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AN ANALYSIS OF THE LOAD CARRYING CAPACITY AND THE STATE OF STRAIN IN FLAT ELEMENTS OF ARBITRARY THICKNESS, WITH CIRCULAR NOTCHES OF VARIOUS DEGREES OF SHARPNESS, SUBJECTED TO TENSION BY AN IMPACT LOAD

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The subject matter of the present analysis is the influence of the degree of notch sharpness on the limiting value of the thickness above which the energy of impact rupture of a structural element with a circular notch has a stable value.

NOTATION

$2h$	breadth of an element in the weakened region,
$2b$	thickness of an element,
a	radius of a circular notch,
$2c$	breadth of the stress region extended into a rigid region,
s	length of the stress region extended into a rigid region,
$\kappa = c/h$	parameter determining the relative depth of the notch,
$\mu = a/h$	parameter determining the relative sharpness of the notch,
$\eta = s/h$	parameter determining the relative optimum distance between the neighbouring notches,
$\lambda = b/h$	parameter determining the relative thickness of an element,
λ_{gr}	limiting value of the parameter λ in the case of impact loading,
λ_{gr}^{m}	limiting value of the parameter λ in the case of quasi-static loading,
$2w$	increase in breadth of the plastic zone,
L	length of the plastic zone,
V	volume of the plastic region of the specimen,
$\omega_p = w/h$	parameter determining the relative increase in breadth of plastic zone,
Ev	unit energy of rupture by impact,
P	limiting load of an element with notches,
P_0	limiting load of an element without notches,
k	yield point of a material in pure shear,
σ^*_{pl}	stress corresponding to the limiting load of an element with notches,
σ^*_m	stress corresponding to the tensile strength of an element with notches,
f	coefficient of increase in load carrying capacity of elements with notches,
f_m	coefficient of increase in destructive loading of elements with notches.

1. INTRODUCTION

The design work of a machine element comes often up against the difficulty of determining its optimum dimensions. In such a case the methods of load carrying capacity described in [1] are very useful for determining or estimating that quantity for a structural element of complicated shape.

To work out a numerical method it is often necessary to introduce some simplifications. In the theory of load carrying capacity it is assumed that the load acting on an element is quasi-static. In reality the load is often dynamic, therefore the usefulness of a theoretical solution can be confirmed by experimental means only. The results of the experiments which have as yet been conducted show good agreement between the theoretical and the real behaviour of structural materials. This concerns static as well as impact or fatigue loads. The results of tests of materials with good plastic properties, subjected to static loads, can be found in the monograph [1] and also in [2] to [11]. Problems of impact loads on such materials are studied in [15] to [17], and fatigue tests are reported in [13] and [14].

On the grounds of the results of those works it may be stated that the solutions obtained by methods of limit load capacity (quasi-static loads being assumed) are also optimum solutions in the case of dynamic loads.

Solutions by the methods of load capacity concern usually three cases, which are those of plane strain, plane stress and axial symmetry. According to the theory, the state of strain is plane, if the thickness $2b$ of an element is infinite as compared with the dimension $2h$ (Fig. 2). As regards the practice, such a state occurs if b is several times greater than h.

The limiting value of the parameter $\lambda_{gr} = b/h$, above which the state of strain becomes plane, depends on a number of factors such as the geometry and the degree of sharpness of the notch, the quality of the material and the type of the load, therefore it is essential that the limiting value of that parameter should be accurately determined before the investigation of the problem of plane strain of an element is approached. The results quoted in [1] to [3] and [8] show considerable dispersion of the estimated limiting values of the parameters λ_{gr}. The reasons are explained in [9] and [10]. In the case of flat bars weakened by rectangular notches [10, 17], the limiting value of the parameter λ_{gr} is contained between 3 and 6 for quasi-static loads [10], and between 2.5 and 4 for impact loads [17].

In the present paper an attempt will be made to determine the relation between the limiting value of the parameter λ_{gr} and the degree of sharpness of the notch $\mu = a/h$ for two aluminium alloys, that is PA2N and PA4N. The analysis of the experimental results will also enable us to determine the conditions, under which theoretical solutions of problems of the load carrying capacity for quasi-static loads are optimum also for impact loading.

