ENGINEERING TRANSACTIONS • Engng. Trans. • 42, 4, 331–337, 1994
Polish Academy of Sciences • Institute of Fundamental Technological Research

EXPERIMENTAL STUDY OF THE PHENOMENON OF STABILITY LOSS OF A HORIZONTAL THIN-WALLED CYLINDRICAL SHELL LOADED BY A LIQUID

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The cylindrical shell constituting the wall of a large horizontal test tank supported at its ends and filled with water is loosing its stability. The compressive stress loading to buckling and failure of the shell, which was loaded by the weight of its liquid contents, has been compared with the critical stress as determined from the empirical formula proposed by Donnell concerning a cylindrical shell subjected to axial compression. The agrement between the two values was found to be satisfactory.

Horizontal cylindrical tanks are manufactured in large quantities and used in masses for production and storage purposes in breveries, fruit and vegetable processing plants or chemical industries. A separate group is formed by railway and automobile tanks for liquid transport. The wall thickness of tanks in present-day production is usually excessive, therefore strength tests of horizontal tanks are essential.

Some tests results of horizontal cylindrical tanks resting on saddle-type supports have been presented in [1, 2, 3] and [4]. An engineer's theory of destruction of tanks supported in this manner enabling the test result to be described with an accuracy sufficient for practical purposes has been presented in [3]. The results of experiments with models of tanks loaded by moments can be found in [5].

Tanks of 100 m^3 which are most often used, the form of which is characterized by a diameter-to-length ratio of 1:1 to 1:5, are usually supported on a two saddle-type supports. The thicknesses of the sheet iron used for the construction of such a tank is decided upon by the local stresses in the regions of reaction of the supports. Those stresses originate from the bending effects occurring along the edges of the saddle supports. In this class of tanks they are high as compared with the membrane stresses. However, in the case of much more elongated tanks, especially those more expediently supported as shown in Fig. 1, the membrane stresses may prove to be decisive for the strength of the structure.

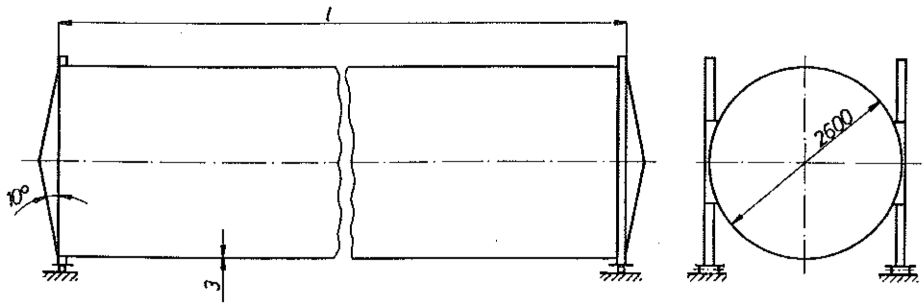

FIG. 1.

The aim of the research work reported here was to determine experimentally the critical values of the compression stresses in a horizontal thin-walled cylindrical shell subject to bending, closed with bottoms, supported at the ends and completely filled with water. If the length of a shell thus loaded is exceeded the shell, is buckled within the region of compression, thus loosing its stability.

Experiments made with a shell loaded by a liquid cannot be performed by using small models, as is the case of axial compression or bending a pure bending moment. The present author had an opportunity to perform failure test on a large horizontal thin-walled tank (Figs. 1, 2, 3) constructed especially for experimental purposes. This test was performed on a test stand which made it possible to increase gradually the length of the tank by transversally cutting the cylindrical shell and inserting short additional segments.

The original dimensions of the tank had been selected to make it approach, in the state of being completely filled with water, the critical stress as determined from the empirical formula (1) proposed by Donnell (and mentioned in [6]) for calculating the critical stress in a thin-walled subjected to axial compression. The form of that formula is as follows

$$\sigma_{\mathrm{cr}} = E\frac{0.6\dfrac{h}{r} - 10^{-7}\dfrac{r}{h}}{1 + 0.004\dfrac{E}{R_e}}, \tag{1}$$

where σ_{cr} is the critical compression, h, r – thickness and radius of the cylindrical shell, respectively, E – Young's modulus and R_e – yield stress.

