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INTERACTIVE ELASTIC BUCKLING OF THIN-WALLED OPEN ORTHOTROPIC BEAM-COLUMNS

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The present paper is a continuation of paper [8]. It deals with an analysis of global and local stabilities and with the investigation of the initial post-buckling equilibrium paths of elastic thin-walled open orthotropic beam-columns. The channel beam-columns, simply supported at the ends, are subject to axial compression, eccentric compression or pure bending. The analytical method of analysis was presented in paper [8]. Numerical calculations are restricted to the nonlinear first order approximation [2, 4]. The principal goal of numerical analysis is to investigate the effect of the wall orthotropy factor upon the critical state (upon the critical stress values and the global and local buckling modes), and upon the initial behaviour in the post-buckling state.

NOTATIONS

The notation used in the present paper is the same as in paper [8]. The most important symbols are as follows:

b_i width of the i-th wall of column,
E_{xi}, E_{yi} Young's moduli of i-th wall along the x and y axes, respectively,
G_i modulus of non-dilatational strain of i-th wall,
h_i thickness of the i-th wall of the column,
l length of the column,
m number of axial half-waves of n-th mode,
$M_{xi}^{(n)}, M_{yi}^{(n)}, M_{xyi}^{(n)}$ bending moment resultants for the i-th wall referring to the n-th buckling mode,
n number of mode,
$N_{xi}^{(n)}, N_{yi}^{(n)}, N_{xyi}^{(n)}$ in-plane stress resultants for the i-th wall referring to the n-th mode in the first approximation,
$u_i^{(n)}, v_i^{(n)}, w_i^{(n)}$ buckling displacement components of middle surface of the i-th wall referring to the n-th buckling mode,
$\beta_i = E_{xi}/E_{yi}$ orthotropy factor of the i-th wall,
ν_{xyi}, ν_{yxi} Poisson's ratio of the i-th wall; the first index indicates transverse direction and the second shows the direction of load,
ξ_n amplitude of n-th buckling mode,
$\overline{\xi}_n$ imperfection amplitude corresponding to ξ_n,
$\sigma_n^* = \sigma_n 10^3/E_{x1}$ dimensionless critical stress of the n-th mode,
σ_s^* limit dimensionless stress for imperfect column (load carrying capacity).

1. Introduction

When dealing with open beam-columns we encounter many different modes of global (flexural, torsional, torsional-flexural and lateral) and local bucklings. It is also known that in such structures the above buckling modes interact with each other, creating a so-called coupled buckling. The problems of the coupled buckling of thin-walled open isotropic beam-columns were discussed in [1, 5–7, 9–14].

The subject of the present paper is the stability of thin-walled beam-columns built of an orthotropic material and their behaviour in the initial elastic post-buckling state. The analysis of a full strain tensor for thin walls [8] and of a plane model of structure enabled us to determine various buckling modes and the related critical load values.

2. Theoretical and numerical solution

The solution of the problem under discussion was presented in [8]. The same article describes capabilities of a computer program prepared. This program can be easily applied in a computer-aided system, CAD/CAM.

Since the analysed cross-sections of beam-columns have a single axis of symmetry, in the present paper, unlike in paper [8] only a two-mode approach is considered.

3. Analytical results

Detailed numerical calculations are carried out for thin-walled channel beam-column (Fig. 1) subject to uniform compression and to pure bending in the plane of the web.

Geometrical dimensions of columns under discussion are assumed to be the same as in papers [1, 5–7]:

$$l = 390\ [\mathrm{mm}], \qquad b_1 = 25\ [\mathrm{mm}], \qquad b_2 = 50\ [\mathrm{mm}], \qquad h_1 = h_2 = 1\ [\mathrm{mm}].$$

All walls are assumed to be made of the same orthotropic material, the principal axes of orthotropy being parallel to the wall edges.

Values of material constants for column walls (13 different cases) are taken from paper [3]. These constants are listed in Table 1.