2. THEORETICAL ANALYSIS

2.1. Solution of the method of slip bands in the case of plane strain

In flat elements weakened on both sides by symmetric notches there occurs, under the action of a tension load, a plastic region in the neighbourhood of a notch. A theoretical solution to the problem of stress distribution in that region has been obtained, for the rigid-plastic model of a solid, assuming that the bar is in a plane state of strain. From the theoretical point of view such a state occurs, as mentioned above, if the parameter $\lambda = b/h$ tends to infinity. In real structures there occur elements for which the ratio b/h is finite. This gives rise to the question as to whether a theoretical solution may be of practical use and how accurate it is. Many scientific works show that theoretical solutions obtained in the case of plane state of strain are confirmed experimentally for flat specimens if the dimension b is several times greater than h. The determination of the limiting load for flat elements weakened by symmetric notches of any configuration is based on the assumption that the narrowest section becomes entirely plastic. In the plastic state the stress distribution in the neighbourhood of the edge depends only on the conditions in that region. The boundary conditions are defined, because the contour of the notch constitutes a free edge. By solving the Cauchy problem we can construct a symmetric network of slip bands on both sides and meeting at the axis of the bar. This solution can be obtained by the Hill method [18], which can enable us to determine numerically the stress distribution and the limit loads for a notch of any form [1]. If this form is not complicated, the limit load can be evaluated by means of certain mathematical formulae.

For round notches (Fig. 2), the formula expressing the limit force is as follows:

$$P = 8kbh\left(1+\frac{a}{h}\right)\ln\left(1+\frac{h}{a}\right) \qquad \text{for} \quad \frac{a}{h} > 0.2624,$$

and

$$P = 8kbh\left(1+\frac{\pi}{2}\right)-\frac{a}{h}\left(e^{\pi/2}-1-\frac{\pi}{2}\right) \qquad \text{for} \quad \frac{a}{h} \leq 0.2624.$$

For a flat bar $2h$ in breadth and $2b$ in thickness, without a notch, this force amounts to

$$P_0 = 8kbh,$$

therefore the coefficient of increase in the load carrying capacity is

$$f = \frac{P}{P_0} = \left(1 + \frac{a}{h}\right) \ln \left(1 + \frac{h}{a}\right) \qquad \text{for} \quad \frac{a}{h} > 0.2624,$$

and

$$f = \left(1 + \frac{\pi}{2}\right) - \frac{a}{h}\left(e^{\pi/2} - 1 - \frac{\pi}{2}\right) \qquad \text{for} \quad \frac{a}{h} \leq 0.2624.$$

If the ratio a/h decreases and, in the extreme case, if a tends to zero, we are concerned with a cut for which the coefficient f of increase in the limit load reaches its maximum value of $f_{\max} = 2.571$, which is also its highest possible value under the conditions of plane strain [1]. A diagram of the coefficient f as a function of the parameter $\mu = a/h$ is shown in Fig. 1. It is seen that f exceeds always the value 1. In the extreme case it exceeds 2.5. This is connected with a three-axial state of stress occurring at internal points of the narrowest section.

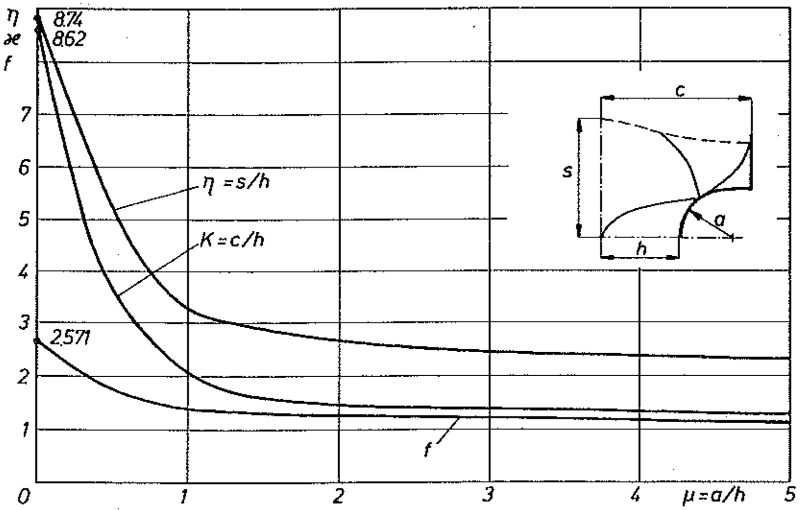

FIG. 1. Diagrams of the parameters f, $\kappa = c/h$ and $\eta = s/h$ as functions of the sharpness of the notch $\mu = a/h$.

