

## PLASTIC ANALYSIS OF METAL SURFACE LAYERS UNDERGOING THE ROLLER BURNISHING PROCESS

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The roller burnishing process as a particular case of the rolling contact problem is considered. The hardening of the burnished surface layers considerably improves the mechanical properties of machine elements. However, in some cases, softening of a thin contact layer underneath the rolled material occurs. It is caused by the overheating due to a high speed of the rolling or by the plastic strain cumulation due to a multiple rolling repetition. As an effect, peeling and cracking of the rolled surface layers appears. The softening effect is taken into account in the present analysis of the roller burnishing process. The proposed slip-line field and the corresponding velocity field are obtained by a modification of the simplified solution proposed by COLLINS [3]. The new solution describes the "seizing effect" observed on some micrographs of metal surface layers. The bounds for the vertical load, applied to the roller and causing this effect, are given as functions of material softening.

### 1. INTRODUCTION

Proper surface treatment of machine elements enables us to increase their resistance to the destructive effect of physical and chemical factors during the exploitation processes. Such a treatment is the roller burnishing of metal surfaces. A high load applied to the roller causes plastic yielding in a thin contact layer. It results in hardening of the layer and appearing of residual stresses in the whole element. The increased yield limit and strong compressive stresses induced in metal surface layers prevent an initiation and propagation of microcracks in the material. As a result, the service life of the burnished machine element increases considerably.

Mechanical properties of the burnished surface layer differ from those of the element core and vary with the distance from the element surface. In the strength calculations of machine elements, the surface layer is neglected because of its small thickness compared to the size of core. However, a detailed analysis of the plastic yield during the roller burnishing process enables us to predict the final properties of the layer, and gives an opportunity to control

the process, to increase the service life, reliability, and to reduce the weight of machine elements.

The roller burnishing process may be considered as a particular case of the rolling contact problem. During this process the highly loaded, force-driven rigid cylinder is rolled without friction over a plane surface of a deformable material. An extensive literature of the problem [1–3] is focussed on the prediction of rolling resistance rather than on the analysis of surface layer properties after the rolling process. According to Collins' model based on the slip-line field theory, the rolled surface undergoes a cumulative horizontal displacement in the direction of the roller motion [3]. This fact is confirmed by many observations of metal surface layers after the roller burnishing process. However, in some cases, the metallographical observations [4–5] show that, while about 90% of the surface layer thickness is displaced ahead of the roller motion, a thin direct contact zone undergoes backward displacements (Fig. 1).

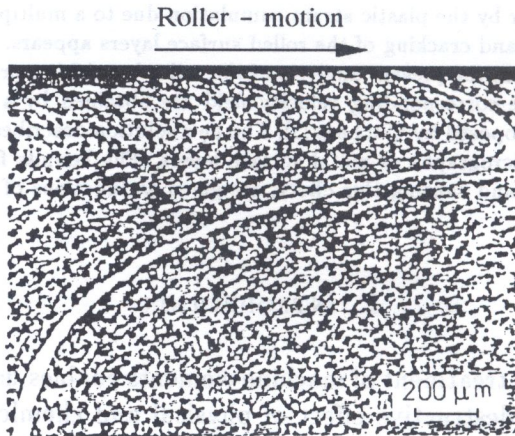


FIG. 1. Micrograph of metal surface layer after 105 cycles of roller burnishing [5].

The above "seizing effect" is connected with a decrease of the material hardness and softening of the material structure in the direct contact zone (Fig. 2).

The softening is caused by:

- cumulation of plastic strain (the highest in that zone) after successive passing of the roller;
- increase of temperature in this zone due to a high speed of the burnishing process (Fig. 3).

The "seizing effect" leads to peeling and cracking of the surface layers and constitutes a serious problem in the engineering practice. The effect does not appear when the load applied to the cylinder is small. It takes place for

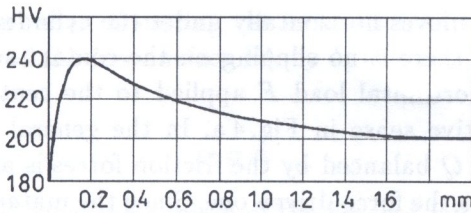


FIG. 2. Hardness of metal surface layer after roller burnishing as a function of distance from the surface [5].

