ENGINEERING TRANSACTIONS • *Engng. Trans.* • **42**, 4, 377–391, 1994
Polish Academy of Sciences • Institute of Fundamental Technological Research

MAGNETOHYDRODYNAMIC NATURAL CONVECTION FLOWS RESULTING FROM THE COMBINED BUOYANCY EFFECTS OF THERMAL AND MASS DIFFUSION

A. A. SAMAAN and F. N. IBRAHIM (CAIRO)

This paper presents a study of laminar doubly diffusive free convection flows of a viscoelastic fluid past an oscillating vertical plate in the presence of a transverse magnetic field. The two buoyant mechanisms are the thermal diffusion and species diffusion. The governing conservation equations of momentum, energy and concentration are nondimensionalized and solved analytically. Effects of Pr (Prandtl number), Sc (Schmidt number), Gr (Grashof number), Gm (modified Grashof number), M (magnetic number), ω (frequency parameter) and k (viscoelastic parameter) upon the velocity field, the shear stress on the plate, the temperature field and the concentration field are discussed. The results show many interesting aspects of the complex interaction of the two buoyant mechanisms.

NOTATION

A, B	quantities given by Eqs. $(2.18)_{1,2}$,
A_i, B_j	$(i = 5, ..., 10;\ j = 1, ..., b)$ quantities given by Eq. (2.31),
B_{15}, B_{16}, Z_1, Z_2	quantities given by Eq. (2.38),
B_0	strength of the applied magnetic field,
C'_p	specific heat at constant pressure,
C'	concentration of the diffusing species,
D'	coefficient of chemical molecular diffusivity,
Gr	Grashof number,
Gm	modified Grashof number,
g	acceleration due to gravity,
k	viscoelastic parameter,
k'	coefficient of the viscoelastic term,
M	magnetic number,
Pr	Prandtl number,
Sc	Schmidt number,
T'	temperature of the fluid,
t'	time variable,
t, u, c	dimensionless time, velocity and concentration, respectively,
u', v'	velocity components in x', y' directions,
u_1, c_1	functions representing the velocity and temperature and depending on η only,
x'	distance along the plate,
y'	distance normal to x'.

Greek symbols

α, α' parameters given by Eqs. $(2.21)_{1,2}$,
β volumetric coefficient of thermal expansion,
β^* volumetric coefficient of expansion with concentration,
γ' thermal conductivity,
θ dimensionless temperature,
θ_1 function that represents the temperature and depends on η only,
μ coefficient of viscosity,
ρ' the density,
$\nu = \frac{\mu}{\rho'}$ kinematic viscosity,
λ complex quantity given by Eq. (2.27),
τ' shear stress, Eq. (2.34),
τ dimensionless shear stress, Eq. (2.35),
η dimensionless distance,
σ electrical conductivity,
ω' frequency of fluctuation,
ω frequency parameter.

Subscripts

i imaginary part,
m maximum value at the plate,
r real part,
w condition at the plate,
∞ condition far from the plate.

1. Introduction

A viscoelastic fluid is a type of fluid which possesses both the appreciable elasticity of shape and also viscous properties. This property may be conferred on the liquid, for example by the addition of long flexible molecules in solution or by dispersing solid or liquid particles. Consequently, when the flow of a viscoelastic fluid in the boundary layer is studied, it is necessary to take into account the variation of concentration with temperature. In many natural and technological processes, temperature and concentration differences occur simultaneously. Such processes occur in cleaning operations, drying, crystal growth, solar ponds and photosynthesis. The term *doubly diffusive convection* is now widely used for all the processes involving simultaneous thermal and concentrations gradients. In a recent survey, Ostrach [10] classified doubly diffusive convection based on the orientation of thermal and concentration gradients with respect to gravity vector. Gebhart and Pera [6] studied laminar natural convection flows driven by thermal and concentration buoyancy adjacent to flat vertical surfaces. They presented an excellent summary of this class of doubly diffusive natural convection. An analytical solution using the local nonsimilarity method for natural convection heat transfer from a vertical surface to a thermally stratified fluid was obtained by CHEN and EICHHORN [4]. They also present experimental results and a summary of related work of other investigators. A numerical study of laminar doubly diffusive free convection flows adjacent to vertical surface in a stable thermally stratified medium has been given by ANGIRASA and SRINIVASAN [1]. Their results show many interesting aspects of the complex interaction of the two buoyant mechanisms. The unsteady free convection flow of a Newtonian fluid in the presence of a magnetic field has been studied by GUPTA [7], CHAWLA [3], SOUNDALGEKAR [11] and MISHRA [9]. The unsteady free convection flow of an incompressible electrically conducting viscoelastic fluid past an oscillating plate in the presence of a transverse magnetic field has been studied by IBRAHIM [8]. ELBASHBESHY and IBRAHIM [5] presented a study for the flow of a Newtonian viscous incompressible fluid along a heated vertical plate, taking into account the variation of the viscosity and thermal diffusivity with temperature.

In this paper a study is made for free convection flow of a Walter's viscoelastic fluid (1964) past a vertical plate, whose velocity, temperature and concentration fluctuate with time harmonically. The effects of the Prandtl number Pr, Schmidt number Sc, Grashof number Gr, modified Grashof number Gm, magnetic number M, frequency ω and the vicoelastic parameter k on the velocity, skin friction, temperature and concentration have been studied. Here we continue the problem discussed by IBRAHIM [8] taking into account the variation of concentration with temperature.