2.2. *Extension of the slip-band network into the rigid region*

An essential parameter which must be taken into consideration, if we want to estimate the load carrying capacity of an element weakened by cuts, is the depth of the notch; its measure is the ratio of the breadth $2c$ of the grip part of the specimen to the minimum distance $2h$ between the notches. The theoretical value (as obtained from the formulae of Sec. 2.1 or the diagram $f = f(\mu)$ of the limit load (Fig. 1)) may be assumed for the analysis, provided that the yield point has not been exceeded at any point in the neighbourhood of the notch (except the plastic region). If this condition should not be satisfied, this would mean that the material is weaker at that point than it is within the region of the notch, therefore the load capacity at the point would be identical with the load carrying capacity of the element as a whole. The safe regions outside the notch are determined by constructing an extension of slip bands into the adjacent rigid regions. Some variants of extension of the slip-band networks constructed by the method proposed by J.F.W. BISHOP [19] can be found in [1]. From those solutions we obtain, for as prescribed sharpness $\mu = a/h$, two characteristic dimensions of the notch, namely $2c$ and $2s$. On solving this problem for various values of the parameter $\mu = a/h$, we obtain a number of limiting values of the dimensions $2c$ and $2s$. In the form of the parameters $\kappa = c/h$ and $\eta = s/h$ those values are represented, as functions of sharpness $\mu = a/h$ of the notch, in Fig. 1. If the contour of the extension region of slip lines, which is determined by the dimensions $2c$ and $2s$, is contained within the contour of the real notch, this means that the limit load may be determined by theoretical means for a prescribed value of sharpness $\mu = a/h$ (making use of the diagram of $f = f(\mu)$). If, however, the breadth $2c$ of the notch or the spacing $2s$ (in the case of an element with more notches) is below the value resulting from the network mentioned above, the limit load can be defined by considering the lower value of sharpness (higher value of the parameter $\mu = a/h$), for which the contour of the network lies within the actual region of the notch.

3. EXPERIMENTAL INVESTIGATION

3.1. *The investigation procedure*

The research work was conducted for two different aluminium alloys, PA2N and PA4N (AlMg1SiMn) of different plastic properties.

Four series of tests were performed for the PA2N alloy. The sharpness parameter $\mu = a/h$ of the notch was constant for each series and amounted to an average of 0.465, 1.125, 1.413 and 3.04. The thickness $2b$ of the element was a variable parameter, the remaining dimensions of the specimens of all the series being the same for all the series.

As regards the PA4N alloy, three series of tests were made, the average of the sharpness parameter of the notch being 0.508, 1.45 and 3.04. The form of the notch and the dimension of the specimens are represented in Fig. 2.

