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IMPACT OF A CYLINDER AGAINST A RIGID TARGET PART II. INITIAL CONDITION

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An analysis of widely known Taylor's experiment that concerns the impact of short deformable cylinders made of a rigid-viscoplastic material against the rigid target, is performed. The case of axi-symmetric geometry with finite deformations and radial inertia is considered. The velocity initial condition given by the jump of the vertical component of the field does not belong to the problem solution as the equations describing the problem do not permit the first order discontinuity. To create the procedure initiating a numerical algorithm for this impact problem, the idea of a thin viscoplastic layer is introduced and a parametric approximation of the velocity field in a power form is proposed. The velocity field obtained from the approximation approches for $t \to t_0$ the profile characteristic for the viscoplastic model.

1. Introduction

There are several attempts to give the theoretical description and further to analyse the so-called "Taylor experimental configuration" (see [1]). The widely known Taylor's experiment concerns the impact of short deformable cylinders made of different materials against the rigid target. Numerous authors carried out the one-dimensional analysis of that experimental configuration assuming a rigid-viscoplastic material (see e.g., [2–5]). In 1991, the first analysis for the case of axi-symmetric geometry with finite deformations and radial inertia using rigid-viscoplastic material has appeared in [6].

One of the crucial points of the problem formulation is the assumption of the initial conditions for that process. Some authors gave initial conditions for the one-dimensional models in terms of velocity or uniaxial stress (cf. [2–5]). However, in their formulation the material model did not take into account the strain rate effects and the initial contact stress was finite (e.g., [1]), or they treated the material as a viscoplastic one and dealt with infinite initial stress.

In the paper the idea of a thin layer near the contact plane in the two-dimensional formulation is proposed. The global energy balance and the global momentum balance equations for the layer have been derived. The discrete form of these equations supplies the numerical code (see [7]) for the first time-step.

2. Formulation of the problem

A short and stress-free cylindrical specimen strikes perpendicularly on a rigid target with velocity v_0. Thermal effects, body forces and friction forces between the target and the specimen are neglected. However, radial inertia and axi-symmetric state of stress only are taken into account. The axial-symmetry condition allows to restrict considerations to the fields determined for two space Euler variables x, r and the time t.

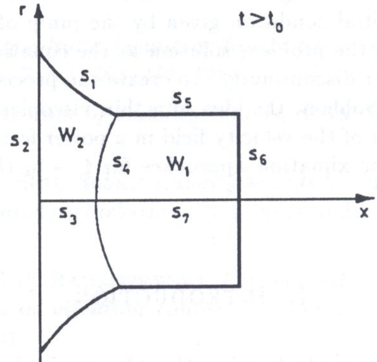

FIG. 1. Specimen under deformation.

In the region W_2 (Fig. 1) the behaviour of the material is described by the rigid-perfectly plastic constitutive equations (2.1) and (2.2) proposed by PERZYNA [8].

$$\text{(2.1)} \qquad \mathbf{d} = \gamma \left(\frac{\sqrt{J_2}}{\kappa} - 1 \right)^3 \frac{\mathbf{s}}{\sqrt{J_2}} \qquad \text{for} \quad \frac{\sqrt{J_2}}{\kappa} - 1 > 0,$$

$$\text{(2.2)} \qquad \mathbf{d} = 0 \qquad \text{for} \quad \frac{\sqrt{J_2}}{\kappa} - 1 \le 0,$$

where $\mathbf{d}$ is the stretching tensor, $\mathbf{s}$ the deviatoric Cauchy stress tensor, J_2 the second invariant of $\mathbf{s}$, γ the viscosity coefficient and k the yield limit in shearing. Hence equation $\sqrt{J_2} = \kappa$ represents the yield condition. In the problem under consideration, tensor $\mathbf{d}$ is equal to its deviatoric part because of the incompressibility constraints $\operatorname{tr} \mathbf{d} = 0$.

On the moving surface S_4 (Fig. 1) separating two parts of the specimen, the plastic one W_2 and the rigid one W_1 (where the condition $\sqrt{J_2} - \kappa \le 0$ is valid), the relation $\sqrt{J_2} - \kappa = 0$ holds. The following equation of the global momentum balance governs the movement of the rigid part W_1

$$\varrho \frac{d}{dt}\left[\mathbf{v}(t) \int\limits_{W_{1(t)}} dv\right] = \int\limits_{\overline{S}_4} \mathbf{t}_n \, ds + \int\limits_{\overline{S_5 \cup S_6}} \mathbf{t}_n \, ds, \tag{2.3}$$

where $\mathbf{t}_n$ (continuity of the traction was proved in [6]) is the traction (Fig. 1) and $\mathbf{v}(t)$ is the velocity of the rigid part W_1.

