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CREEP INVESTIGATIONS OF THE LIGNOSTONE UNDER TENSION AND TORSION

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Specimens of the beech-wood lignostone are tested for creep under simple tension along the grains and torsion at room temperature. The creep strains $2\bar{\varepsilon}_{12}(t)$ generated by pure torsion in the shearing stress range $\sigma_{12} = (0.4 - 0.6)R_{12}$ (where R_{12} is the shearing strength) are found to be nonlinear. Similarly, the creep strains $\varepsilon_{11}(t)$ generated by simple tension in the normal stress range $\sigma_{11} = (0.2 \div 0.35)R_{11}$ where R_{11} is the tensile strength, are also nonlinear. To describe those nonlinear behaviour the nonlinear viscoelasticity theory is employed and the similarity of the creep curves is used. Correct description of the considered phenomena is obtained.

1. Introduction

Solutions of the initial-boundary value problems require the knowledge of physico-mechanical properties of the materials involved, both short-term and long-term ones [2]. Systematic investigations in the field of wood rheology have been conducted over a number of decades [3–5]. These tests dealt with the creep properties in a uniaxial stress state and only the strains along the applied forces were measured.

This paper is devoted to the tests and the description of creep under pure tension and torsion.

2. Description of experiments

The creep was examined on the 5 KN creep-testing machine manufactured by IMP and suitably adapted to investigate tension and torsion. The creep strains ε_{11} under tension were measured in the direction of the applied force by means of induction strain gauges with the accuracy of 0.0001 mm. The creep strains $2\bar{\varepsilon}_{12}$ under torsion were obtained by measuring the angle of twist, where $\bar{\varepsilon}_{12}$ denotes an average of shearing strains $\bar{\varepsilon}_{12}$ and $\bar{\varepsilon}_{13}$ in the orthotropic material. The angle of twist was measured with by dial gauges with 0.01 mm accuracy. Suitable pattern of measurements is shown in Fig. 1.

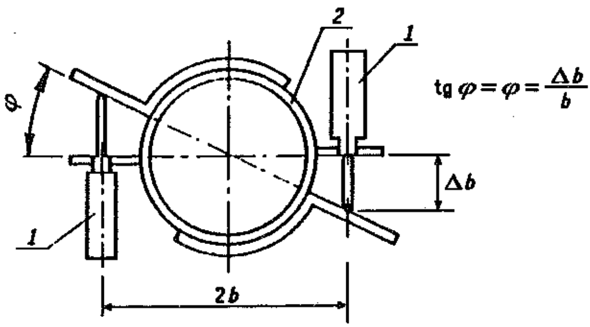

FIG. 1. Measurement of the angle of twist: *1* – dial gauge, *2* – hollow cylindrical specimen.

Specimens were made of the beech-wood lignostone with the density $\varrho = 990\,\mathrm{kg/m^3}$ and the compression ratio $n = 1.45$. They were prepared in the Plywood Industries Factory in Białystok.The technological conditions of the production of samples are given in [1]. The shape and dimensions of the specimens are presented in Fig. 2.

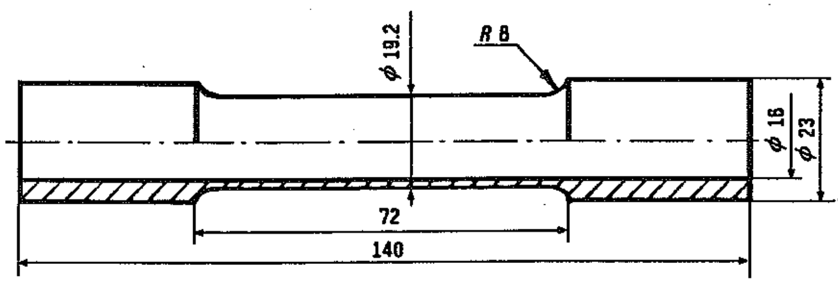

FIG. 2. Test specimen.

Prior to testing the specimens were seasoned in the Feutron 3001 – 10 climatic chamber with the air humidity $(65 \pm 2.5)\%$ and the temperature of $(293 \pm 3)\,\mathrm{K}$. The creep tests were performed under identical climatic conditions.