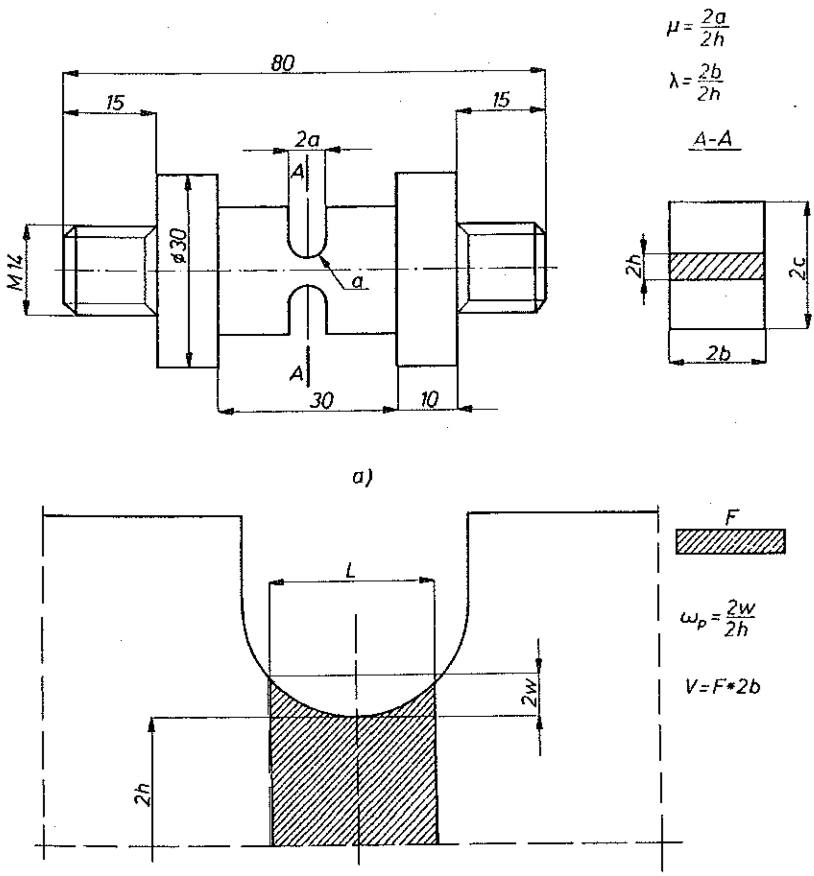

FIG. 2. The form of the notch and the dimensions of notched specimens.

The tests were performed using a Charpy pendulum machine manufactured by the VEB Werkstoffprüfmaschinen, Leipzig (Germany). The normal applicaton of that device is for impact testing of structural materials. With a small modification, by introducing elongated supports and constructing a complementary grip element for the test specimen and a transverse element to be mounted at the other end of the specimen, the device could be used to perform impact tension tests. The parameter to be measured was the energy of rupture of the specimen. Detailed description of the method of impact tests can be found in [20] and [21].

3.2. The results of the tests of specimens of PA2N and PA4N alloys

The tests reported here were conducted in four series for PA2N, and in three series for PA4N alloy. The differences between the particular series were those of sharpness of the notches expressed by the parameter $\mu = a/h$ (Fig. 1). Thus, specimens belonging to the same series had identical notches, with identical radius a and different thicknesses $2b$.

For static tests, the quantity to be measured is the force as referred to the minimum section of the specimen. As regards the impact tests, the measured quantity is the energy. In the course of the breaking process, this energy is dispersed over a certain volume of the specimen. Part of this energy (negligibly small) is spent on producing elastic stresses in the region of the grip of the specimen, the remaining part being lost at large strains in the plastic zone adjacent to the notch. The volume of reference for the destruction energy was evaluated, for each specimen, on the basis of measurement of the notch geometry. In view of the differences in the degree of sharpness of the notches (parameter μ) in particular series, and small differences in the remaining dimensions, an additional parameter, $\omega_p = 2w/2h$, was introduced. This parameter, for which the same breadth of the plastic zone is assumed (for each particular series), makes it possible to compare the volumes of specimens used in different series in which the destruction energy is dispersed. The way of determining that parameter is illustrated in Fig. 2a. As regards the PA2N alloy, the maximum value of the parameter ω_p for the series of specimens with $\mu = 0.465$ is $\omega_p = 0.465$ (for a higher value the region of the notch is exceeded). Preserving the breadth of the plastic zone, the same value of ω_p was also assumed for the remaining three series. For the material PA4N, the maximum value of the parameter ω_p for the series of specimens with $\mu_{\text{śr}} = 508$ is $\omega_p = 0.508$. Following the same principle as that used for PA2N, the value $\omega_p = 0.508$ was also assumed for the remaining two series.

It is worthwhile to emphasize that the values of the breadth of the plastic zone assumed for evaluation of the parameter ω_p, do not differ essentially from the breadth of the zones observed in the specimens broken by impact.

The variation of the energy of destruction of specimens made of PA2N aluminium alloy as a function of thickness of the specimen ($\lambda = b/h$) is illustrated, for particular series differing by the sharpness of the notch, in Fig. 3. The analogous results for specimens made of PA4N aluminium alloy are shown in Fig. 4.

