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TRANSIENT FREE CONVECTION FLOW OF AN ELASTICO-VISCOUS FLUID PAST AN INFINITE VERTICAL PLATE

R. DEKA, U.N. DAS (GUWAHATI)
and V.M. SOUNDALGEKAR (THANE)

A solution to the problem of an unsteady, non-isothermal flow of an elastico-viscous fluid in a half-space is presented. The flow is induced by a temperature jump on the limiting plane. The constitutive equations of Walters' liquid B' are used.

NOTATIONS

C_p heat capacity J/kgK,
g acceleration due to gravity m/s^2,
K thermal conductivity W/(mK),
Pr Prandtl number,
t dimensionless time,
T thermodynamic temperature, K,
T'_w plate temperature K,
T_∞ temperature of fluid far away from the plate K,
u dimensionless velocity in x-direction,
y dimensionless coordinate normal to the plate,
β volumetric expansion coefficient 1/K,
θ dimensionless temperature,
μ dynamic viscosity, kg/(sm),
ν kinetic viscosity, m^2/s,
ϱ density, kg/m^3,
κ dimensionless elastic parameter,
T_{xy} non-dimensional shearing stress.

1. INTRODUCTION

SCHETZ and EICHHORN [1] studied transient free convection flow of a viscous fluid past an infinite vertical isothermal plate. Many other papers of this topic are quoted in that paper. Flow of Newtonian fluids was considered. However, in modern technology, many new fluids (such as polymers) are discovered. Blood is another fluid which, like polymer fluids, exhibits elastic properties. Some fluids are only partly elastic and, based on this assumption, Walters proposed new constitutive equation which are now called Walters liquid B'. The flow of Walters' liquid B' past an impulsively started horizontal infinite plate was studied by SOUNDALGEKAR [2], and the case of an impulsively started infinite vertical plate was also studied by SOUNDALGEKAR [3]. It is now proposed to study the transient free convection flow of Walters' liquid B' past an infinite vertical isothermal plate.

2. Mathematical analysis

We consider the flow of an elastico-viscous Walters' liquid B' past an infinite vertical isothermal plate. The x'-axis is taken along the plate in the vertical direction, and the y'-axis is assumed to be normal to the plate. The constitutive equations of Walters' liquid B' are given by SOUNDALGEKAR and PURI [4]. Hence, under the usual Boussinesq approximation, the unsteady free convection flow of this fluid past an infinite vertical plate is governed by the following equations in non-dimensional form:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \theta + \frac{\partial^2 u}{\partial y^2} - \kappa \frac{\partial^3 u}{\partial y^2 \partial t}, \\ \Pr \frac{\partial \theta}{\partial t} &= \frac{\partial^2 \theta}{\partial y^2}.\end{aligned} \tag{2.1}$$

The initial and boundary conditions are

$$\begin{aligned} u &= 0, \quad \theta = 0 \quad \text{for all } y,\ t \le 0, \\ u &= 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \\ u &= 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.2}$$

The non-dimensional quantities are defined as follows:

$$\begin{aligned} &L = \left(g\beta\Delta T/\nu^2\right)^{-1/3}, \qquad U = (\nu g \beta \Delta T)^{1/3}, \qquad \Delta T = T'_w - T'_\infty, \\ &t^* = (g\beta\Delta T)^{-2/3}\nu^{1/3}, \qquad u = u'/U, \qquad y = y'/L, \qquad t = t'/t^*, \\ &\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \qquad \Pr = \mu C_p/\kappa, \qquad \kappa = \kappa_0/\varrho L^2. \end{aligned} \tag{2.3}$$

All the quantities are defined in the Notations.

Following SOUNDALGEKAR and PURI [4], we assume

$$u = u_0 + \kappa u_1, \tag{2.4}$$

because κ is very small ($\ll 1$) for Walters' liquid B'. Substituting (2.4) in Eq. $(2.1)_1$, equating the coefficients of equal powers of κ and neglecting those of κ^2, we have

$$\frac{\partial u_0}{\partial t} = \theta + \frac{\partial^2 u_0}{\partial y^2}, \tag{2.5}$$

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^3 u_0}{\partial t \partial y^2}, \tag{2.6}$$

with the following initial and boundary conditions:

$$\begin{array}{llll} u_0 = 0, & u_1 = 0, & \theta = 0 & \text{for all } y,\ t \le 0, \\ u_0 = 0, & u_1 = 0, & \theta = 1 & \text{at } \quad y = 0, \\ u_0 = 0, & u_1 = 0, & \theta \to 0 & \text{as } \quad y \to \infty . \end{array} \tag{2.7}$$

The solutions of Eqs. $(2.1)_1$, (2.5) and (2.6) satisfying the initial and boundary conditions (2.7) are derived by the Laplace transform technique.

$$\theta = \operatorname{erfc}\left(\eta\sqrt{\Pr}\right), \tag{2.8}$$

$$\begin{aligned} u = \frac{t}{\Pr - 1}\Bigg[&(1+2\eta^2)\operatorname{erfc}(\eta) - (1+2\Pr\eta^2)\operatorname{erfc}(\eta\sqrt{\Pr}) \\ &- \frac{2\eta}{\sqrt{\pi}}\left(e^{-\eta^2} - \sqrt{\Pr}\, e^{-\Pr\eta^2}\right)\Bigg] \\ &+ \kappa\left[\frac{\Pr}{(\Pr-1)^2}\operatorname{erfc}(\eta) - \frac{1}{\Pr-1}\left\{\frac{\eta}{\sqrt{\pi}}e^{-\eta^2} \right.\right. \\ &\left.\left. + \frac{\Pr}{\Pr-1}\operatorname{erfc}\left(\eta\sqrt{\Pr}\right)\right\}\right], \end{aligned} \tag{2.9}$$

where $\eta = y/2\sqrt{t}$.