2. MATHEMATICAL ANALYSIS

We consider two-dimensional, unsteady, magnetohydrodynamic free-convection and mass-transfer flow of a viscoelastic and electrically-conducting fluid along a vertical flat plate. The velocity, temperature and the concentration of the fluid along the plate fluctuate with time harmonically. All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. A magnetic field of uniform strength is applied transversally to the direction of the flow. The magnetic Reynold's number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The origin of the coordinate system is taken to be at any point of the flat vertical infinite plate, the x'-axis is chosen along the plate vertically upwards, and the y'-axis perpendicular to the plate, as shown in Fig. 1. In the special case

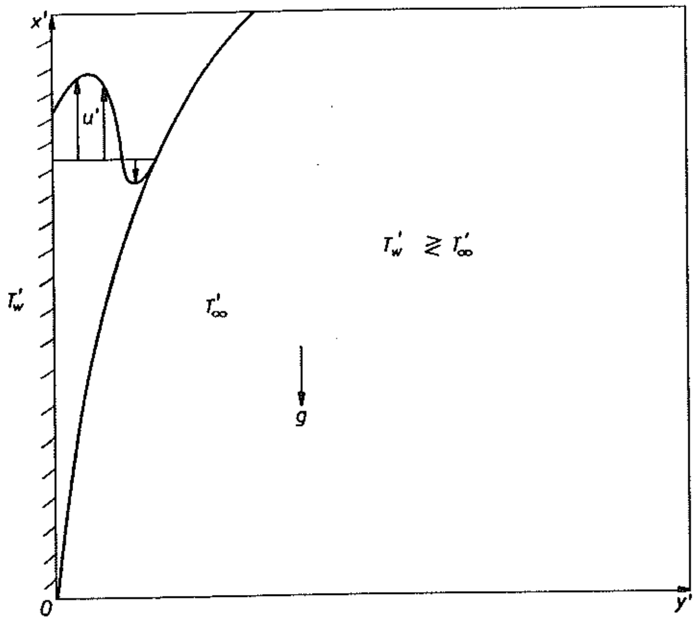

FIG. 1. Physical coordinate system.

when the flow is independent of x' and the velocity normal to the plate v' vanishes ewerywhere; the unsteady free convection flow of an incompressible viscoelastic fluid in the presence of a magnetic field is governed by the following equations of momentum, energy and mass transfer (IBRAHIM [8], ANGIRASA and SRINIVASAN [1])

$$\rho'\frac{\partial u'}{\partial t'} = g(\rho'_\infty - \rho') - \sigma B_0^2 u' + \mu\frac{\partial^2 u'}{\partial y'2} - k'\frac{\partial^3 u'}{\partial t'\partial y'^2}, \tag{2.1}$$

$$\rho' C'_p \frac{\partial T'}{\partial t'} = \gamma'\frac{\partial^2 T'}{\partial y'^2}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D'\frac{\partial^2 C'}{\partial y'^2}, \tag{2.3}$$

where ρ' is the density in the boundary layer, ρ'_∞ is the density far away from the plate, u' is the velocity in the x'-direction, t' is the time variable, σ is the electrical conductivity of the medium, B_0 is the strength of the applied magnetic field, μ is the coefficient of viscosity, k' is the coefficient of the viscoelastic term, g is the acceleration due to gravity, C'_p is the specific heat at constant pressure, γ' is the thermal conductivity, T' is the temperature of the fluid, C' is the concentration of the diffusing species, and D' is the coefficient of chemical molecular diffusivity. In the energy equation (2.2) the terms representing viscous and Joule dissipation are neglected as they are really very small in free convection flows.

The appropriate boundary conditions are

at $y' = 0$:

$$\begin{aligned} u' &= u'_m e^{i\omega' t'}, \\ T' &= T'_w = T'_\infty + (T'_m - T'_\infty)e^{i\omega' t'}, \\ C' &= C'_w = C'_\infty + (C'_m - C'_\infty)e^{i\omega' t'}, \end{aligned} \tag{2.4}$$

as $y' \to \infty$:

$$u' \to 0, \qquad T' \to T'_\infty, \qquad C' \to C'_\infty, \tag{2.5}$$

where u'_m, T'_m and C'_m are the maximum velocity, temperature and concentration of the fluid at the plate, respectively, T'_w and C'_w are the corresponding values at the plate and ω' is the frequency of fluctuation. To eliminate the term $g(\rho'_\infty - \rho')$ from Eq. (2.1), we use the equation of state (GEBHART and PERA [6])

$$(\rho'_\infty - \rho') = \rho'\beta(T' - T'_\infty) + \rho'\beta^*(C' - C'_\infty), \tag{2.6}$$

where β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration. Consequently, Eqs. (2.1) and (2.6) give

$$\rho'\frac{\partial u'}{\partial t'} = \rho' g\beta(T'-T'_\infty)+\rho' g\beta^*(C'-C'_\infty)-\sigma B_0^2 u'+\mu\frac{\partial^2 u'}{\partial y'^2}-k'\frac{\partial^3 u'}{\partial t'\partial y'^2}. \tag{2.7}$$

Hence we observe from Eq. (2.7) that the two buoyant mechanisms aid each other when the quantities $\beta(T' - T'_\infty)$ and $\beta^*(C' - C'_\infty)$ have the same sign, and oppose each other when they have opposite signs.