The variation of the unit energy of destruction as a function of the thickness parameter λ was used to determine the limiting value λ^u_{gr} for which

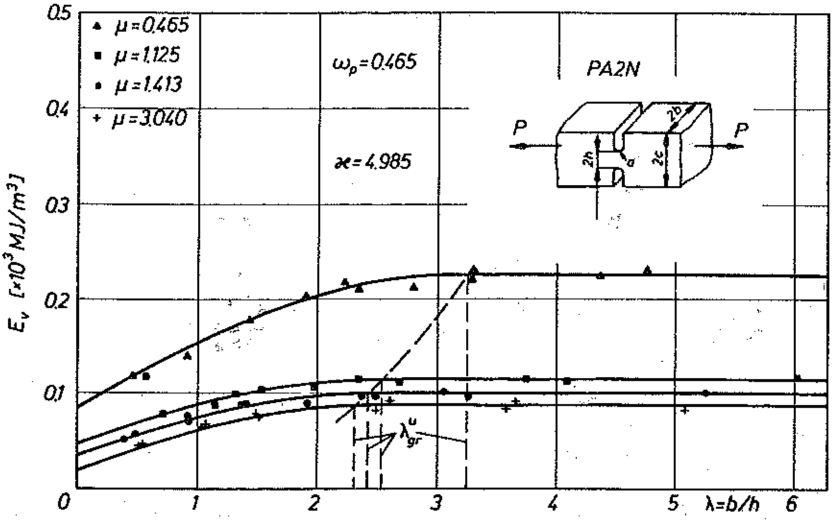

FIG. 3. Diagrams of rupture energy E_v of specimens with notches of different degrees of sharpness $\mu = a/h$ subjected to impact tension, as a function of thickness $\lambda = b/h$ for PA2N aluminium alloy.

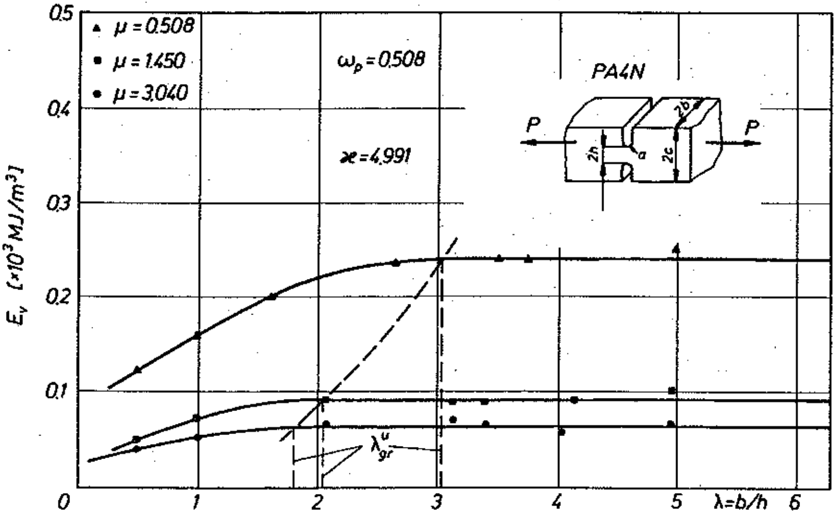

FIG. 4. Diagrams of destruction energy E_v of specimens with notches of different degrees of sharpness $\mu = a/h$ subjected to impact tension, as a function of thickness $\lambda = b/h$ for PA4N aluminium alloy.

this energy becomes stable. The variation of the limiting values λ^u_{gr} as a function of the sharpness parameter μ of the notch is represented for PA2N and PA4n in Figs. 5 and 8, respectively. Those figures show also (for the purpose of comparison) diagrams of the limiting parameters λ^m_{gr} obtained