The main purpose of numerical calculations is to analyse how the wall orthotropy factor, $\beta = \beta_i = E_{xi}/E_{yi} = E_x/E_y$, influences the global and

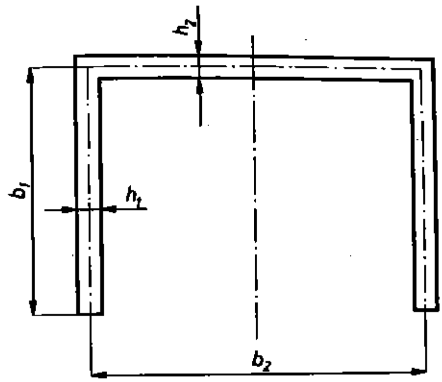

FIG. 1. Open cross-section considered.

Table 1. Elastic constants for various cases of composite beam-columns.

Spec. no.	$\beta = E_x/E_y$	ν_{xy}	ν_{yx}	G/E_x
1	0.0728	0.02184	0.3	0.4065
2	0.1315	0.03945	0.3	0.4091
3	0.3031	0.09093	0.3	0.4002
4	0.5064	0.15192	0.3	0.3937
5	0.7041	0.21123	0.3	0.4009
6	0.8358	0.25074	0.3	0.3882
7	1.0000	0.3	0.3	0.3846
8	1.1964	0.3	0.25074	0.3245
9	1.4202	0.3	0.21123	0.2823
10	1.9747	0.3	0.15192	0.1994
11	3.2992	0.3	0.09093	0.1213
12	7.6045	0.3	0.03945	0.0538
13	13.7362	0.3	0.02184	0.0296

local critical stress values, buckling modes and load carrying capacity as determined in the first order approximation.

Figure 2 presents the lowest values of critical stress σ_n^* in cases of global and local buckling of the channel column subject to axial compression and bending in the plane of the web, plotted as a function of a wall orthotropy factor, β. Index n assumes the following values: 1 – for the first or the second global buckling mode, 2 – for the lowest local buckling mode.

The presented relations show that the values of global dimensionless critical stresses in case of global buckling, $\sigma_1^* = \sigma_1 10^3/E_x$ of a column under axial compression (curve *1*) at $\beta \leq 1$, are practically independent of factor β or, to be more precise, are only slightly dependent on the value of modulus

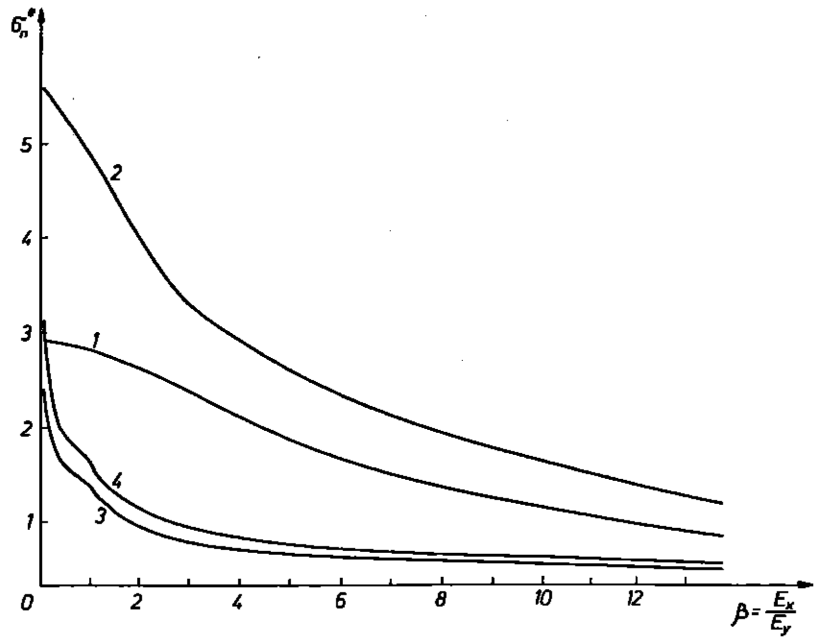

FIG. 2. Dimensionless stress σ_n^* versus the orthotropy factor β. Curves: *1* – global buckling for the uniformly compressed column, *2* – lateral buckling for the beam-column in bending, *3* – local buckling for the compressed column, *4* – local buckling for the beam-column in bending.