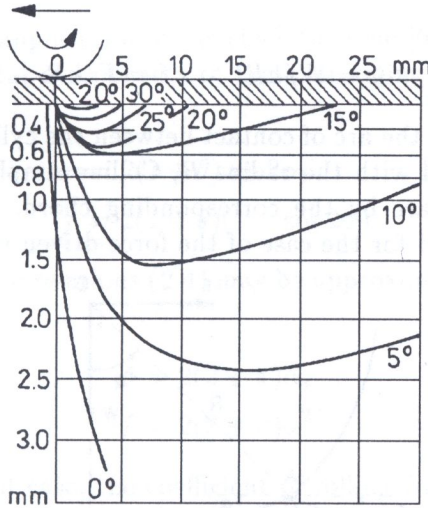


FIG. 3. Distribution of relative temperature in the metal surface layer after roller burnishing [5].

higher loads, when the softening of material due to rolling exceeds a certain critical value. The description of the “seizing effect” and determination of loads which cause this effect is the aim of the paper. In the next section, the frictionless rolling of a force-driven rigid cylinder on a rigid/perfectly plastic half-space is reconsidered. Next, a new slip-line field and the associated velocity field that describe the effect are constructed. Finally, the bounds for the proposed solution are discussed.

## 2. COLLINS SOLUTION FOR FRICTIONLESS ROLLING

Let us specify Collins’ solution [3] for the case of frictionless loading. Consider a rigid cylinder rolling on a rigid/perfectly plastic half-space with a constant speed  $V$ . For convenience, the problem will be regarded as a problem of steady motion in which the axis of the cylinder remains at rest,

and the half-space moves horizontally under the cylinder with the speed  $V$ . It is assumed that there is no slipping on the contact surface. The vertical load  $W$  and the horizontal load  $F$  applied to the centre of the roller are shown in the positive sense in Fig. 4a. In the general case considered by Collins, the torque  $Q$  balanced by the friction forces is applied to the roller. Here, the cylinder is the force-driven one. Since the material of the half-space is described by the yield shear stress  $k$  only, and the cylinder radius  $R$  is the only scale-defining parameter, one can introduce the following dimensionless load variables:

$$(2.1) \quad w = W/kR, \quad f = F/kR.$$

Because length of the arc of contact between the cylinder and the surface is small as compared with the radius  $R$ , Collins simplified the problem by approximating this arc by the corresponding chord. One of his solutions called Mode I is valid for the case of the force-driven cylinder (Fig. 4a, b).

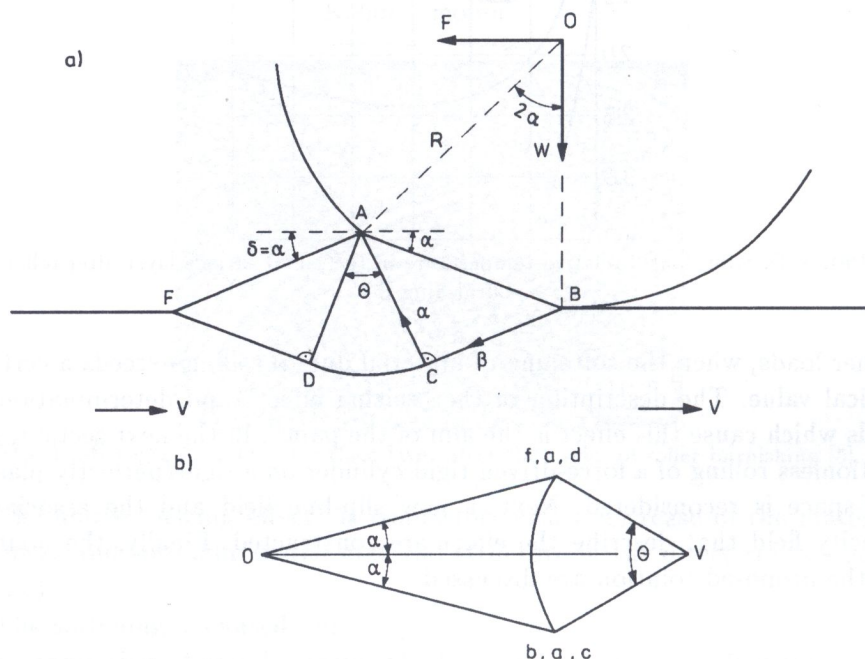


FIG. 4. Mode I of Collins' solution for frictionless rolling, a) slip-line field, b) hodograph [3].