We introduce now the dimensionless quantities

$$\begin{aligned} &\eta = \frac{y' u'_m}{\nu}, \quad t = \frac{u'^2 t'}{4\nu}, \quad \omega = \frac{4\nu\omega'}{u'^2_m}, \quad u = \frac{u'}{u'_m}, \\ &\theta = \frac{T' - T'_\infty}{T'_m - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_m - C'_\infty}, \quad \mathrm{Pr} = \frac{\mu C'_p}{\gamma'}, \\ &\mathrm{Sc} = \frac{\nu}{D'}, \quad \mathrm{Gr} = \frac{\nu g\beta(T'_m - T'_\infty)}{u'^3_m}, \quad \mathrm{Gm} = \frac{\nu g\beta^*(C'_m - C'_\infty)}{u'^3_m}, \\ &\mathrm{M} = \frac{\sigma B_0^2 \nu}{\rho' u'^2_m}, \quad k = \frac{k' u'^2_m}{4\nu^2\rho'}, \quad \nu = \frac{\mu}{\rho'}. \end{aligned} \tag{2.8}$$

Equations (2.2), (2.3) and (2.7) together with the boundary conditions (2.4) and (2.5) under the transformation (2.8) reduce to

$$\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{4}\frac{\partial u}{\partial t} - \mathrm{M}u - k\frac{\partial^3 u}{\partial t \partial \eta^2} = -\mathrm{Gr}\theta - \mathrm{Gm}C, \tag{2.9}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} - \frac{1}{4}\mathrm{Pr}\frac{\partial \theta}{\partial t} = 0, \tag{2.10}$$

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{1}{4}\mathrm{Sc}\frac{\partial C}{\partial t} = 0, \tag{2.11}$$

associated with the boundary conditions.

$$\text{at} \quad \eta = 0: \quad u = e^{i\omega t}, \qquad \theta = e^{i\omega t}, \qquad C = e^{i\omega t}, \tag{2.12}$$

$$\text{as} \quad \eta \to \infty: \quad u = 0, \qquad \theta = 0, \qquad C = 0. \tag{2.13}$$

In order to solve the partial differential equations (2.9)–(2.11) subject to the boundary conditions (2.12) and (2.13), we assume that (IBRAHIM [8])

$$u(\eta, t) = u_1(\eta)e^{i\omega t}, \qquad \theta(\eta, t) = \theta_1(\eta)e^{i\omega t}, \qquad c(\eta, t) = C_1(\eta)e^{i\omega t}. \tag{2.14}$$

Substituting Eqs. (2.14) into Eqs. (2.9)–(2.13), we get

$$(1 - ik\omega)\frac{d^2 u_1}{d\eta^2} - \left(\frac{i\omega}{4} + \mathrm{M}\right)u_1 = -\mathrm{Gr}\theta - \mathrm{Gm}C_1, \tag{2.15}$$

$$\frac{d^2 \theta_1}{d\eta^2} - \frac{i\omega \mathrm{Pr}}{4}\theta_1 = 0, \tag{2.16}$$

$$\frac{d^2 C_1}{d\eta^2} - \frac{i\omega \mathrm{Sc}}{4}C_1 = 0. \tag{2.17}$$

The boundary conditions for u_1, θ_1 and C_1 are

$$\text{at} \quad \eta = 0: \quad u_1 = 1, \qquad \theta_1 = 1, \qquad C_1 = 1, \tag{2.18}$$

$$\text{as} \quad \eta \to \infty: \quad u_1 = 0, \qquad \theta = 0, \qquad C_1 = 0. \tag{2.19}$$

The solution of Eqs. (2.16) and (2.17) subject to the boundary conditions $(2.18)_{2,3}$ and $(2.19)_{2,3}$ is given by

$$\theta_1 = e^{-\alpha\eta} \qquad \text{and} \qquad C_1 = e^{\alpha'\eta}, \tag{2.20}$$

where

$$\alpha = \frac{1}{2}\sqrt{i\omega \mathrm{Pr}} \qquad \text{and} \qquad \alpha' = \frac{1}{2}\sqrt{i\omega \mathrm{Sc}}, \tag{2.21}$$

the real and imaginary parts of $\alpha = \alpha_r + i\alpha_i$ and $\alpha' = \alpha'_r + i\alpha'_i$ are

$$\alpha_r = \alpha_i = \sqrt{\frac{\omega \mathrm{Pr}}{8}} \qquad \text{and} \qquad \alpha'_r = \alpha'_i = \sqrt{\frac{\omega \mathrm{Sc}}{8}}. \tag{2.22}$$

From Eqs. $(2.14)_{2,3}$ and (2.20)

$$\theta = e^{(i\omega t - \alpha\eta)} \qquad \text{and} \qquad C = e^{(i\omega t - \alpha'\eta)}, \tag{2.23}$$

the real and imaginary parts of $\theta = \theta_r + i\theta_i$ and $C = C_r + iC_i$ are given by

$$\theta_r = e^{-\alpha_r \eta} \cos(\omega t - \alpha_i \eta) \qquad \text{and} \qquad \theta_i = e^{-\alpha_r \eta} \sin(\omega t - \alpha_i \eta), \tag{2.24}$$

$$C_r = e^{-\alpha'_r \eta} \cos(\omega t - \alpha'_i \eta) \qquad \text{and} \qquad C_i = e^{-\alpha'_r \eta} \sin(\omega t - \alpha'_i \eta). \tag{2.25}$$