E_y in circumferential direction. It is so because the dimensionless critical stresses, as measured along the vertical axis, $\sigma_1^* = \sigma_1\, 10^3/E_x$ are a function of modulus E_x along the axis of the column. For higher β values ($\beta > 1$), the global critical stress value of the column under axial compression decreases; at $\beta = 13.736$ it is nearly five times less than for the column made of an isotropic material ($\beta = 1$). It should be kept in mind, however, that dimensional critical stresses vary proportionally to the value of modulus E_x since $\sigma_n = 0.001\sigma_n^* E_x$. The numerical analysis carried out shows that the variation of coefficient β changes not only the global critical stress values of the channel under axial compression but also the buckling modes. The analysis of the influence of β factor upon the critical stress values and global buckling modes is possible due to the fact that the plate model (and not the beam-bar model) of the column is used in the considerations.

Figures 3 present two global and local buckling modes of the orthotropic channel (wall orthotropy factor $\beta = 0.0728$, 1.0 and 13.736, respectively). The first global mode concerns the lowest global load value, and the second one – the greater global load.

As can be seen in Figs. 3, in the channel with $\beta = 0.0728$ and $\beta = 1.0$

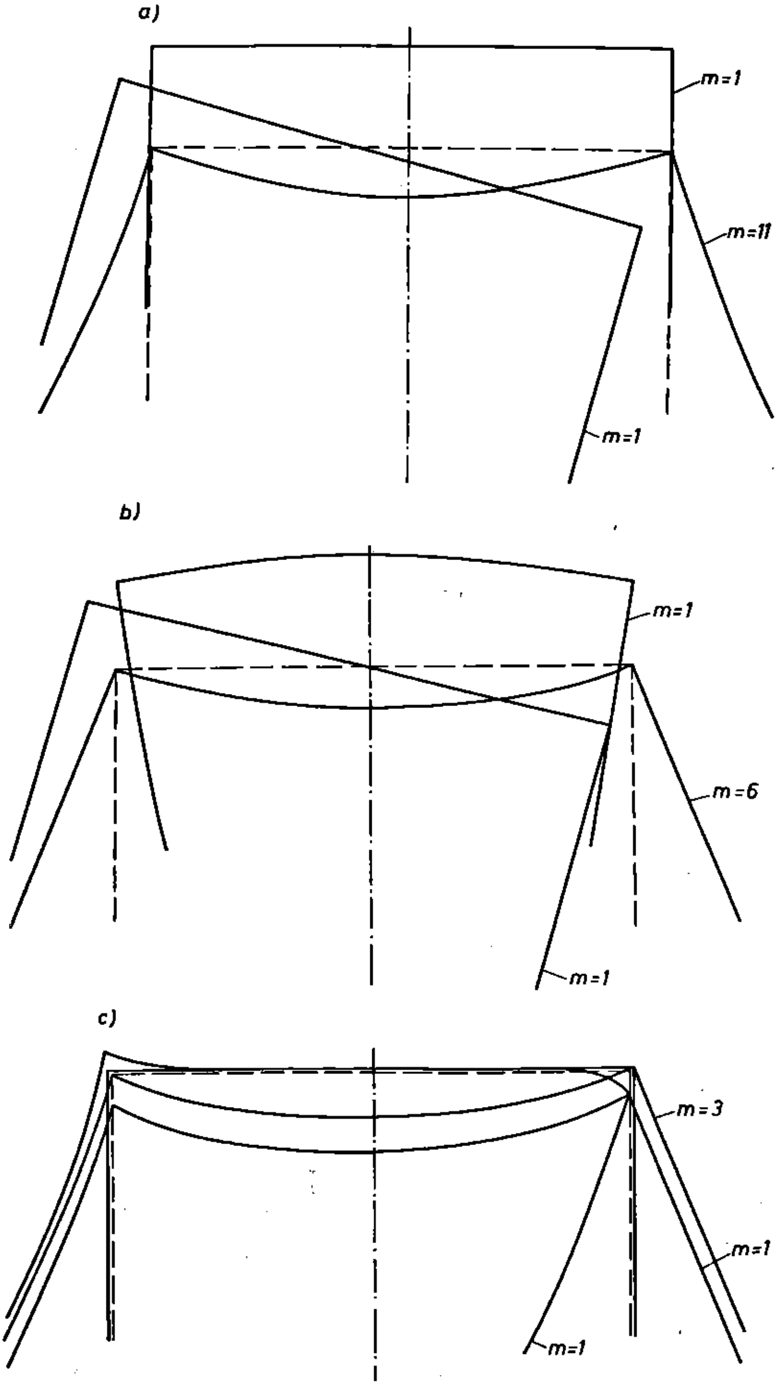

FIG. 3. Two global and local buckling modes for the compressed column corresponding to: a – $\beta = 0.0728$; b – $\beta = 1.0$; c – $\beta = 13.736$.