According to this solution, the slip-line field is composed of two regions ADF and ABC, in which  $\alpha$  and  $\beta$  slip-lines are straight, and a centred fan ACD exists with straight  $\alpha$ -lines. Incompressibility of the material and plane deformation require that in the steady state, the level of the surface

is unchanged by the roller. Denoting the inclination of the chord AB by  $\alpha$ , and the inclination of the free surface FA by  $\delta$ , this requirement is satisfied when

$$(2.2) \quad \alpha = \delta.$$

In the considered case, the angle  $\theta$  of the centred fan ADC is given by

$$(2.3) \quad \theta = \pi/2 - 2\alpha.$$

After a standard application of Hencky's theorem [6], it is found that the vertical and horizontal components of the load acting on the cylinder take the form:

$$(2.4) \quad \begin{aligned} w &= 2(1 + \theta) \sin 2\alpha, \\ f &= 2(1 + \theta)(1 - \cos 2\alpha). \end{aligned}$$

For small angles  $\alpha$ , the relations (2.4) may be approximated by the following formulae:

$$(2.5) \quad \begin{aligned} w &= 2(2 + \pi)\alpha, \\ f &= 2(2 + \pi)\alpha^2. \end{aligned}$$

For the considered case, the coefficient of rolling friction  $\mu_R \equiv f/w$  proposed by Collins is proportional to the vertical load  $w$  according to the rule

$$(2.6) \quad \mu_R = w/2(2 + \pi).$$

The rigid material below FDCB is not overstressed provided the angle  $\alpha$  is non-negative.

According to the hodograph shown in Fig. 4 b, the regions ABC and ADF move rigidly, regarding the rigid part of the half-space, in the direction BC and DF, respectively. In the centred fan ACD, the velocity is constant along  $\beta$ -lines and the velocity discontinuity appears along FDCB. Because there is no slipping on the chord AB, the velocity  $U$  of the rigid material in ABC is the same as the peripheral speed of the cylinder. From the hodograph diagram, it follows that this speed is smaller than the advance speed of the material below the line FDCB

$$(2.7) \quad U = \frac{\cos(\alpha + \pi/4)}{\cos \pi/4} V \leq V.$$

It means that the surface layer is displaced always in front of the roller motion and the "seizing effect" cannot be described within Collins' model.

### 3. SOLUTION FOR SOFTENING MATERIAL

After too many cycles of rolling, a cumulation of plastic strains initiates a great number of microcracks in a thin contact layer. On the other hand, a high speed of the burnishing process causes an increase of temperature in this layer (see Fig. 3). Both the effects lead to softening of the material in the considered zone. Thickness of this zone is small as compared to the depth of the whole plastified layer. To simplify the further analysis, assume that the yield shear stress of the softened zone  $k_0$  is lower than the yield shear stress of the whole plastified layer  $k$ . Concluding, we assume that behind a certain front line dividing the contact surface AB shown in Fig. 4 a, within a thin layer, a lower yield shear stress  $k_0$  appears, namely

$$(3.1) \quad k_0 \leq k.$$

Let us modify the model shown in Fig. 4 a penetrating slightly the cylinder into the half-space as it is shown in Fig. 5 a. The penetration is determined by the angle  $\varepsilon$ , which describes the inclination of the tangent to the cylinder at point G.