The tensile creep tests were made for three stress levels: $\sigma_{11}/R_{11} = 0.2;\ 0.287;\ 0.35$, where $R_{11} = 195.1\,\mathrm{MPa}$ denotes the tensile strength along the grains. The torsional creep tests were also made for three stress levels: $\sigma_{12}/R_{12} = 0.4;\ 0.5;\ 0.6$, where $R_{12} = 15.56\,\mathrm{MPa}$ denotes the technical shearing strength. The loading rate amounted to 2 MPa/s. The first reading was made 10 s later than the load was applied.

3. Tests results and their description

The elongations ε_{11} are shown in Fig. 3 while the shearing strains $2\bar{\varepsilon}_{12}$ accompanying torsion can be seen in Fig. 4.

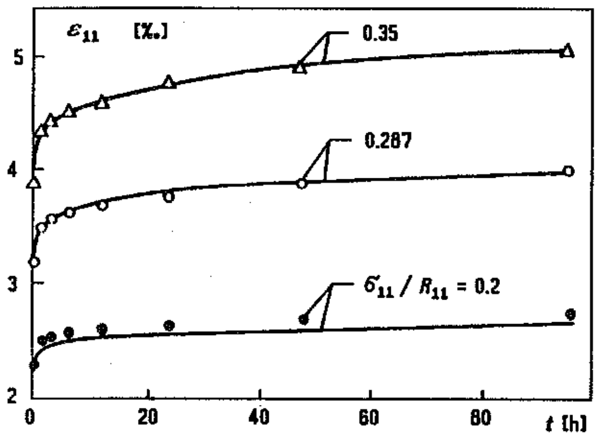

FIG. 3. Creep curves ε_{11}: dots, circles and triangles represent experimental data acc. to Eqs. (3.1), (3.2), (3.7).

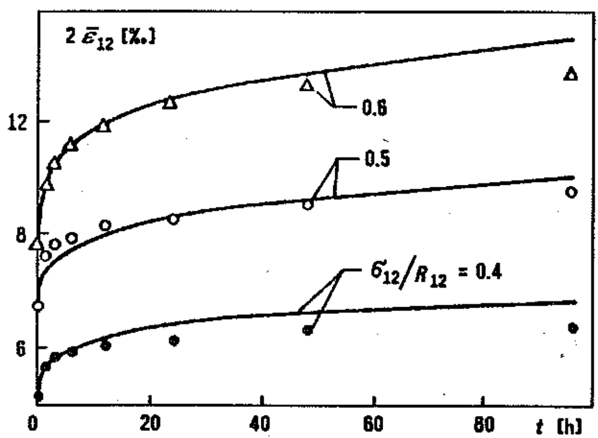

FIG. 4. Creep curves $2\bar{\varepsilon}_{12}$: dots, circles and triangles represent experimental data acc. to Eqs. (3.9), (3.18), (3.23).

3.1. Description of the tension test-strains ε_{11}

Examining the ratios $\varepsilon^c_{11}(\sigma_{11})/\varepsilon^c_{11}(\sigma_{11} = 0.35R_{11})$ for various σ_{11} it has been statistically proved that these ratios are constant for different times and depend on the stress level in a nonlinear manner. However, the creep curves ε^c_{11} are similar due to the constancy of the above ratios. Thus the creep strains can be represented by a product of a stress function $f_{11}(\sigma_{11})$ and the time function $g_{11}(t)$. The global strain ε_{11} will be a sum of the short-term strain and the creep strain

$$\varepsilon_{11}(t,\sigma_{11}) = \varepsilon_{11}(0,\sigma_{11}) + \varepsilon^c_{11}(t,\sigma_{11}). \tag{3.1}$$

The short-term strains are described by the formula

$$\begin{aligned} \varepsilon_{11}(0,\sigma_{11}) &= a_{11kl}\sigma_{kl} + a_{11klmnop}\sigma_{kl}\sigma_{mn}\sigma_{op} \\ &= a_{1111}\sigma_{11} + a_{11111111}\sigma^3_{11} = A_{1111}\sigma_{11}/R_{11} + A_{11111111}(\sigma_{11}/R_{11})^3 . \end{aligned} \tag{3.2}$$