the first global buckling mode is the torsional-flexural buckling, while the second one has a flexural character. In case of the column with $\beta = 13.736$, the second global buckling mode represents a flexural buckling, where the weaker part of the column (regarding stability) is its web (web buckling causes a rotation of flanges), whereas the first mode refers to a torsional-flexural buckling, the weaker parts which regard the stability being the flanges. All global buckling modes have one buckling half-wave (sinusoidal half-wave) at column length. All local buckling modes, regardless of the value of β factor, are similar to the second global buckling mode with $\beta = 13.736$ (Fig. 3c). They differ in the number of half-waves along the column.

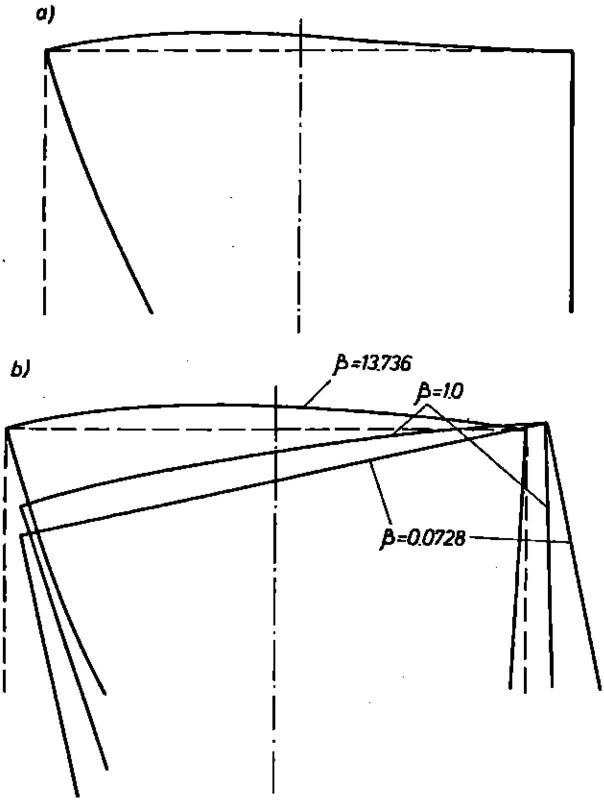

FIG. 4. Global (a) and local (b) buckling modes for the beam-column in bending at different $\beta = 0.0728$; 1.0; 13.736.

The following illustration presents global (Fig. 4a) and local (Fig. 4b) buckling modes of the channel subject to pure bending in the web plane (so-called lateral buckling), the wall orthotropy factor being $\beta = 0.0708$, 1.0 and 13.736. In case of a column with $\beta = 13.736$, longitudinal edges of adjacent walls undergo slight displacements in the global buckling mode

(Fig. 4a) which is characteristic for the local buckling modes (Fig. 4b). All local modes of the bending channel have nearly identical buckling modes (as in the case of a compressed channel) which differ only in their numbers of half-waves, m.

Figure 5 shows the change in the number of half-waves, m, being formed during local buckling along the axially compressed channel, plotted as a function of β.