Substituting from Eqs. (2.20) into Eq. (2.15) and using the boundary conditions $(2.18)_1$ and (2.19), we get

$$u_1 = e^{-\lambda\eta} + \frac{\mathrm{Gr}\left(e^{-\lambda\eta} - e^{-\alpha\eta}\right)}{\left[\alpha^2(1 - i\omega k) - \mathrm{M} - \frac{1}{4} i\omega\right]} + \frac{\mathrm{Gm}\left(e^{-\lambda\eta} - e^{-\alpha'\eta}\right)}{\left[\alpha'^2(1 - i\omega k) - \mathrm{M} - \frac{1}{4} i\omega\right]}, \tag{2.26}$$

where

$$\lambda = \lambda_r + i\lambda_i,$$

$$\lambda_r = \left[\frac{1}{2}\left(A + \sqrt{A^2 + 4B}\right)\right]^{1/2}, \qquad \lambda_i = \frac{\omega(1 + 4\mathrm{M}k)}{8\lambda_r(1 + \omega^2 k^2)}, \tag{2.27}$$

$$A = \frac{\mathrm{M} - \frac{1}{4}\omega^2 k}{1 + \omega^2 k^2}, \qquad B = \left[\frac{\omega(1 + 4\mathrm{M}k)}{8(1 + \omega^2 k^2)}\right]^2. \tag{2.28}$$

The real and imaginary parts of $u_1 = u_{1r} + iu_{1i}$ are given by

$$u_{1r} = e^{-\lambda_r \eta} \cos \lambda_i \eta + \left(\frac{A_5 B_5 + A_6 B_6}{A_5^2 + A_6^2}\right), \tag{2.29}$$

$$u_{1i} = -e^{-\lambda_r \eta} \sin \lambda_i \eta + \left(\frac{A_5 B_6 - A_6 B_5}{A_5^2 + A_6^2}\right), \tag{2.30}$$

where

$$\begin{aligned}
A_5 &= -4\alpha_r^2\alpha_r'^2(1-k^2\omega)+\frac{\omega}{2}(\alpha_r^2+\alpha_r'^2)(1-4\mathrm{M}k)+\mathrm{M}^2-\frac{\omega^2}{16},\\
A_6 &= 8\alpha_r^2\alpha_r'^2k\omega-(\alpha_r^2+\alpha_r'^2)\left(2\mathrm{M}+\frac{1}{2}\omega^2k\right)+\frac{1}{2}\mathrm{M}\omega,\\
B_5 &= \mathrm{Gr}(A_7B_1+A_8B_2)+\mathrm{Gm}(A_9B_3+A_{10}B_4),\\
B_6 &= \mathrm{Gr}(A_7B_2-A_8B_1)+\mathrm{Gm}(A_9B_4-A_{10}B_3),\\
A_7 &= e^{-\lambda_r\eta}\cos\lambda_i\eta-e^{-\alpha_r\eta}\cos\alpha_i\eta,\\
A_8 &= e^{-\lambda_r\eta}\sin\lambda_i\eta-e^{-\alpha_r\eta}\sin\alpha_i\eta,\\
A_9 &= e^{-\lambda_r\eta}\cos\lambda_i\eta-e^{-\alpha_r'\eta}\cos\alpha_i'\eta,\\
A_{10} &= e^{-\lambda_r\eta}\sin\lambda_i\eta-e^{-\alpha_r'\eta}\sin\alpha_i'\eta,\\
B_1 &= 2\alpha_r'^2k\omega-\mathrm{M},\qquad B_2=2\alpha_r'^2-\frac{\omega}{4},\\
B_3 &= 2\alpha_r^2k\omega-\mathrm{M},\qquad B_4=2\alpha_r^2-\frac{\omega}{4}.
\end{aligned} \tag{2.31}$$

From Eq. $(2.14)_1$ the real and imaginary parts of the velocity $u=u_r+iu_i$ are given by

$$u_r = u_{1r}\cos\omega t-u_{1i}\sin\omega t, \tag{2.32}$$

$$u_i = u_{1i}\cos\omega t+u_{1r}\sin\omega t. \tag{2.33}$$

The physical quantity of primary interest is the shear stress on the plate τ', which is defined by

$$\tau=\left[\mu\frac{\partial u'}{\partial y'}-k'\frac{\partial^2u'}{\partial t'\partial y'}\right]_{y'=0}. \tag{2.34}$$

Equation (2.34) can be written in the dimensionless form by using Eqs. (2.8) and $(2.14)_1$, as

$$\tau=\frac{\tau'}{\rho'u_m'^2}=\left(\frac{\partial u}{\partial\eta}-k\frac{\partial^2u}{\partial t\partial\eta}\right)_{\eta=0}=e^{i\omega t}(1-ik\omega)\left(\frac{du_1}{d\eta}\right)_{\eta=0}. \tag{2.35}$$