The arc of contact is fixed by the chord AB with the inclination  $\alpha$  to the rolled surface. This arc will be approximated by two chords AC and CB with the inclination  $\alpha + \varepsilon/2$  and  $\varepsilon/2$ , respectively. The inclination of the free surface AF in front of the cylinder is determined by the angle  $\delta$ . The transient contact surface BG is assumed to be not loaded and is regarded as a free surface. The angles  $\alpha$ ,  $\delta$  and  $\varepsilon$  are non-negative, and the angle  $2\alpha$  is not smaller than  $\varepsilon$  (see Fig. 5 a). For high loads applied to the roller (the case of the hardening burnishing process), point C is close to B and angle  $\varepsilon$  is much less than  $2\alpha$ .

The slip-line field is composed of two independent parts:  $AFD_1D_2C$  and  $BCE_1E_2G$ . Each of them is constructed in the same way as in Collins' model and contains two triangles and one centred fan. The angles of the fans  $AD_1D_2$  and  $BE_1E_2$  are denoted by  $\theta$  and  $\psi$ , respectively. The thickness of the first plastic zone is  $\sqrt{2}R \sin \alpha$ , and for the second one it is equal to  $\sqrt{2}R \sin \varepsilon/2$ , where  $R$  is the roller radius. For the hardening burnishing process the thickness of the second part of the slip-line field is much smaller than the thickness of the first one.

In the general case, when the torque  $Q$  is applied to the cylinder, a friction prescribed by the inclination  $\beta$  of  $\alpha$  slip-lines to the contact surface appears on this surface. In this case, the problem has two degrees of freedom and the slip-line field is completely described once the angles  $\alpha$  and  $\beta$  are specified. In our case, the friction does not appear and  $\beta = \pi/4$ . Then, we need four

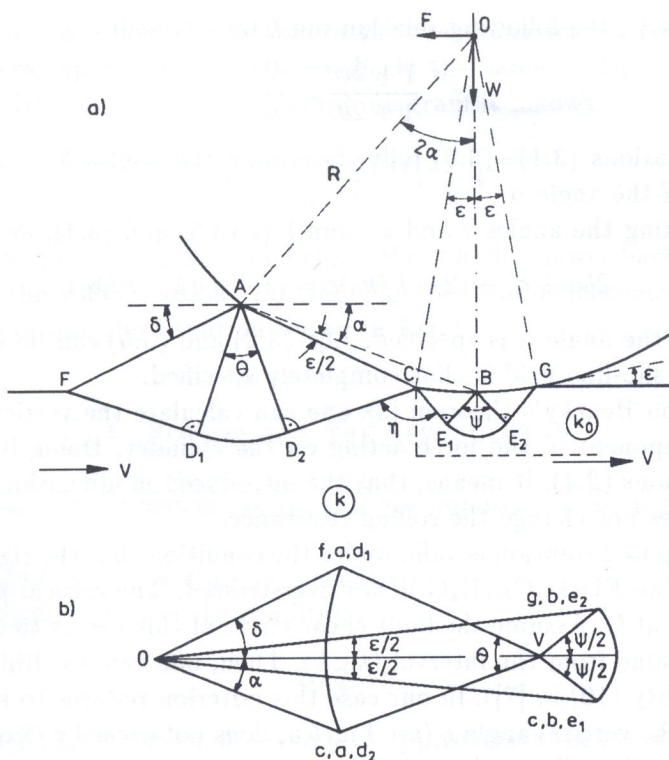


FIG. 5. Proposed solution for material softening during rolling, a) slip-line field, b) hodograph.

relations to determine the angles  $\delta$ ,  $\epsilon$ ,  $\theta$  and  $\psi$  as functions of the angle  $\alpha$ . Moreover, the solution should depend on the ratio  $k/k_0$ , which describes the softening of the material during the passage of the roller.

The first relation results from the condition that the length of the chord AB is equal to the length of the section AF, thus

$$(3.2) \quad \delta = \alpha + \epsilon/2.$$

Summing up the angles centred at points A and B, we obtain

$$(3.3) \quad \alpha + \epsilon/2 + \delta + \theta = \pi/2$$

and

$$(3.4) \quad \psi - \epsilon = \pi/2,$$

respectively.