The creep strains can be expressed by

$$\varepsilon^c_{11}(t,\sigma_{11}) = f_{11}(\sigma_{11})g_{11}(t) . \tag{3.3}$$

The function $f_{11}(\sigma_{11})$ is assumed to have the form

$$f_{11}(\sigma_{11}) = E_{1111}\sigma_{11}/R_{11} + E_{11111111}(\sigma_{11}/R_{11})^3 , \tag{3.4}$$

and the function $g_{11}(t)$ to be

$$g_{11}(t) = \int\limits_0^t K_{1111}(\tau)\, d\tau, \tag{3.5}$$

where $K_{1111}(\tau)$ denotes a creep kernel having the form

$$\begin{aligned} K_{1111}(\tau) &= t_0^{-D_{1111}}\overline{B}_{1111}\tau^{D_{1111}-1}e^{-C_{1111}(\tau/t_0)^{D_{1111}}} , \\ \overline{B}_{1111} &= B_{1111}C_{1111}D_{1111}, \qquad t_0 = 1\,\mathrm{h} . \end{aligned} \tag{3.6}$$

Substituting Eqs. (3.6) into Eq. (3.5), integrating and inserting Eqs. (3.4) and (3.5) into Eq. (3.3), we obtain

$$\begin{aligned} \varepsilon^c_{11}(t,\sigma_{11}) = &\left[E_{1111}\sigma_{11}/R_{11} + E_{11111111}(\sigma_{11}/R_{11})^3\right] \\ &\times \left\{B_{1111}\left[1 - e^{-C_{1111}(t/t_0)^{D_{1111}}}\right]\right\} . \end{aligned} \tag{3.7}$$

The function in brackets is called the Kohlrausch function and can be also obtained from the modified standard model [3]. The constants in the formulae (3.2) and (3.7) are determined by the least square procedure and have the values

$$
(3.8)\qquad \begin{aligned} &A_{1111} = 11.19 \cdot 10^{-3}, \quad A_{11111111} = 0, \quad B_{1111} = 3.612 \cdot 10^{-3},\\ &C_{1111} = 0.118, \quad D_{1111} = 0.283, \quad E_{1111} = 1.531. \end{aligned}
$$

The description of creep by the formula (3.1) with the constants (3.8) is visualized in Fig. 3.

3.2. Description of the torsion test-strains $2\bar{\varepsilon}_{12}$

The strains $2\bar{\varepsilon}_{12}$ accompanying torsion are described similarly to the formula (3.1). The global strain $2\bar{\varepsilon}_{12}$ is a sum of the short-term strain and the creep strain,

$$
(3.9)\qquad 2\bar{\varepsilon}_{12}(t, \sigma_{12}) = 2\bar{\varepsilon}_{12}(0, \sigma_{12}) + 2\bar{\varepsilon}^c_{12}(t, \sigma_{12}).
$$

In order to determine $2\bar{\varepsilon}_{12}(0, \sigma_{12})$ let us employ the general relationship for a nonlinear anisotropic body in the form

$$
(3.10)\qquad 2\bar{\varepsilon}_{12} = 2\left(a'_{12kl}\sigma'_{kl} + a'_{12klmn}\sigma'_{kl}\sigma'_{mn} + a_{12klmnop}\sigma'_{kl}\sigma'_{mn}\sigma'_{op} + \ldots\right).
$$

Taking the first and the third term for an orthotropic material we obtain

$$
(3.11)\qquad 2\bar{\varepsilon}_{12} = 4a'_{1212}\sigma'_{12} + 16a'_{12121212}{\sigma'_{12}}^3 .
$$

Incremental angle of twist has, in the presence of Eq. (3.11), the form

$$
(3.12)\qquad d\,(2\bar{\varepsilon}_{12}) = \left[4a'_{1212}(\alpha) + 48a'_{12121212}(\alpha){\sigma'_{12}}^2\right] d\sigma'_{12} .
$$

The determination of $d\sigma'_{12}$ as a function of α can be, in view of Fig. 5, expressed by

$$
(3.13)\qquad d\sigma'_{12} = \frac{dM \cdot r}{J_0} = \frac{\sigma'_{12} \cdot r\, d\alpha \cdot g \cdot r \cdot r}{2\pi g r^3} = \frac{\sigma'_{12} d\alpha}{2\pi} .
$$