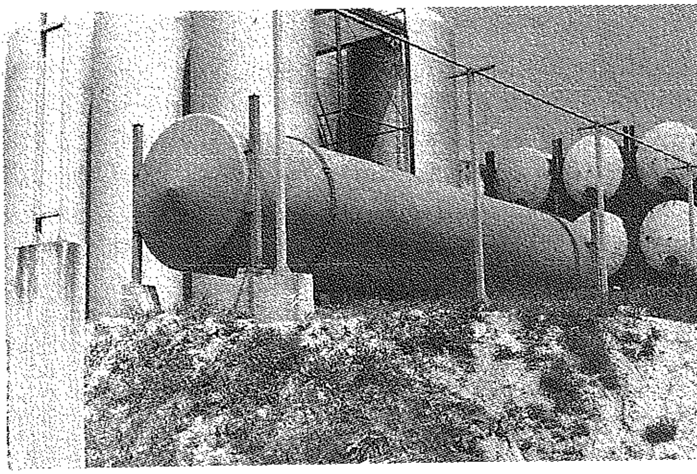

FIG. 2.

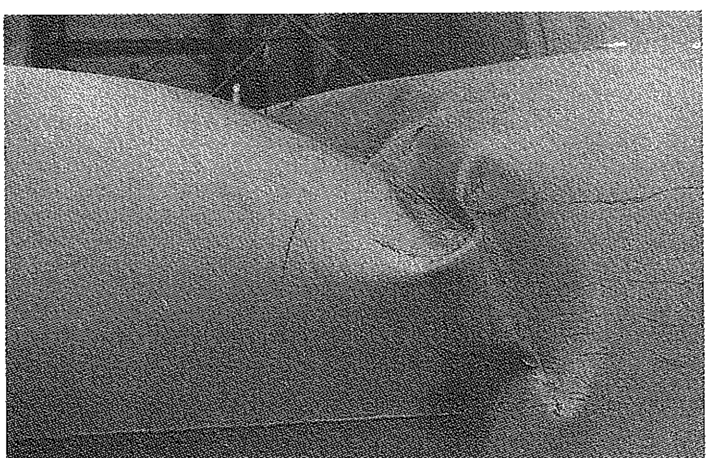

FIG. 3.

It was found that this formula may be used for determining the critical compression stress due to bending of a horizontal cylindrical shell filled with water, despite the fact that only a small part of the cross-section is subjected to high compressive stress while, the formula has been constructed for a shell uniformly compressed along the periphery, in the direction of the generating lines of the cylinder.

The test tank was made of sheet steel, the principal data of which were $R_e = 220\,\mathrm{MPa}$ and $E = 2.1 \cdot 10^5\,\mathrm{MPa}$. The diameter of the tank was $2r = 2.6\,\mathrm{m}$, and the wall thickness $h = 3\,\mathrm{mm}$. The critical stress calculated from the formula (1) was $\sigma_{\mathrm{cr}} = 58.46\,\mathrm{MPa}$. The tank which was filled with water lost its stability when its length reached the value $l = 12.5\,\mathrm{m}$.

The membrane state of stress in the shell tank was determined. Bearing in mind the possibility of small error due to the mass of the tank itself having not been taken into consideration, the internal forces were found from the following relations presented in [7]

$$
\begin{aligned}
N_x &= \frac{1}{2}\gamma r^2 + \frac{1}{8}\gamma(4x^2 - l^2 - 2r^2)\cos\varphi, \\
N_\varphi &= \gamma r^2(1 - \cos\varphi), \\
N_{x\varphi} &= -\gamma r x \sin\varphi,
\end{aligned}
\tag{2}
$$

where N_x, N_φ, $N_{x\varphi}$ – membrane forces, γ – specific weight of the liquid and l, r, x, φ – dimensions of the shell and the coordinates according to Fig. 4, respectively.

The membrane stresses calculated for some selected points of the cylindrical shell are presented in Table 1.

Table 1.