From Eqs. (2.29) and (2.30) the real and imaginary parts of $\tau=\tau_r+i\tau_i$ are

$$\tau_r = Z_1(\cos\omega t+\omega k\sin\omega t)-Z_2(\sin\omega t-\omega k\cos\omega t), \tag{2.36}$$

$$\tau_i = Z_2(\cos\omega t+\omega k\sin\omega t)+Z_1(\sin\omega t-\omega k\cos\omega t), \tag{2.37}$$

where

$$Z_1=\left(\frac{A_5B_{15}+A_6B_{16}}{A_5^2+A_6^2}\right)-\lambda_r,\qquad Z_2=\left(\frac{A_5B_{16}-A_6B_{15}}{A_5^2+A_6^2}\right)-\lambda_i,$$

$$
\begin{aligned}
B_{15} &= \mathrm{Gr}\,[B_1(\alpha_r - \lambda_r) + B_2(\lambda_i - \alpha_r)] \\
&\qquad + \mathrm{Gm}\,[B_3(\alpha'_r - \lambda_r) + B_4(\lambda_i - \alpha'_r)], \\
B_{16} &= \mathrm{Gr}\,[B_2(\alpha_r - \lambda_r) - B_1(\lambda_i - \alpha_r)] \\
&\qquad + \mathrm{Gm}\,[B_4(\alpha'_r - \lambda_r) - B_3(\lambda_i - \alpha'_r)].
\end{aligned}
\tag{2.38}
$$

3. Results and discussion

In order to have a physical point of view of the problem, numerical calculations are carried out for different values of the dimensionless parameters (or numbers) of the flow. These parameters of the flow are k (viscoelastic parameter), M (magnetic number), Gr (Grashoff number), Gm (modified Grashoff number), Pr (Prandtl number), Sc (Schmidt number) and ω (frequency parameter). The vicoelastic parameter k represents both the appreciable elasticity of shape and also the viscous properties of the fluid. The Grashoff number Gr represents here the effects of the free convection currents due to the difference between the maximum temperature of the flat plate and the temperature of the free stream. The modified Grashoff number Gm represents the effects of the free convection currents due to the difference between the maximum concentration at the flat plate and the concentration of the free stream.

The results of calculations are presented in Figs. 2–8 and Table 1.

Figure 2 shows the variation of u_r with k when Pr = Sc = 2, ω = 10,

Table 1. The variation of τ_r with the parameters of the flow.

Pr = Sc = 2, Gr = Gm = 10, M = 3, $\omega t = \frac{\pi}{4}$		Pr = Sc = 2, Gr = Gm = 10, k = 0.1, $\omega t = \frac{\pi}{4}$		Pr = Sc = 2, Gr = 10, k = 0.1, M = 4, $\omega t = \frac{\pi}{4}$		Sc = 2, Gr = Gm = 10, k = 0.1, M = 4, $\omega t = \frac{\pi}{4}$		Pr = Sc = 2, Gr = Gm = 10, k = 0.1, M = 3	
k	τ_r	M	τ_r	Gm	τ_r	Pr	τ_r	ωt	τ_r
0	3.9235	0	4.9315	0	0.4411	1	3.3751	0	1.3510
0.1	3.3961	1	4.4672	10	2.9034	2	2.9034	$\pi/4$	3.3961
0.2	3.1523	2	3.9282	20	5.3658	3	2.6328	$\pi/2$	3.4518
0.3	2.9166	3	3.3961	30	7.8281	4	2.4469	$3\pi/4$	1.4855
0.4	2.6731	4	2.9034	40	10.2904	5	2.3074	π	−1.3510

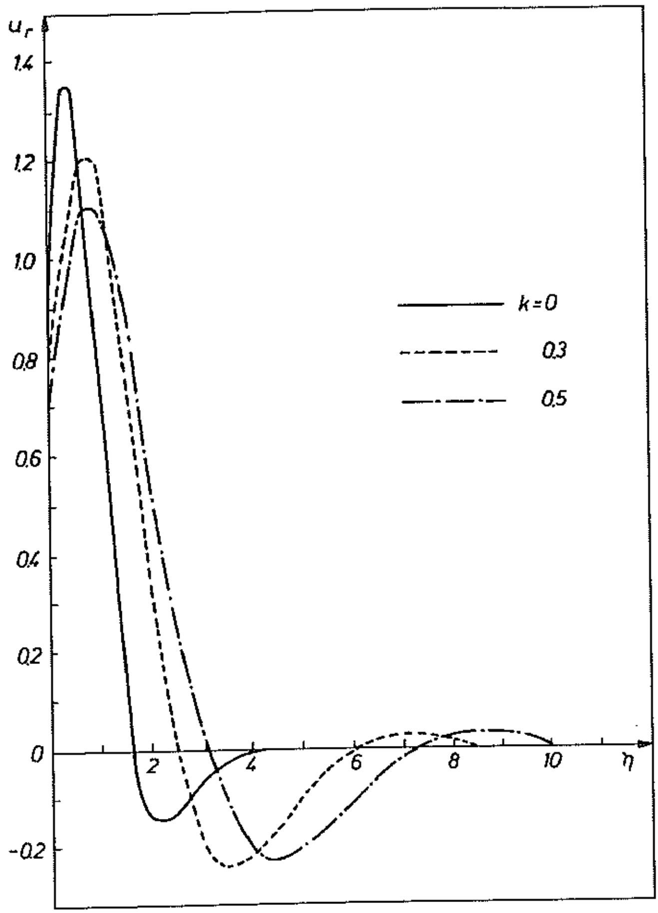

FIG. 2. Velocity field u_r against η for different values of k for $\mathrm{Pr} = \mathrm{Sc} = 2$, $\omega = 10$, $\omega t = \frac{\pi}{4}$, $\mathrm{Gr} = \mathrm{Gm} = 10$ and $\mathrm{M} = 3$.