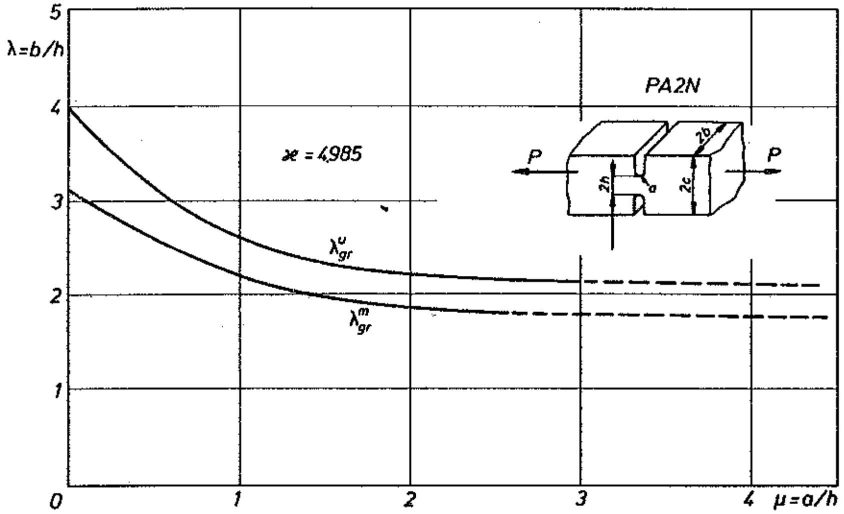

FIG. 5. Diagrams of the limiting thickness λ^u_{gr} and λ^m_{gr} for impact and quasi-static rupture tests, respectively, of specimens of PA2N aluminium alloy as a function of the degree of sharpness $\mu = a/h$ of the notch.

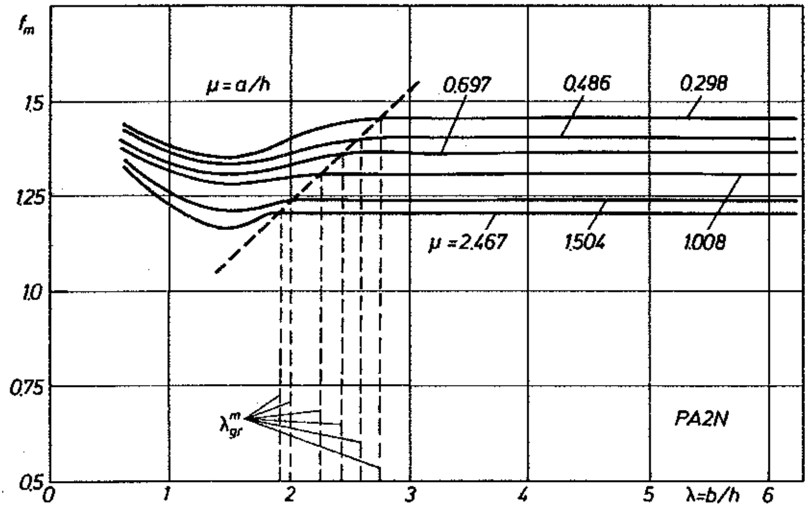

FIG. 6. Diagrams of the parameter f_m as a function of the thickness $\lambda = b/h$ for specimens with notches of various degrees of sharpness $\mu = a/h$, made of PA2N aluminium alloy.

as a result of static tension tests and taken from [9] (Figs. 6 and 7). The parameter λ^m_{gr} has been determined for breaking stresses σ^*_m (Fig. 7).

By analysing the diagrams of the parameters λ^u_{gr} and λ^m_{gr} (Fig. 5 for PA2N and Fig. 8 for PA4N) we can observe similarity of their form, independently of the kind of the material, despite the fact that the diagrams have been

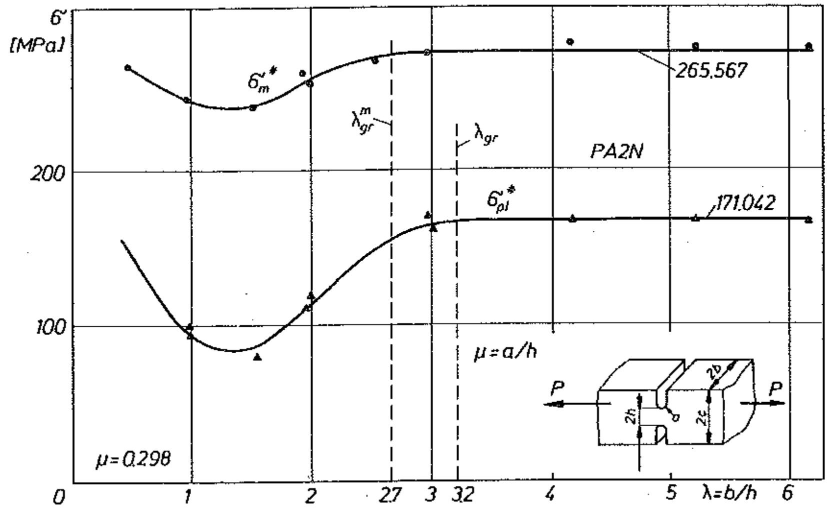

FIG. 7. Diagram of tensile strength σ_m^* of specimens with notches and diagram of limiting stress σ_{pl}^* as a function of thickness $\lambda = b/h$ for the PA2N aluminium alloy.