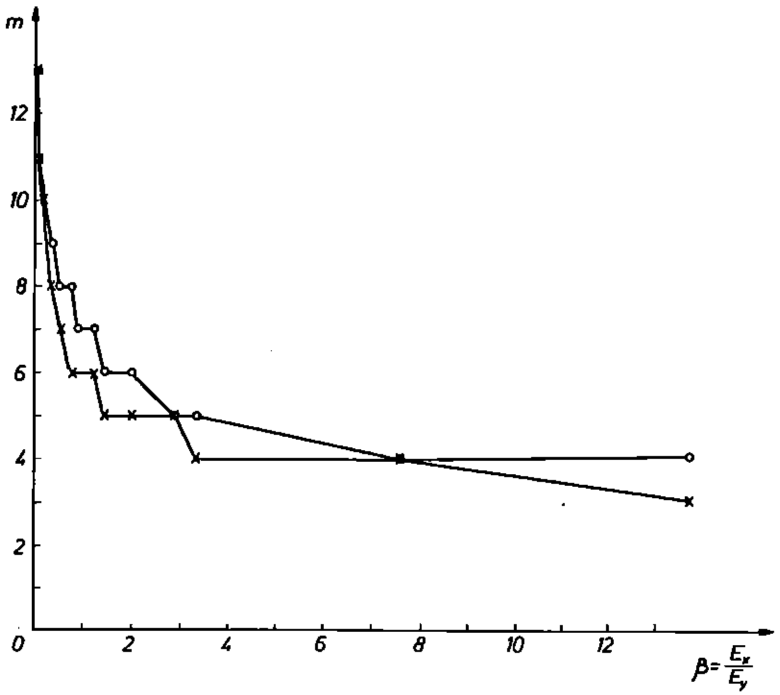

FIG. 5. The number of half-waves m versus the orthotropy factor β. o – compressed channel, × – channel subject to bending.

Figure 5 indicates that for the column with $\beta = 0.0728$ the number of half-waves, m, is almost twice as great as that for a column built of an isotropic material ($\beta = 1.0$). For $\beta > 3.0$ the number of half-waves is practically independent of the value of β.

Figure 6 shows plots of the bending moments, $\overline{M}_{yi}^{(1)} = M_{yi}^{(1)} b_i^2 / D_{xi}$, for the first (Fig. 6a) and the second (Fig. 6b) global buckling modes of axially compressed channel, the orthotropy factors of column material being $\beta =$ 0.0728, 1.0 and 13.736. Plots illustrating the first global buckling mode show that, with increasing β, the values of bending moments $\overline{M}_{yi}^{(1)}$ decrease. It means that, at a constant value of modulus E_x, moments $\overline{M}_{yi}^{(1)}$ decrease

