_{12} + (\tilde{E}_{1212}/R_{12})B_{1212}\left[1 - e^{-C_{1212}(t/t_0)^{D_{1212}}}\right],\\ G_2(t) &= A_{12121212}/R_{12}^3 + (\tilde{E}_{12121212}/R_{12}^3)B_{1212}\left[1 - e^{-C_{1212}(t/t_0)^{D_{1212}}}\right]. \end{aligned}$$

From the analysis of isochronous curves and the strain difference $2\bar{\varepsilon}_{12}(t) - G_1\sigma_{12} - G_2\sigma_{12}^3 = 2\hat{\bar{\varepsilon}}_{12}$ it follows that in the $2\hat{\bar{\varepsilon}}_{12} - \sigma_{11}$ coordinate system these curves become straight lines (coefficients of linear correlation for these straight lines are in the range $0.9700 - 9992$).

Thus in the formula (3.4) we have

$$G_4(t) = 0. \tag{3.6}$$

On account of Eq. (3.6) the formula (3.4) takes the form

$$2\bar{\varepsilon}_{12}(t) = G_1(t)\sigma_{12} + G_2(t)\sigma_{12}^3 + G_3(t)\sigma_{11}\sigma_{12}. \tag{3.7}$$

Assuming the function $G_3(t)$ in the form similar to $G_1(t)$ (under the assumption that the time function representing the creep kernel is the same) we can write

$$G_3(t) = \tilde{A}_{121211} + B_{121211}\left[1 - e^{-C_{121211}(t/t_0)^{D_{121211}}}\right], \tag{3.8}$$

where $t_0 = 1$ h. The values of constants determined with the use of the least square procedure, amount to

$$\begin{aligned} \tilde{A}_{121211} &= 1.725 \cdot 10^{-3}, & B_{121211} &= 6.43 \cdot 10^{-3}, \\ C_{121211} &= 0.105, & D_{121211} &= 0.242. \end{aligned} \tag{3.9}$$

The description of creep with the use of Eq. (3.7) and the help of Eq. (2.1), (3.5), (3.8), (3.9) is visualized in Fig. 1.

3.2. Description of creep strain ε_{11}

The component ε_{11} of the global creep strain tensor for an orthotropic material under tension and torsion can be described by taking the first three terms and introducing the notation as in [2], namely

$$\begin{aligned} \varepsilon_{11}(t) = (\sigma_{11}/R_{11})&\left(A_{1111} + \int_0^t K_{1111}(\tau)\,d\tau\right) \\ &+ (\sigma_{11}/R_{11})^2\left(A_{111111} + \int_0^t K_{111111}(\tau)\,d\tau\right) \\ &+ (\sigma_{11}/R_{11})^3\left(A_{11111111} + \int_0^t K_{11111111}(\tau)\,d\tau\right) \\ &+ (\sigma_{11}\sigma_{12}^2)/R_{12}^3\left(A_{11111212} + \int_0^t K_{11111212}(\tau)\,d\tau\right) \\ &+ (\sigma_{12}/R_{12})^2\left(A_{111212} + \int_0^t K_{111212}(\tau)\,d\tau\right). \end{aligned} \tag{3.10}$$

With suitable notation for the functions of time, the formula (3.10) becomes

$$(3.11) \qquad \varepsilon_{11}(\tau) = F_1(t)\sigma_{11} + F_2(t)\sigma_{11}^2 + F_3(t)\sigma_{11}^3 + F_4(t)\sigma_{12}^2\sigma_{11} + F_5(t)\sigma_{12}^2.$$

On comparing Eqs. (3.11) and (2.2) we obtain

$$(3.12) \qquad F_2(t) = 0.$$

Let us analyse an influence of the stress σ_{12} on the strain $\varepsilon_{11}(t)$. The results for simple tension under $\sigma_{11}/R_{11} = 0.287$ and $\varepsilon_{11}(t)$ and under the same tensile stress with the addition of the statical stress $\sigma_{12} = 0.6R_{12}$, are depicted in Fig. 3. A slight effect increasing the influence of shearing stresses

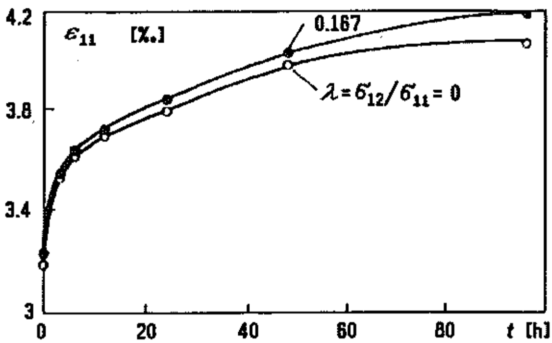

FIG. 3. Creep curves for ε_{11}: *1* – simple tension at $\sigma_{11} = 0.287R_{11}$, *2* – tension and torsion for $\lambda = \sigma_{12}/\sigma_{11} = 0.167$ and $\sigma_{12} = 0.6R_{12}$.