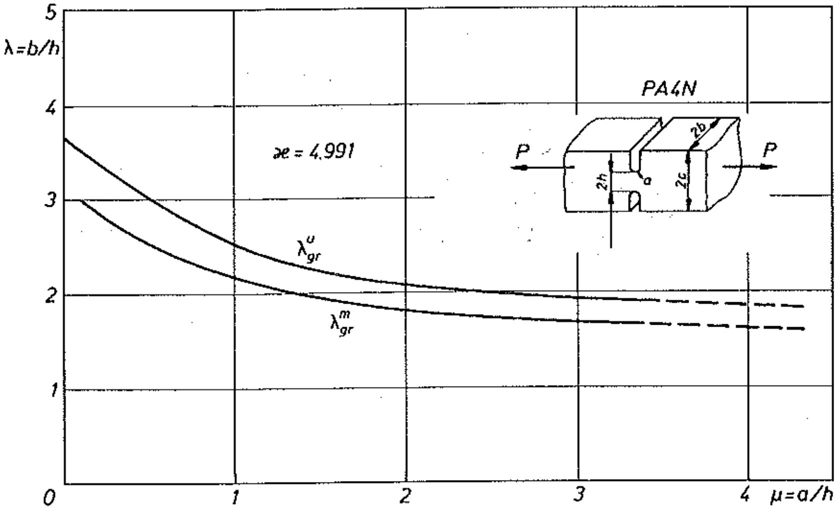

FIG. 8. Diagrams of limit thickness λ_{gr}^u and λ_{gr}^m for impact and quasi-static rupture tests, respectively, of specimens made of PA4N aluminium alloy as a function of sharpness $\mu = a/h$ of the notch.

obtained for different types of loading. The parameter λ_{gr}^m was obtained by uni-axial tension tests of specimens under quasi-static loading, up the rupture, and λ_{gr}^u by impact tests.

Confrontation of the diagrams for PA2N (Fig. 5) shows that the limit thickness for impact loading (as determined by the parameter λ_{gr}^u) is by about 16% greater than the analogous thickness determined for quasi-static loading (parameter λ^m_{gr}). A similar result was obtained for PA4N (Fig. 8). It follows that, if we want to transfer the results of optimization of the parameter λ^m_{gr} obtained by quasi-static tension tests to the conditions of impact rupture, the values of the parameter λ^m_{gr} should be increased by about 16%, thus yielding λ^u_{gr} ($\lambda^u_{\mathrm{gr}} = 1.16\lambda^m_{\mathrm{gr}}$).

It should be emphasized that those results are contrary to those which have been obtained for elements with rectangular notches [17]. For, it was shown in [17] that, in the case of plane elements with rectangular notches, the limit thickness λ^m_{gr} as determined for quasi-static destructive tension loading have, for a given material, higher values (by about 16%) than the analogous thickness λ^u_{gr} as determined under conditions of impact tension.

The above results show clearly that the role played by the form of the notch is essential for the establishment of a plane state of strain in a structural element of finite thickness, subjected to a quasi-static or impact tension load.