The boundary conditions for $t > t_0$ are given by

$$\mathbf{T}\,\mathbf{n} = 0 \qquad \text{on} \quad \overline{S}_1 \cup \overline{S}_5 \cup \overline{S}_6\,, \tag{2.4}$$

$$\mathbf{v}\cdot\mathbf{n} = 0, \qquad \mathbf{T}\,\mathbf{n}\cdot\boldsymbol{\tau} = 0 \qquad \text{on} \quad \overline{S}_2\,, \tag{2.5}$$

$$\mathbf{v}\cdot\mathbf{n} = 0, \qquad \mathbf{T}\,\mathbf{n}\cdot\boldsymbol{\tau} = 0 \qquad \text{on} \quad \overline{S}_3 \cup \overline{S}_7\,, \tag{2.6}$$

where $\mathbf{T}$ is the Cauchy stress tensor, $\boldsymbol{\tau}$ and $\mathbf{n}$ denote tangent and normal versors, respectively, outside the region $W_1 \cup W_2$ (Fig. 1).

The initial configuration of the specimen χ_0 and the striking velocity v_0 describe the initial conditions at $t = t_0$ as follows:

$$\chi = \chi_0\,, \tag{2.7}$$

$$\mathbf{v}\cdot\mathbf{i} = v_0\,, \qquad \mathbf{v}\cdot\mathbf{j} = 0 \qquad \text{on} \quad \overline{W}_1, \tag{2.8}$$

where χ is the function of motion and χ_0 describes the initial specimen configuration, $\mathbf{i}$ and $\mathbf{j}$ are versors of the x and r axes, respectively. Hence for $t = t_0$ the velocity jump $[\![v]\!] = 0 - v_0$ at the contact plane occurs.

The viscoplastic process for $t > t_0$ when $W_2 \neq \emptyset$ is governed by the system of equations

$$\varrho\dot{\mathbf{v}} = \operatorname{div}\mathbf{T}, \tag{2.9}$$

$$\mathbf{d} = \frac{1}{2}\left[\operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^T\right], \tag{2.10}$$

$$\operatorname{tr}\left[\operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^T\right] = 0, \tag{2.11}$$

together with Eqs. (2.1) – (2.3) and initial-boundary conditions (2.4) – (2.8).

3. Discussion of the type of the PDE system

The determination of the type of the system of equations is an essential point in solving the initial-boundary-value problem. It allows to impose the admissible initial and boundary conditions for that problem.

Let us make the inverse transformation of the constitutive equation (2.1).

$$\mathbf{s} = \kappa\left[\left(\frac{\sqrt{J_d}}{\gamma}\right)^{1/3} + 1\right]\frac{\mathbf{d}}{\sqrt{J_d}}, \qquad J_d > 0. \tag{3.1}$$

Note that $\mathbf{s}$ is some nonlinear tensor function of the velocity gradient. If we denote that function by $\widehat{\mathbf{s}}$ and recall the geometric relation (2.10), then the constitutive equation (3.1) has the following form:

$$\mathbf{s} = \widehat{\mathbf{s}}(\operatorname{grad}\mathbf{v}). \tag{3.2}$$

Recall the definition of the material derivative; then the system of Eqs. (2.9)–(2.11) and (3.1) can take the form, with $\mathbf{T} = \mathbf{s} + \sigma\mathbf{1}$,

$$\operatorname{div}\left[\widehat{\mathbf{s}}(\operatorname{grad}\mathbf{v}) + \sigma\mathbf{1}\right] - \varrho\frac{\partial\mathbf{v}}{\partial t} - \varrho\operatorname{grad}\mathbf{v}\,\mathbf{v} = \mathbf{0}, \tag{3.3}$$

$$\operatorname{tr}\left[\operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^T\right] = 0. \tag{3.4}$$

The equation (3.3) after transformation can be written as follows:

$$\frac{\partial\mathbf{v}}{\partial t} + \operatorname{grad}\mathbf{v}\,\mathbf{v} - \frac{1}{\varrho}\operatorname{grad}\sigma - \frac{1}{\varrho}\operatorname{div}\widehat{\mathbf{s}}(\operatorname{grad}\mathbf{v}) = \mathbf{0}. \tag{3.5}$$