The hydrostatic pressure in the triangle  $AD_2C$  and the triangle  $BE_1C$  is equal to  $k(1 + 2\theta)$  and  $k_0(1 + 2\psi)$ , respectively. Since it should be the same

at the point C, the following relation must be satisfied:

$$(3.5) \quad \frac{1 + 2\psi}{1 + 2\theta} = \frac{k}{k_0}.$$

Four relations (3.1)–(3.5) fully determine the angles  $\delta$ ,  $\varepsilon$ ,  $\theta$  and  $\psi$  as functions of the angle  $\alpha$ .

Eliminating the angles  $\theta$  and  $\psi$  from Eqs. (3.3) and (3.4), we obtain

$$(3.6) \quad 2(\alpha + \delta) + (2 + k/k_0)\varepsilon = (1 + \pi)(1 - k/k_0).$$

Thus, if the angle  $\alpha$  is specified, Eqs. (3.2) and (3.5) can be solved for  $\delta$  and  $\varepsilon$ . The slip-line field is thus completely specified.

Basing on Hencky's theorem [6], one can calculate the vertical and horizontal component of the force acting on the cylinder. Doing it we obtain the expressions (2.4). It means, that the introduced modification of Collins' solution does not change the rolling resistance.

The proposed solution is valid under the condition that the rigid material below the line  $FD_1D_2CE_1E_2G$  is not overstressed. The critical point is the rigid vertex at C. Assume the limit shear stress at this vertex to reach a certain fixed value from the interval  $(k_0, k)$ . Then, one can use Hill's criterion (see inequality (10) in [7]). In our case this criterion reduces to the requirement that the vertical angle  $\eta$  (see Fig. 5 a) does not exceed  $\pi/2$  or the angle  $\alpha$  is non-negative. Then, the rigid vertex at C is never overstressed.

Taking into account that the angles  $\alpha$ ,  $\delta$  and  $\varepsilon$  are non-negative, from Eq. (3.6) it follows that if  $k_0$  approaches  $k$ , the angles  $\alpha$ ,  $\delta$  and  $\varepsilon$  tend to zero. Comparing the proposed model with that given by Collins, we see that the only common solution for both models is the trivial case of the vanishing load (see Fig. 6).

Looking at the hodograph shown in Fig. 5 b, we see that the plastic region in front of the cylinder moves ahead of the roller motion like in Collins'

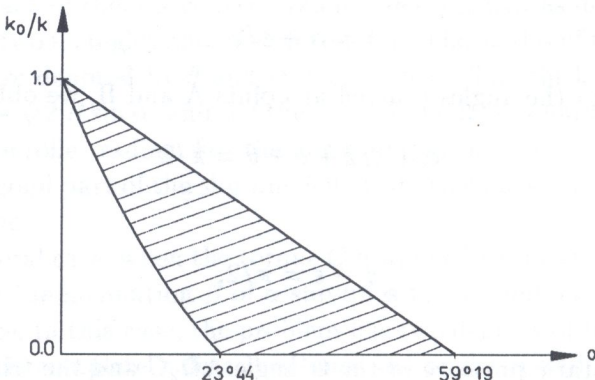


FIG. 6. Validity domain of the solution describing "seizing effect" (shaded area).



solution. Now, consider the motion of the remaining plastic zone. Because the peripheral speed of the cylinder  $U_0$  is the same as that of the rigid material in  $BE_2G$ , from the hodograph diagram it follows:

$$(3.7) \quad U_0 = \frac{\sin(\varepsilon + \pi/4)}{\sin \pi/4} V \geq V.$$

It means that the plastic zone behind the cylinder moves backwards with respect to the roller motion. Putting together the displacements of both regions, we obtain the situation shown in Fig. 1.

#### 4. BOUNDS FOR THE NEW SOLUTION

Recall that the following restrictions are imposed on the proposed solution:

- the angles  $\alpha$ ,  $\delta$  and  $\varepsilon$  are non-negative;
- the angle  $\alpha$  satisfies the condition:

$$(4.1) \quad 2\alpha \geq \varepsilon;$$

- the angle  $2\alpha + \varepsilon$  is small enough to approximate the arc of contact by the corresponding chord;
- the yield shear stress  $k_0$  is smaller than or equal to the initial yield shear stress  $k$ .