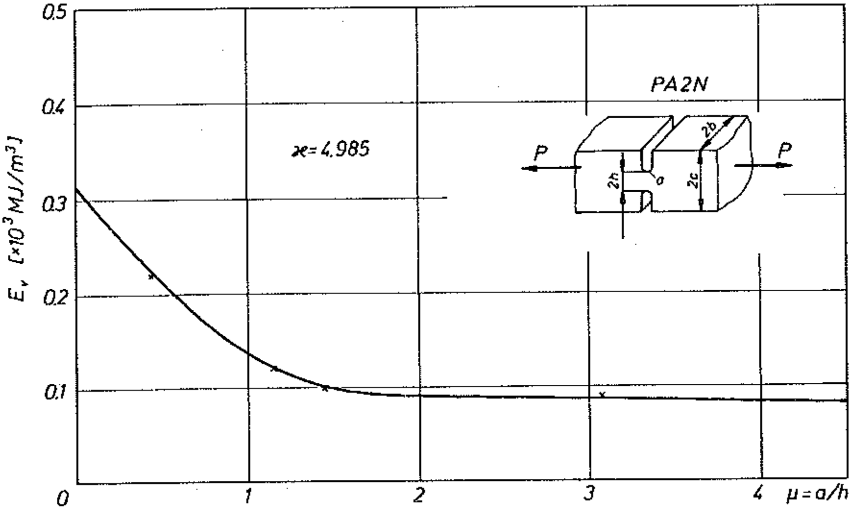

FIG. 9. Diagram of maximum unit energy of rupture E_v as a function of the degree of sharpness $\mu = a/h$ of the notches for specimens of PA2N aluminium alloy subjected to impact tension.

Figures 9 and 10 represent diagrams of maximum (stabilized) unit destruction energy as a function of the parameter $\mu = a/h$ for the materials PA2N and PA4N. These values of energy were determined from the diagrams in Figs. 3 and 4. On the basis of the diagrams in Figs. 9 and 10 it can be found that the influence of the degree of sharpness of the notch on the value of the unit destruction energy under impact tension is essential. The value

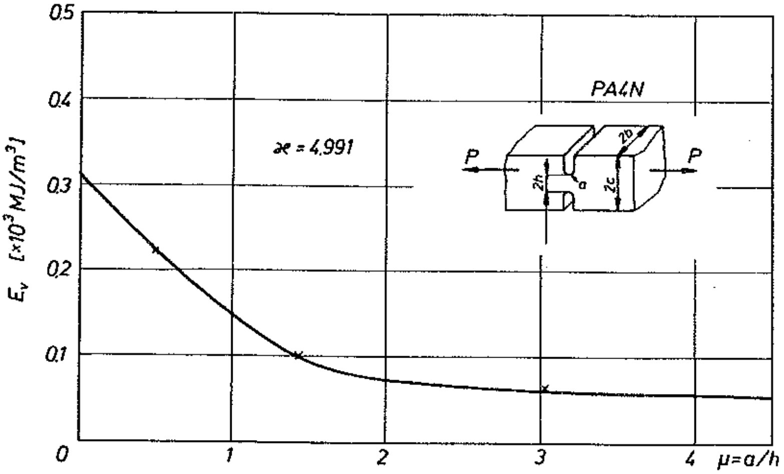

FIG. 10. Diagram of maximum unit energy of rupture E_v as a function of sharpness $\mu = a/h$ of the notches for specimens of PA4N aluminium alloy subjected to impact tension.

of this energy is maximum for a very small radius a (sharp cuts). If the radius a increases, the unit energy of impact rupture decreases very rapidly. For less sharp notches, for which $\mu \geq 2$, the influence of degree of the notch sharpness on the value of the unit energy of destruction as a result of impact tension dies out gradually, so that the kind of the material of the element tested is of minor importance.

4. INFERENCES

The unit energy of rupture under impact tension of a specimen with circular notches increases with increasing thickness of the specimen until the thickness of the specimen reaches its limiting value λ_{gr}^{u}.

The value of the stabilized unit energy of rupture depends on the thickness of the notch, and is higher for sharper notches (that is for lower values of the parameter μ).

The limiting value of the thickness parameter λ_{gr}^{u} depends on the degree of sharpness of the notch and increases, similarly to the energy, with increasing sharpness (decreasing μ). The value of the parameter λ_{gr}^{u} is maximum in the case of sharp notches. Depending on the kind of the material, the expected maximum value of the limit thickness parameter λ_{gr}^{u} is about 3.8 or 4.

In the case of destructive loads the variation of the limiting thickness λ_{gr}^{u} or λ_{gr}^{m} as a function of the notch sharpness μ is similar for the impact and quasi-static tension.

By contrast with bars having rectangular notches, the limiting values of the parameter $\lambda^{u}_{\mathrm{gr}}$ for elements with rounded notches are, in the case of a uni-axial impact load, by about 16% higher than the analogous values $\lambda^{m}_{\mathrm{gr}}$ of the thickness of the same elements in the case of quasi-static destruction as a result of a uni-axial tension test.

The geometry of the notch plays an essential role as regards the conditions for a plane state of strain to appear in an element of complex form subjected to quasi-static or impact tension.

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