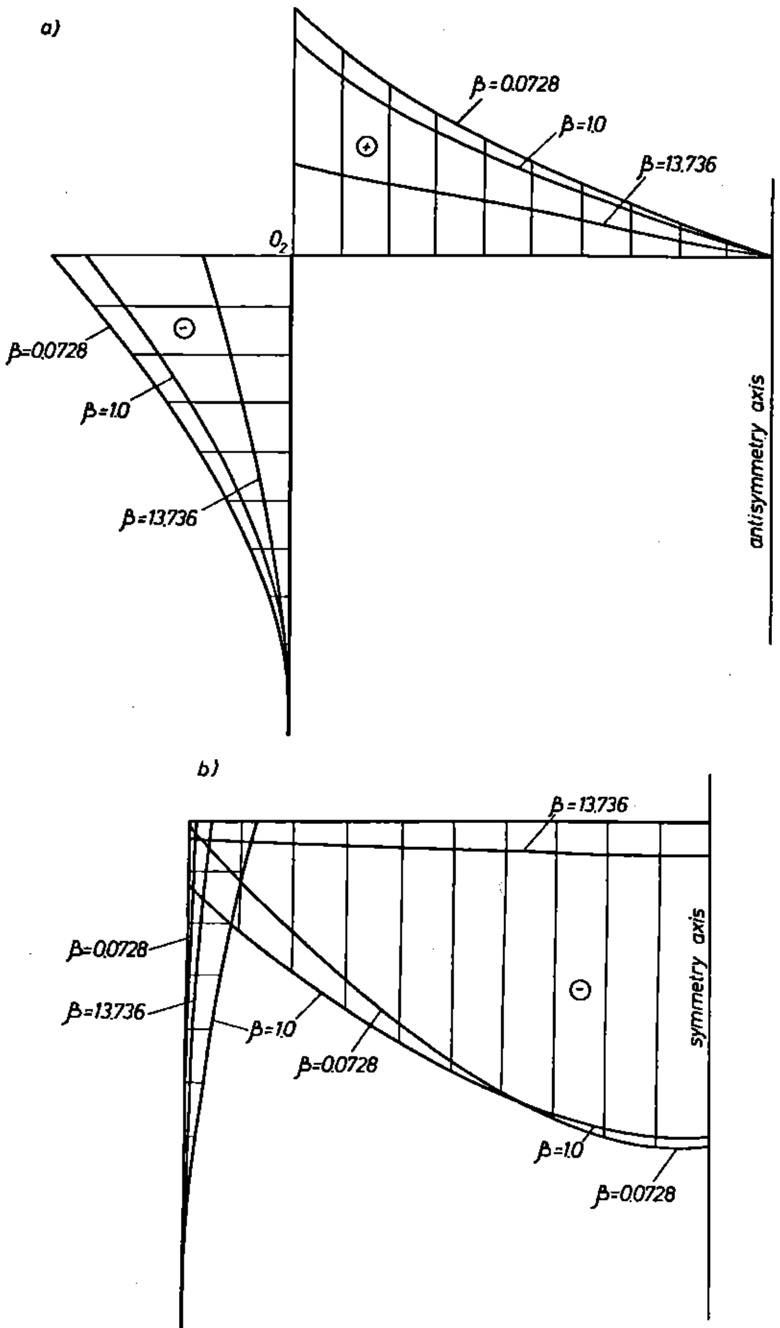

FIG. 6. Bending moments $\overline{M}_{yi}^{(1)}$ for the first (a) and the second global buckling mode of the compressed column.

together with a reduction of modulus E_y. Diagrams of moments $\overline{M}_{yi}^{(1)}$ shown in Fig. 6 confirm that during the compression of the column with $\beta = 13.736$, the first global buckling mode results from the buckling of flanges, while the second one is a consequence of buckling of the web.

Plot of normal forces, $\overline{N}_{yi}^{(2)} = N_{yi}^{(2)} b_1 / K_{x1}$, for the first local buckling mode of the compressed column are presented in Fig. 7. Diagrams show a rapid decrease of the absolute value of force $\overline{N}_{yi}^{(2)}$ with increasing β. For fixed values of modulus E_x this means a very quick increase in the absolute value of force $\overline{N}_{yi}^{(2)}$ accompanying an increase in modulus E_y.

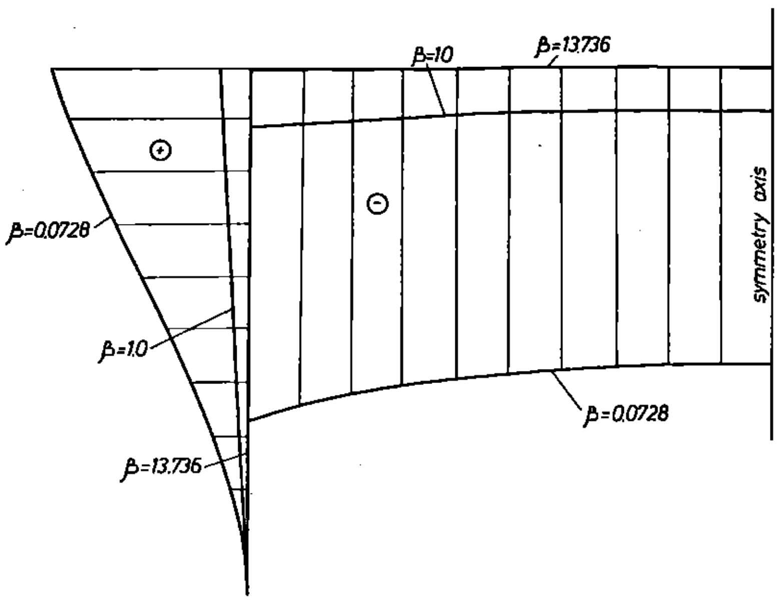

FIG. 7. Plot of in-plane stress resultant $\overline{N}_{yi}^{(2)}$ for local buckling mode of the compressed column.