From Eqs. (3.12), (3.13) it follows that

$$
(3.14)\qquad 2\bar{\varepsilon}_{12} = 4\int\limits_0^{\pi/2} \left[4a'_{1212}(\alpha) + 48a'_{12121212}(\alpha){\sigma'_{12}}^2\right] \frac{\sigma'_{12} d\alpha}{2\pi} .
$$

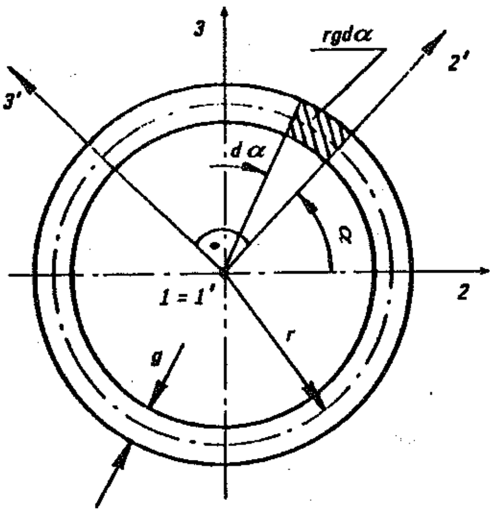

FIG. 5. Cross-section of a specimen and the coordinate axes.

The tensorial transformation formulae for the orthotropic body lead to

$$
(3.15)\qquad
\begin{aligned}
a'_{1212} &= \alpha_{1i}\alpha_{2j}\alpha_{1k}\alpha_{2l}a_{ijkl},\\
a'_{12121212} &= \alpha_{1i}\alpha_{2j}\alpha_{1k}\alpha_{2l}\alpha_{1m}\alpha_{2n}\alpha_{1o}\alpha_{2p}a_{ijklmnop},\\
a'_{1212} &= a_{1212}\cos^2\alpha + a_{1313}\sin^2\alpha,\\
a'_{12121212} &= a_{12121212}\cos^4\alpha + 6a_{12121313}\cos^2\alpha\sin^2\alpha\\
&\quad + a_{13131313}\sin^4\alpha.
\end{aligned}
$$

Substituting Eq. (3.15) into Eq. (3.14), we get

$$
(3.16)\qquad 2\bar{\varepsilon}_{12}|0,\sigma_{12}| = 2(a_{1212}+a_{1313})\sigma_{12} + 18(a_{12121212}+2a_{12121313} + a_{13131313})\sigma_{12}^3.
$$

Introducing the notation

$$
(3.17)\qquad
\begin{aligned}
2(a_{1212}+a_{1313})R_{12} &= \tilde{A}_{1212},\\
18(a_{12121212}+2a_{12121313}+a_{13131313})R_{12}^3 &= \tilde{A}_{12121212},
\end{aligned}
$$

we obtain an alternative form

$$
(3.18)\qquad 2\bar{\varepsilon}_{12}(0,\sigma_{12}) = \tilde{A}_{1212}\sigma_{12}/R_{12} + \tilde{A}_{12121212}(\sigma_{12}/R_{12})^3.
$$

The creep strain is described by the formula resulting from the similarity of the creep curves:

$$
(3.19)\qquad 2\bar{\varepsilon}_{12}^c(t,\sigma_{12}) = f_{12}(\sigma_{12})\int_0^t K_{1212}(\tau)\,d\tau.
$$

The creep kernel $K_{1212}(\tau)$ is assumed in the form

$$(3.20)\qquad \begin{aligned} K_{1212}(\tau) &= t_0^{-D_{1212}}\overline{B}_{1212}\tau^{D_{1212}-1}e^{-C_{1212}(\tau/t_0)^{D_{1212}}}, \\ \overline{B}_{1212} &= B_{1212}C_{1212}D_{1212}, \qquad t_0 = 1\,\text{h}. \end{aligned}$$

Substituting Eqs. (3.20) into Eq. (3.19) and integrating, we arrive at

$$(3.21)\qquad 2\varepsilon^c_{12}(t,\sigma_{12}) = f_{12}(\sigma_{12})B_{1212}\left[1 - e^{-C_{1212}(t/t_0)^{D_{1212}}}\right].$$