on the strains ε_{11} can be seen. A statistical hypothesis $\mu_1 = \mu_2$ on the equality of average strains is postulated (μ_1 does not allow for an additional stress σ_{12}). To evaluate the difference between two averages the following formula is used:

$$(3.13) \qquad t = |\bar{x}_1 - \bar{x}_2|\sqrt{n}\Big/\sqrt{s_1^2 + s_2^2},$$

where $\bar{x}_1$, $\bar{x}_2$ – two different averages, n – number of specimens from which $\bar{x}_1$ and $\bar{x}_2$ are calculated, s_1, s_2 – standard deviations. Before using the test t from the formula (3.13), equality of variances was verified by means of Hartley's test. The value of t test, from formula (3.13), was contained for particular creep times in the interval $0.077 - 0.505$. The value t_{crit} for $\alpha = 0.05$ was 2.776. Thus the equality of the averages is confirmed. That is why the influence of pure torsion on the longitudinal strains can be neglected. It follows that

$$(3.14) \qquad F_5(t) = 0.$$

From the formulae (3.11), (3.12), (3.14) we have

$$\varepsilon_{11}^{d} - F_1\sigma_{11} - F_3\sigma_{11}^3 = F_4\sigma_{12}\sigma_{11}^2. \tag{3.15}$$

Since the left-hand side of Eq. (3.15) is close to zero, we assume that

$$F_4(t) = 0. \tag{3.16}$$

Finally, the formula (3.11) takes the form

$$\varepsilon_{11}(t) = F_1(t)\sigma_{11} + F_3(t)\sigma_1^3, \tag{3.17}$$

where the functions $F_1(t)$ and $F_3(t)$ are defined according to Eq. (2.2). The description is visualized in Fig. 2.

4. Statistical analysis of test results

To verify the correctness of the description let us calculate the mean absolute and relative square errors:

$$\begin{aligned} r_1 &= \left\{\left[\sum_{i=1}^{N}(y_i - y_t)^2\right]/N\right\}^{1/2}, \\ r_2 &= \left\{\left[\sum_{i=1}^{N}(y_i - y_t)^2/y_i^2\right]/N\right\}^{1/2}, \end{aligned} \tag{4.1}$$

where y_i – measured strains, y_t – theoretical strains, N – number of measurements. The results of calculations are given in Table 1. Certain auxiliary values are also shown necessary to estimate the confidence intervals, referring to the mean value and to particular observations. The necessary formulae are:

for the absolute error

$$\Delta_1 = t_{\alpha,\nu}\left\{\left[\sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij} - y_i)^2\right]/\nu\right]^{1/2}, \tag{4.2}$$

for the relative error

$$\Delta_2^w = t_{\alpha,\nu}\left\{\frac{1}{\nu}\sum_{i=1}^{n}\sum_{j=1}^{m}[(y_{ij} - y_i)/y_i]^2\right]^{1/2}, \tag{4.3}$$

where $\alpha = 0.05$, $\nu = nm - n$ – number of the degrees of freedom, n - number of stress levels, m – number of instants at which the strain reading were taken. The values calculated from the formulae (4.2), (4.3) corresponds to single observations. To obtain the mean value they must be divided by $\sqrt{p}$, where p is the number of repeats for a given time (number of specimens for a given stress state). Before these values were calculated, the Cochran test was performed to check the variance homogeneity.

Table 1. Results of calculations of r_1, r_2, Δ_1, Δ_2^w.

Statistical magnitude	Type of strain	
	$2\varepsilon_{12}$	ε_{11}
r_1 [‰]	0.388	0.085
r_2 [%]	2.99	4.53
Δ_1 [‰]	0.193	0.175
Δ_2^w [%]	6.16	11.74

5. Conclusions

1. The global creep strain $2\bar{\varepsilon}_{12}(t)$ corresponding to tension and torsion is correctly described by the nonlinear theory of viscoelasticity (Fig. 1 and Table 1, where $r_2 < \Delta_2^w$ and $r_1 > \Delta_1$).

2. The global creep strain $\varepsilon_{11}(t)$ corresponding to tension and torsion is also properly described by the nonlinear theory of viscoelasticity (Fig. 2 and Table 1, where $r_2 < \Delta_2^w$ and $r_1 < \Delta_1$).

References

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