$\omega t = \frac{\pi}{4}$, $\mathrm{Gr} = \mathrm{Gm} = 10$ and $\mathrm{M} = 3$. As shown in Fig. 2, the velocity increases near the plate with a decrease in k. This result contradicts that obtained by IBRAHIM [8] for the case when the concetration is neglected. This shows that the concentration currents increase the velocity of a Newtonian fluid ($k = 0$) more than that corresponding to a viscoelastic fluid ($k > 0$). Numerical studies show that the variation of u_r with Gr is similar to the variation of u_r with Gm given in Fig. 4. From Figs. 3 – 5 it is clear that the effects of the magnetic number M, the Grashof number Gr and the Prandtl number Pr on the velocity field for the viscoelastic fluid are similar to that for a Newtonian fluid (MISHRA and MOHAPATRA [9]). The positive value of the velocity u_r at any plane parallel to the plate and near to it

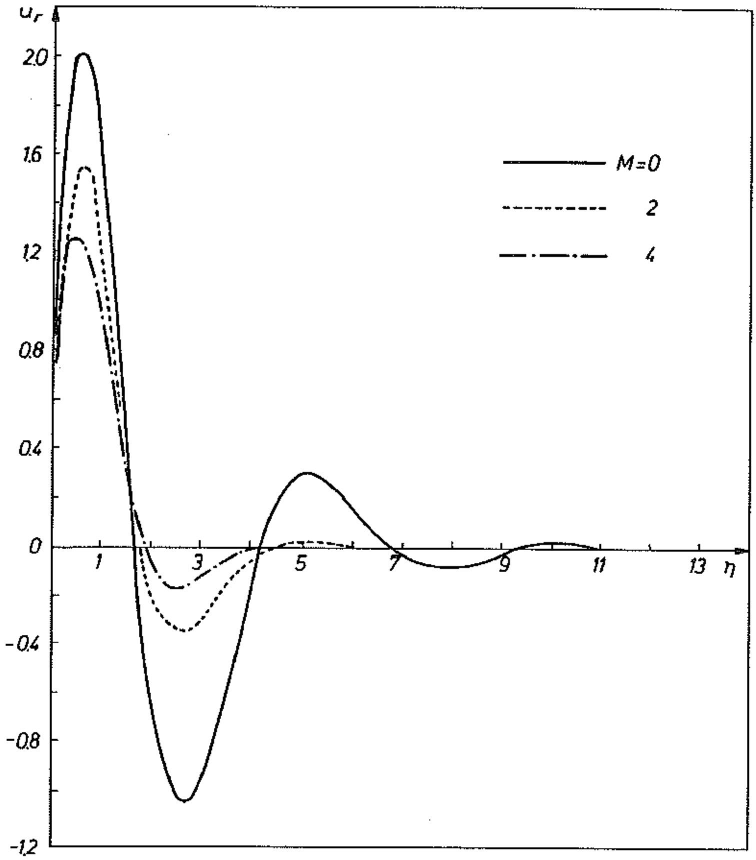

FIG. 3. Velocity field u_r against η for different values of M with Pr = Sc = 2, $\omega = 10$, $\omega t = \frac{\pi}{4}$, Gr = Gm = 10 and $k = 0.1$.

increases as either Gr or Gm increase and either M or Pr decrease, respectively. From Figs. 6 and 7, we can see that as the time t increases, both the temperature of the plate θ_r and the velocity of the plate u_r decrease. Consequently the fluid adjacent to the plate will be heated and its velocity in the direction of the plate will be greater than the velocity of the plate itself. It can also be seen that regions of high velocity correspond to regions to high temperature and concentration and *vice versa*. From Fig. 8 it can be seen that the temperature of the fluid θ_r increases as the Prandtl number Pr decreases. Figures 5 and 8 emphasize the fact that regions of high velocities correspond to regions of high temperature and concentration.

Table 1 gives the variation of the dimensionless shear stress τ_r with the parameters of the flow k, M, Gm, Pr and ωt for $\omega = 10$. Thus τ_r decreases as one of the parameters k, M and Pr increases, keeping the other two

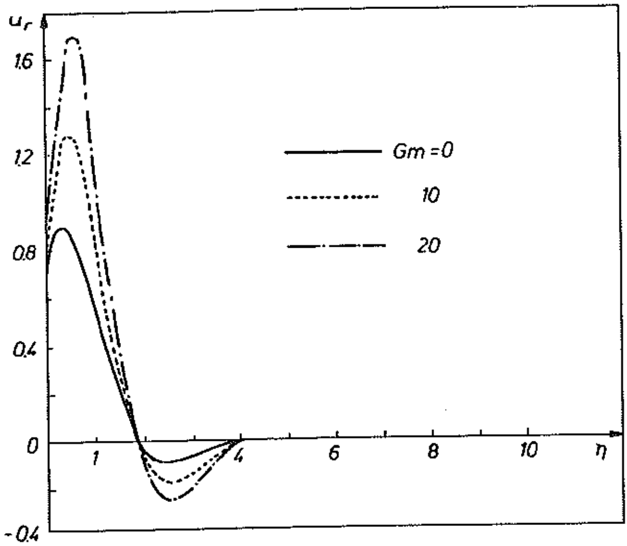

FIG. 4. Velocity field u_r against η for different values of Gm with $\mathrm{Pr} = \mathrm{Sc} = 2$, $\omega = 10$, $\omega t = \pi/4$, $\mathrm{Gr} = 10$, $\mathrm{M} = 4$ and $k = 0.1$.