Let us introduce the parameter

$$(4.2) \quad \kappa \equiv k_0/k,$$

where  $0 < \kappa \leq 1$ . Eliminating the angle  $\delta$  from Eqs. (3.3) and (3.6), we can express the angle  $\varepsilon$ , which describes the range of the new plastic zone, by the angle  $\alpha$  determining the applied load

$$(4.3) \quad \varepsilon = \frac{1}{3 + \kappa} [(1 + \pi)(1 - \kappa) - 4\alpha].$$

The restriction (4.1) and the condition  $\varepsilon \geq 0$  give the following bounds for the angle  $\alpha$ :

$$(4.4) \quad \frac{1}{2}(1 + \pi) \frac{1 - \kappa}{5 + \kappa} \leq \alpha \leq \frac{1}{4}(1 + \pi)(1 - \kappa).$$

The bounds (4.4), plotted in Fig. 6, determine the validity domain of the proposed solution.

The approximate relations for the vertical and horizontal components of the load acting on the cylinder are given by equations (2.5). Note, that they do not depend on the angle  $\varepsilon$ , and then they are the same as in the case of rolling without the "seizing effect". Equations (2.1) and (2.5)<sub>1</sub> enable us to express the inequalities (4.4) for the vertical force  $W$

$$(4.5) \quad (1 + \pi)(2 + \pi)kR \frac{1 - \kappa}{5 + \kappa} \leq W \leq \frac{1}{2}(1 + \pi)(2 + \pi)kR(1 - \kappa),$$

where  $k$  is the yield shear stress of the material before the burnishing process, and  $R$  is the roller radius.

Thus if, for a prescribed load  $W$ , the softening of the material is too high, the "seizing effect" occurs. Decrease of this softening by reduction of the speed of the burnishing process or by reduction of the number of rolling cycles, diminishes the size of the plastic zone under the bottom of the roller. If the softening is small enough, the "seizing effect" disappears.

## 5. CONCLUSIONS

During the roller burnishing process with a high speed or after many cycles of rolling, softening of a thin surface layer of the rolled material may appear. The plastic analysis of the process shown above, based on Collins' solution for the frictionless rolling, takes this fact into account. According to this analysis, the relations between the applied loads and the length of the contact zone are the same as those given by Collins. However, the shape of the slip-line field and the corresponding kinematics are different. Besides the bulk plastic region in front of the rolling cylinder, a small plastic zone of softened material under the bottom of the roller appears. The softened material is seized by the roller and is displaced backwards with respect to the roller motion. The large plastic strains of opposite signs cause a cumulation of microcracks in the material. Finally it leads to peeling and cracking of the metal surface layers in the burnished machine elements.

According to the proposed model, if we know the softening of the material during the roller burnishing process, we can avoid the undesirable effects keeping the vertical load applied to the roller out of the bounds prescribed by inequalities (4.5).

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#### REFERENCES

1. J.E. MERWIN and K.L. JOHNSON, *An analysis of plastic deformation in rolling contact*, Proc. Inst. Mech. Engng., 177, 676-690, 1963.
2. J. MANDEL, *Résistance au roulement d'un cylindre indéformable sur un massif parfaitement plastique*, Le Frottement et l'Usure, 25-33, 1966.
3. I.F. COLLINS, *A simplified analysis of the rolling of a cylinder on a rigid/perfectly plastic half-space*, Int. J. Mech. Sci., 14, 1-14, 1972.
4. G. PAHLITZSCH and P. KROHN, *Über das Glattwalzen zylindrischer Werkstücke im Einstechverfahren*, Werkstattstechnik, 1, 1966.
5. W. PRZYBYLSKI, *Burnishing technology* [in Polish], WNT, Warszawa 1987.
6. R. HILL, *The mathematical theory of plasticity*, Clarendon Press, Oxford 1950.
7. R. HILL, *On the limits set by plastic yielding to the intensity of singularities of stress*, J. Mech. Phys. Solids, 2, 278-285, 1954.

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