The continuity conditions in column corners [8] imply that $N_{yi}^{*(n)} = Q_{yi+1}^{*(n)}$ and $Q_{yi}^{*(n)} = N_{yi+1}^{*(n)}$ (Fig. 7); this suggests a strong influence of Kirchhoff's forces, $Q_{yi}^{*(n)}$, upon the critical stress values and the buckling modes.

Curves presented in Fig. 8 illustrate the ratio of limiting stress, σ_s^*, to the minimum critical local stress, σ_2^*, as a function of the factor β for the assumed initial imperfections $|\overline{\xi}_1| = 1.0$ and $|\overline{\xi}_2| = 0.2$ ([8]). In case of axial compression of a column, the considerations included the interaction of the first or the second global buckling modes with the basic local buckling mode. In the channel subject to axial compression, the interaction between the global flexural buckling mode and the local one yields lower values of

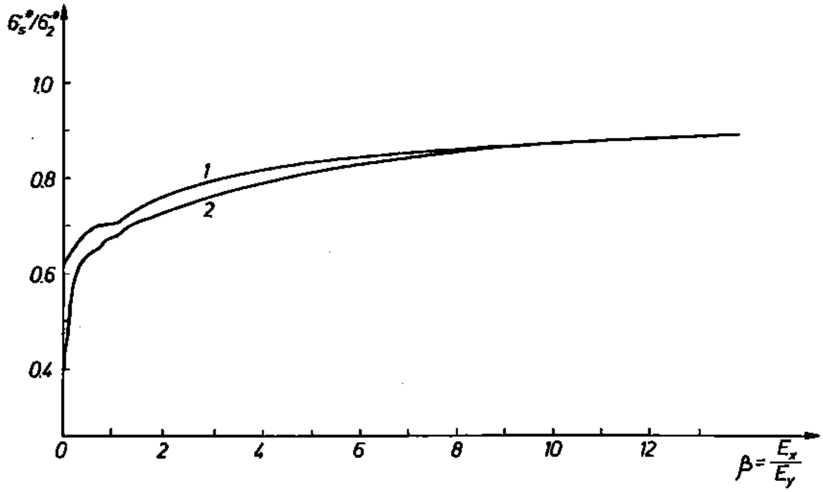

FIG. 8. Load carrying capacity σ_s^*/σ_2^* carried by orthotropy factor β. Curves: *1* – the compressed column, *2* – the beam-column in bending.

the limiting load, σ_s^*/σ_2^*, than those following from the interaction between the torsional-flexural mode and the same local buckling mode.

In the range of variability of β discussed in this paper the value of limiting load, σ_s^*/σ_2^*, in case of an interaction between a global torsional-flexural buckling mode and a local mode, is not less than 0.9. Therefore a more dangerous interaction is the one between the global flexural buckling mode and the local one.

Diagrams presented in Fig. 8 indicate that the channel structure built of low-β material ($\beta < 5$) are more sensitive to initial imperfections; the highest imperfection sensitivity is exhibited by the channel subject to bending in the web plane for very low values of β ($\beta < 0.5$).

4. Conclusions

The solution obtained and the computer program prepared enabled us to carry out a detailed analysis of the behaviour of thin-walled orthotropic open beam-columns in the critical and initial elastic post-buckling state; considerations included different kinds of load – from axial compression through eccentric compression to pure bending. The computer program prepared allows to carry out a parametric analysis of stability in case of orthotropic thin-walled columns of different cross-sections, including the columns with longitudinal stiffeners.

Numerical calculations prove that in case of orthotropic structures under compression, the global modes obtained in the assumed plate model (Fig. 3) may significantly differ from the results obtained using the earlier beam-bar model.

Calculations referring to interactions of the most dangerous buckling modes of orthotropic thin-walled columns are often very similar to those obtained by analysing isotropic structures [5, 10].

The present analysis has to be completed by including the second approximation in order to investigate the post-buckling behaviour in the case when the first order interaction is weak.

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