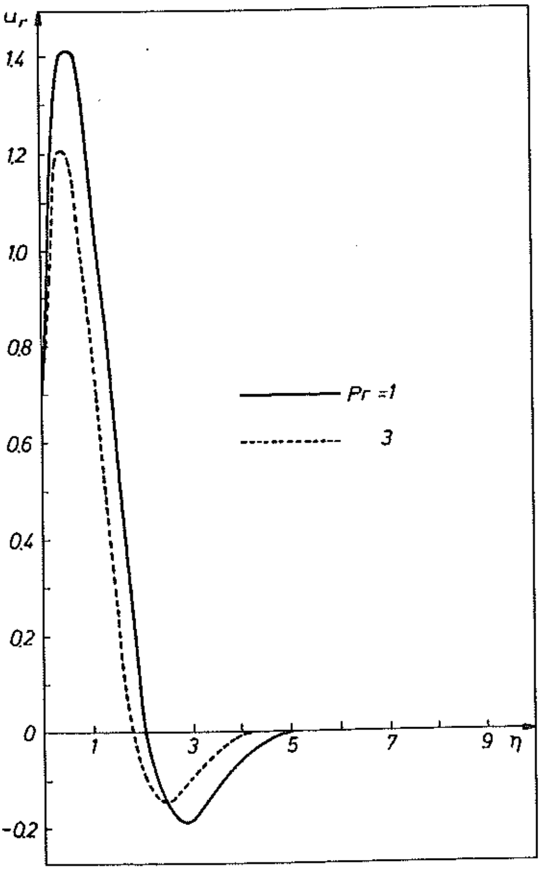

FIG. 5. Velocity field u_r against η for different values of Pr with $\mathrm{Sc} = 2$, $\omega = 10$, $\omega t = \pi/4$, $\mathrm{Gr} = \mathrm{Gm} = 10$, $\mathrm{M} = 4$ and $k = 0.1$.

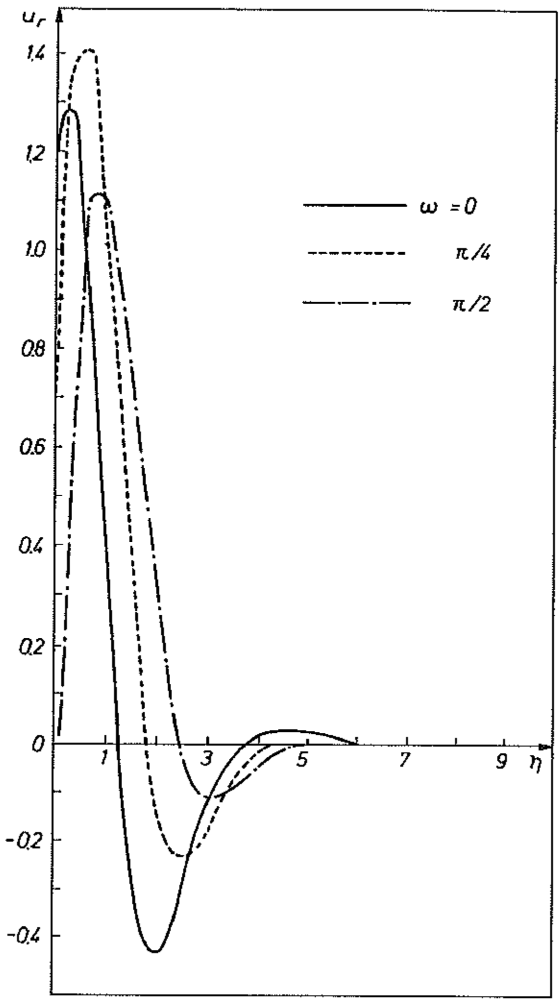

FIG. 6. Velocity field u_r against η for different values of ωt with Pr = Sc = 2, $\omega = 10$, Gr = Gm = 10, M = 3 and $k = 0.1$.

parameters constant. τ_r increases with the increase of either Gr or Gm, keping the other parameter constant. For $0 \leq \omega t \leq \frac{\pi}{2}$, τ_r increases with the increase of ωt, while for $\frac{\pi}{2} \leq \omega t \leq \pi$, τ_r decreases with the increase of ωt. The negative sign of τ_r when $\omega t = \pi$ means that the velocity of the fluid particle on the plate will be in the negative direction of the x'-axis. The magnetohydrodynamic unsteady free convection flow of a viscoelastic fluid along a vertical plate, when the concentration is not taken into account, can be derived from the above analysis by taking Gm = 0.