The function $f_{12}(\sigma_{12})$ is expressed by the formula of the type

$$(3.22)\qquad f_{12}(\sigma_{12}) = \tilde{E}_{1212}\sigma_{12}/R_{12} + \tilde{E}_{12121212}(\sigma_{12}/R_{12})^3.$$

Remembering Eq. (3.22), from Eq. (3.21) we obtain

$$(3.23)\qquad \begin{aligned} 2\bar{\varepsilon}^c_{12}(t,\sigma_{12}) = &\left[\tilde{E}_{1212}\sigma_{12}/R_{12} + \tilde{E}_{12121212}(\sigma_{12}/R_{12})^3\right] \\ &\times B_{1212}\left[1 - e^{-C_{1212}(t/t_0)^{D_{1212}}}\right]. \end{aligned}$$

The constants appearing in the formulae (3.18) and (3.23) are determined by means of the least square procedure and are:

$$(3.24)\qquad \begin{aligned} &\tilde{A}_{1212} = 13.18\cdot 10^{-3}, \quad \tilde{A}_{12121212} = 0, \quad B_{1212} = 11.31\cdot 10^{-3}, \\ &C_{1212} = 0.0806, \quad D_{1212} = 0.340, \quad \tilde{E}_{1212} = 0.771, \\ &\tilde{E}_{12121212} = 2.557. \end{aligned}$$

The description of the creep by the formula (3.9) taking into account Eqs. (3.18), (3.23) and the constants (3.24) is visualized in Fig. 4.

4. Statistical verification of the mathematical models

To assess the exactness of the description, mean absolute and relative square errors are calculated from the formulae

$$(4.1)\qquad r_1 = \left\{\left[\sum_{i=1}^{N}(y_i - y_t)^2\right]/N\right\}^{1/2}, \qquad r_2 = \left\{\left[\sum_{i=1}^{N}((y_i - y_t)/y_i)^2\right]/N\right\}^{1/2},$$

where y_i – measured strain, y_t – theoretical strain, N – number of measurements. The results of calculations are given in Table 1 where some auxiliary values are also shown necessary to calculate the confidence intervals of the mean value and specific observations.

Table 1. Results of calculations of r_1, r_2, Δ_1, Δ_2^w.

statistical magnitude	type of strain	
	ε_{11}	$2\varepsilon_{12}$
r_1 [‰]	0.061	0.212
r_2 [%]	2.27	2.77
Δ_1 [‰]	0.497	1.42
Δ_2^w [%]	14.62	16.8

The following formulae were used:

for the absolute error

$$\Delta_1 = t_{\alpha,\nu} \left\{ \left[\sum_{i=1}^{n} \sum_{j=1}^{m} (y_{ij} - \bar{y}_i)^2 \right] / \nu \right\}^{1/2}, \tag{4.2}$$

for the relative error

$$\Delta_2^w = t_{\alpha,\nu} \left\{ \frac{1}{\nu} \sum_{i=1}^{n} \sum_{j=1}^{m} [(y_{ij} - \bar{y}_i)/y_i]^2 \right\}^{1/2}, \tag{4.3}$$

where $\alpha = 0.05$, $\nu = nm - n$ – number of the degrees of freedom, n – number of stress levels, m – number of instants at which the strains were measured. The values calculated according to the formulae (4.2) and (4.3) correspond to single observations. To obtain mean values they must be divided by $\sqrt{p}$, where p is a number of repeats for a given time instant (the number of specimens for a given stress level). Before these values were calculated, the Cochran test was used to check the variance homogeneity.

5. Conclusions

The creep strains ε_{11} under torsion and the creep strains $2\bar{\varepsilon}_{12}$ under torsion are found to be nonlinear with respect to time. The creep curves are similar. Their correct description can be obtained by representing the short-term strains in the form of a third-degree polynomial in stresses and the creep strains as a product of that polynomial and a time function of the Kohlrausch type. This is confirmed by Fig. 3 and Table 1 in which $r_2 < \Delta_2^w$ and $r_1 < \Delta_1$ for ε_{11}, and by Fig. 4 and Table 1 in which $r_2 < \Delta_2^w$ and $r_1 < \Delta_1$ for $2\bar{\varepsilon}_{12}$.

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