φ	0		$\pi/2$		π	
x [m]	0	±6.25	0	±6.25	0	±6.25
σ_x [MN/m²]	−62.5	1.38	2.76	2.76	68	4.15
σ_φ [MN/m²]	0	0	5.53	5.53	11.05	11.05
$\sigma_{x\varphi}$ [MN/m²]	0	0	0	26.57	0	0

The variation of the membrane stresses in some selected cross-sections of the cylindrical shell at the instant of stability loss is shown in Fig. 4. Then, the maximum compressive stress occurring in the shell was $\sigma_x = 62.5\,\mathrm{MPa}$. This value is close to the critical compression stress as determined from the formula (1) for a cylindrical shell under axial compression $\sigma_{\mathrm{cr}} = 58.46\,\mathrm{Mpa}$.

Figure 5 shows the dependence of the critical stress on the geometrical form of the cross-section of the cylindrical shell, according to the formula (1), then properties of the material being used the same as those of the material used for construction of the test tank. The compression stress $\sigma_x = 62.5\,\mathrm{MPa}$ and the proportions of the shell tested correspond to the point z in that figure.

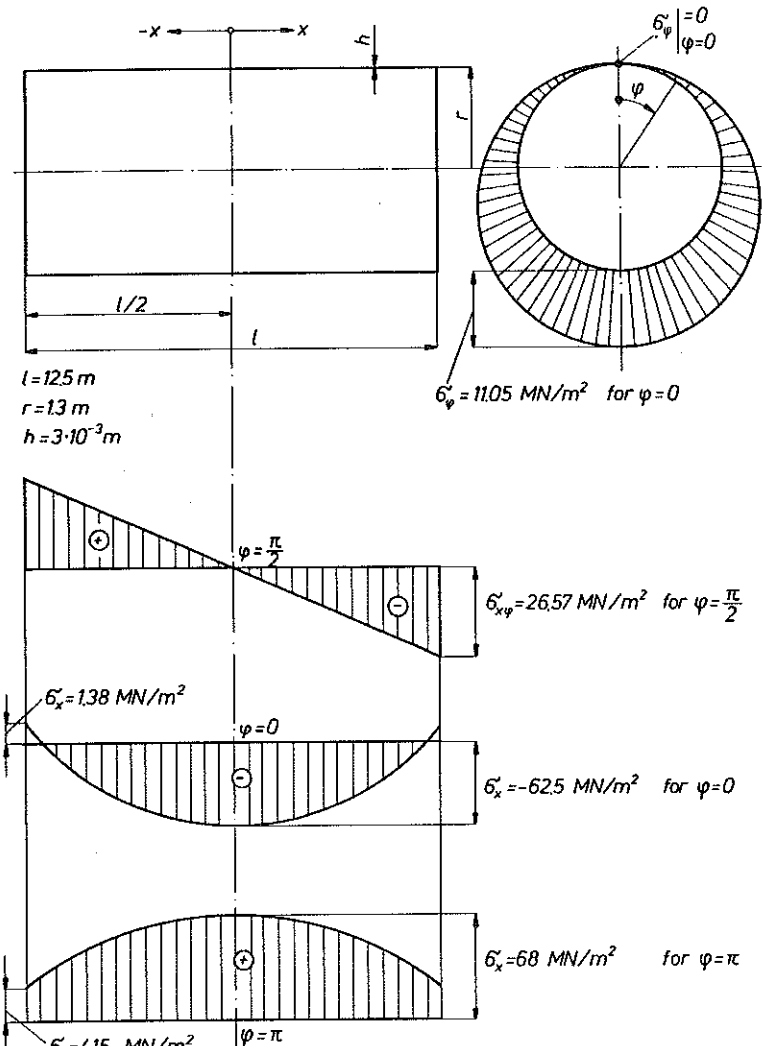

FIG. 4.