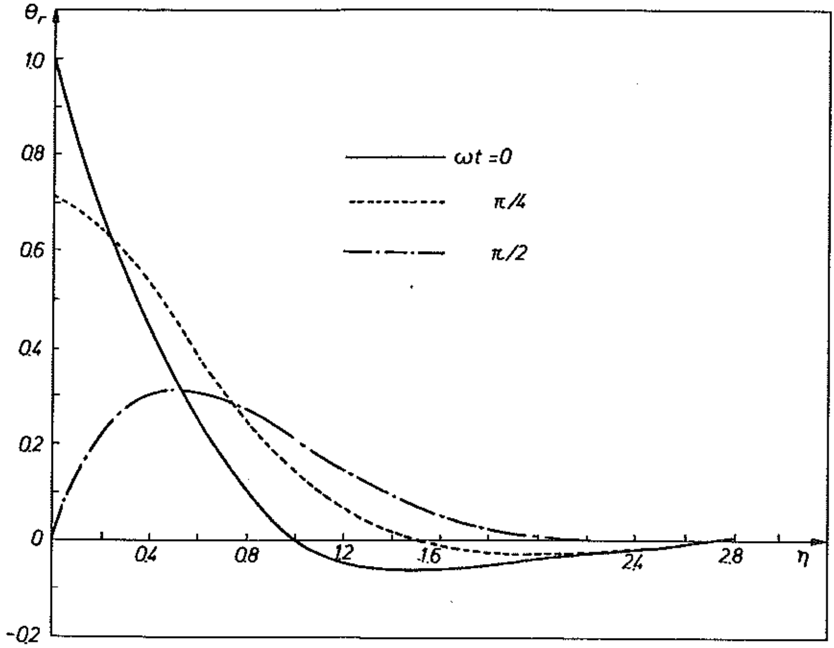

FIG. 7. The variation of θ_r with η at different time instants ωt with $\mathrm{Pr} = \mathrm{Sc} = 2$, $\omega = 10$, $\mathrm{Gr} = \mathrm{Gm} = 10$, $\mathrm{M} = 3$ and $k = 0.1$.

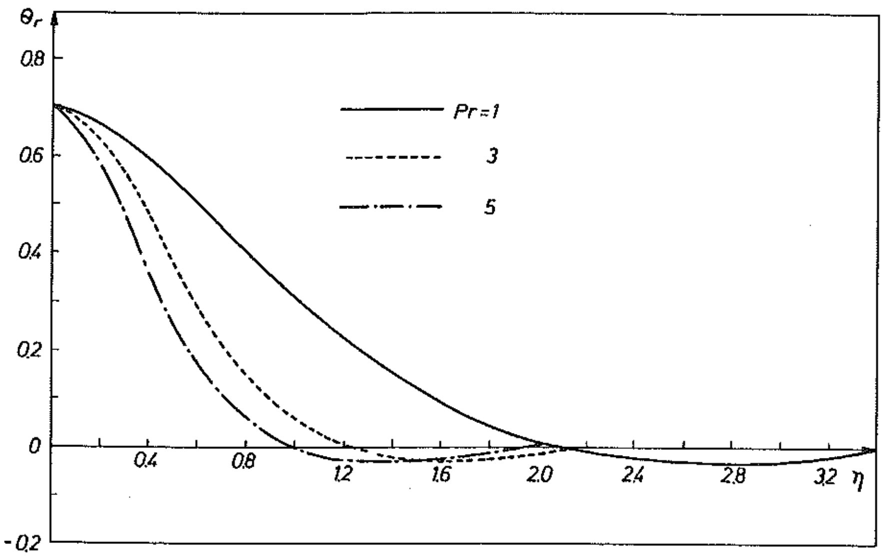

FIG. 8. The variation of θ_r with η for different values of Pr with $\mathrm{Sc} = 2$, $\omega = 10$, $\omega t = \frac{\pi}{4}$, $\mathrm{Gr} = \mathrm{Gm} = 10$, $\mathrm{M} = 4$ and $k = 0.1$.

References

1. D. Angirasa and J. Srinivasan, *Natural convection flows due to the combined buoyancy of heat and mass diffusion in a thermally stratified medium*, ASME J. Heat Transfer, **111**, 657–663, 1989.

2. D.W. Beard and K. Walters, *Elastic-viscous boundary-layer flow. Part 1. Two-dimensional flow near a stagnation point*, Proc. Cambridge Phil. Soc., **60**, 667–674, 1964.

3. S.S. Chawla, *Magnetohydrodynamic unsteady free convection*, Z. Angew. Math. Mech., **47**, 499, 1967.

4. C.C. Chen and R. Eichhorn, *Natural convection from a vertical surface to a thermally stratified fluid*, ASME J. Heat Transfer, **98**, 446–451, 1976.

5. E.M.A. Elbashbeshy and F.N. Ibrahim, *Steady free convection flow with a variable viscosity and thermal diffusivity along a vertical plate*, Accepted for publication in J. Phys. D: Appl. Phys., 1993.

6. B.Gebhart and L. Pera, *The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion*, Int. J. Heat Mass Transfer, **14**, 2025–2050, 1971.

7. A.S. Gupta, *Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field*, Appl. Scient. Pes., A9, 319, 1960.

8. F.N. Ibrahim, *Magnetohydrodynamic unsteady free convection flow of a viscoelastic fluid along a vertical plate*, Engng. Trans., **39**, 1, 111–121, 1991.

9. S.P. Mishra and P. Mohapatra, *Magnetohydrodynamic unsteady free convection flow past a vertical porous plate*, Indian J. Pure and Appl.Math., **6**, 8, 901–917, 1975.

10. S. Ostrach, *Natural convection with combined driving forces*, Physico-Chemical Hydrodynamics, **1**, 233–247, 1980.

11. V.M. Soundalgekar, *Unsteady MHD free convection flow past an infinite vertical flat plate with variable suction*, Indian J. Pure and Appl. Math., **3**, 426, 1972.

DEPARTMENT OF MATHEMATICS, FACULTY OF GIRLS
AIN SHAMS UNIVERSITY, CAIRO
and
DEPARTMENT OF MATHEMATICS, FACULTY OF SCIENCES
AIN SHAMS UNIVERSITY, CAIRO, EGYPT.

Received August 17, 1994.