The shearing stress is given by

$$P'_{x'y'} = \eta_0 \frac{\partial u'}{\partial y'} - \kappa_0 \frac{\partial^2 u'}{\partial y' \partial t}, \tag{2.10}$$

and, in view of (2.3), (2.11), (2.10), we have

$$\begin{aligned} T_{xy} = \frac{\sqrt{t}}{\Pr-1}&\left[\left(\frac{1}{\sqrt{\pi}} + \frac{1}{\sqrt{\Pr}}\right)\left(\sqrt{\Pr}-1\right)\right] \\ &+ \frac{\kappa}{2\sqrt{\pi}}\left[\frac{2\Pr(\sqrt{\Pr}-1)}{(\Pr-1)^2} - \frac{1}{\Pr-1}\right]. \end{aligned} \tag{2.11}$$

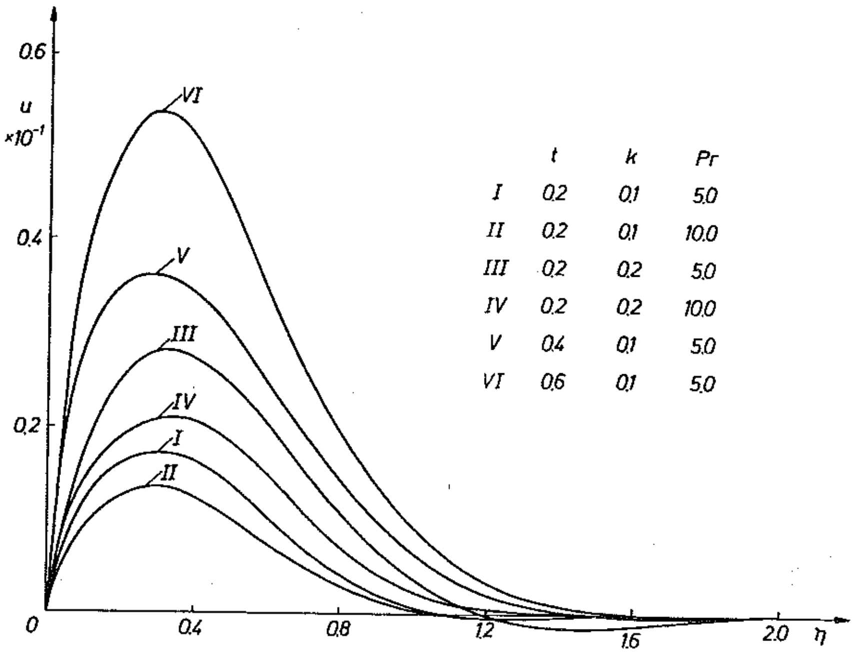

FIG. 1.

In order to gain physical insight into the problem, we have computed numerical values of u from Eq. (2.9) and these are plotted in Fig. 1, for $\mathrm{Pr} = 5$ and 10. We observe from this figure that the velocity increases with increasing t, κ, but decreases with increasing Prandtl number.

To understand the effect of the elastic parameter κ on the shearing stress, we observe from Eq. (2.1) that for a steady state $\partial(\ )/\partial t = 0$, and hence the fluid does not behave like an elastic-viscous fluid in the steady state. In case of a Newtonian fluid, we study the time required to reach the steady state since it is a one-dimensional conduction process and the value of time needed to reach the steady state time can be calculated. However, in the present case, the concept of steady-state is meaningless and the shearing stress always depends on time. So we cannot find the steady-state time in the elastic-viscous fluids in one-dimensional flow.

However, we have computed the numerical values of T_{xy} and these are listed in Table 1.

We conclude from this Table 1 that the shearing stress decreases with increasing Prandtl number, but an increase in the elastic parametr leads to an increase in the value of shearing stress. The shearing stress also increases with increasing time.

Table 1. Values of T_{xy}.

t	κ/Pr	5.0	10.0
0.2	0.1	0.1457	0.0982
0.2	0.2	0.1518	0.1018
0.4	0.1	0.2019	
0.6	0.1	0.2455	

3. Conclusion

We observe that the velocity and the shearing stress decreases with increasing Prandtl number, but both the magnitudes increase with increasing time or the elastic parameter.

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DEPARTMENT OF MATHEMATICS
GAUHATI UNIVERSITY, GUWAHATI, ASSAM
and
BRINDAVAN SOCIETY, THANE, INDIA.

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