In the course of the tank loading process, once one half of the tank was filled, some small concavities appeared at the upper part of the shell, in the middle section of the tank. Similar concavities appeared near the bottoms, close to the generator line located at an angle $\varphi = 45°$. As the filling process of the tank was continued, those deformations decreased or vanished. The generator line of the lateral surfaces in the upper part of the tank ($\varphi = 0$) did not change its form during the entire filling period until the moment of stability loss of the shell was reached. When the tank was completely

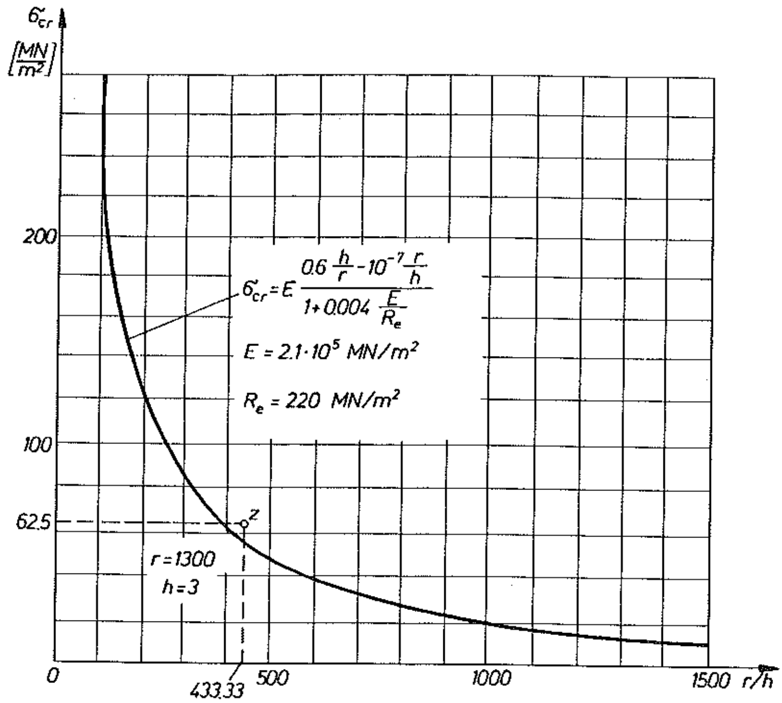

FIG. 5.

filled, what was indicated by water appearing at the outlet of the upper stub pipe, the stability of the shell was lost, which happened only after a score of seconds or so when it was no more expected, in the neighbourhood of the middle section of the tank. This process lasted until the lower part of the tank rested on the supoports located on the foundation. The tank did not loose its tightness in the course of the destruction process. It was completely filled with water, the flow of which, through the relatively small outlet of the stub pipe, was intensely throttled, therefore the reduction in capacity of the deformed tank could not be rapid. This coincidence of the stability loss and the fact of the tank being completely filled with water was, of course, accidental. The value of the critical compression stress producing buckling and stability loss of the shell depends on its geometrical accuracy, the rectilinearity of the generator lines of the shell within the region of high compression stresses being decisive. The test tank used had been manufactured with extreme accuracy for a welded shell structure made of thin sheet steel. The manufacturing deviations of the shell from a perfect cylindrical form were of the order of shell thickness. In reality, even tanks belonging to the same production series have different imperfections of form, which means operation with different safety margins.

References

1. T. Kacperski, *Investigation into the problem of load carrying capacity of tanks resting on saddle type supports* [in Polish], Rozpr. Inż., **19**, 4, 637–645, 1971.
2. T. Kacperski, *The influence of spacing of saddle-type supports on the failure mechanism of cylindrical tanks for liquids* [in Polish], A Doctor's Thesis, Warsaw University of Technology 1972.
3. T. Kacperski, *An engineer's theory of destruction or circular cylindrical tanks subjected to transverse compression by saddle-type supports* [in Polish], Rozpr. Inż., **24**, 1, 135–148, 1976.
4. T. Kacperski, *The load-carrying capacity of multistage tanks* [in Polish], Przegl. Mech., **13**, 437–440, 1976.
5. S. Danis and T. Kacperski, *Horizontal cylindrical tanks loaded by moments* [in Polish], Rozpr. Inż., **32**, 1, 87–97, 1984.
6. S.P. Timoshenko and J.M. Gere, *Theory of elastic stability* (Polish translation), Arkady, Warszawa 1963.
7. W. Flügge, *Statical analysis of shells* (Polish translation), Arkady, Warszawa 1972.

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Received April 13, 1993; new version May 11, 1994.