The last vectorial term of the above equation can be expressed in the indicial notation

$$\frac{1}{\varrho}\left[\operatorname{div}\widehat{\mathbf{s}}(\operatorname{grad}\mathbf{v})\right]_k = \frac{1}{\varrho}\frac{\partial\widehat{s}_{kl}}{\partial v_{ij}}v_{ij;l}, \tag{3.6}$$

where $v_{ij} \equiv v_{i;j}$, and the semicolon denotes the covariant differentiation. If we introduce the nonlinear matrix function $\mathbf{A}$ with the components $\mathbf{A}_{klij} = \partial\widehat{s}_{kl}/\partial v_{ij}$ and denote by $\oplus$ multiplication with contraction related to the above indicial notation, we get

$$\frac{1}{\varrho}\operatorname{div}\widehat{\mathbf{s}}(\operatorname{grad}\mathbf{v}) = \frac{1}{\varrho}\mathbf{A}(\operatorname{grad}\mathbf{v}) \oplus \operatorname{grad}\operatorname{grad}\mathbf{v}. \tag{3.7}$$

Finally, the system of Eqs. (2.9)–(2.11) and (3.1) can be expressed in the following matrix form

$$\mathbf{P}\begin{bmatrix}\dfrac{\partial\mathbf{v}}{\partial t}\\[2mm] \dfrac{\partial\boldsymbol{\sigma}}{\partial t}\end{bmatrix} + \mathbf{Q}\begin{bmatrix}\operatorname{grad}\mathbf{v}\\ \operatorname{grad}\boldsymbol{\sigma}\end{bmatrix} + \frac{1}{\varrho}\mathbf{R}\begin{bmatrix}\operatorname{grad}\operatorname{grad}\mathbf{v}\\ \operatorname{grad}\operatorname{grad}\boldsymbol{\sigma}\end{bmatrix} = \mathbf{0}, \tag{3.8}$$

where the matrices **P**, **Q**, **R** are to be specified from the Eqs. (3.3)–(3.4) after the specification of the matrix **A**.

The above system of equations is unclassifiable as the first order system (2.9)–(2.11) and (3.1); reduced to the form (3.3)–(3.4), it is a mixed system of different order equations and verges upon the parabolic system, because two scalar equations (3.5) are, for a fixed s, parabolic ones if the matrix **A** (see (3.7)) is nonsingular.

Because of the unclassifiable character of the system, one can not determine *a priori* the initial-boundary conditions but it can be done on the basis of heuristic considerations and physical interpretations.

4. Velocity field approximation

Now we propose the velocity field approximation for the discrete initial condition formulation investigated in the paper.

The impact, as a physical event, implies a finite initial stress in the contact plane. The initial conditions for the impact problem can be posed by introducing into the problem the initial stress or velocity jump at this plane. In the paper [6] it was proved that the initial condition (2.8) does not belong to the problem solution because, as the analysis on the moving surface shows ([6]), the equations describing the problem do not permit the first order discontinuity of the velocity field.

Taking into account the discontinuity of the physical initial conditions (2.8), one should look for the solution of this problem in a one-sided open interval $(t_0, t_k]$. Hence, the numerical solution of this initial value in the whole interval $(t_0, t_k]$ needs an extra treatment of the initial conditions. Similar difficulties take place in the one-dimensional case for the rigid-viscoplastic rod impact. The papers [3, 5] and [9] related to the Taylor's experimental configuration for the one-dimensional formulation and rigid-viscoplastic material, give some interesting treatments of initial conditions. In [3] e.g., the non-physical instantaneous cross-section change is assumed to achieve the finite initial stress at the impact plane. The contact plane is the shock wave front, because the artificially introduced strain jump implies the velocity jump, what can not occur for the rigid-viscoplastic model.

In [5] and [9] the one-dimensional rigid-viscoplastic model is considered and consistently, the initial stress at the contact plane tends to infinity. In the numerical solution of [9], the authors have assumed that the initial stress is 200 s_0 (where s_0 is the yield limit), and after four time steps the proper distribution takes place.

The present paper proposes the velocity field approximation for the first time step. Let us consider the first stage of the cylinder impact. The velocity of the elastic wave in the rigid-viscoplastic material is formally infinite, so the elastic loading and unloading of the stress-free specimen take place instantaneously after it gets in contact with the rigid target at $t = t_0$. If the velocity of striking is high enough, yielding of the specimen near the contact plate takes place. The surface bounding the viscoplastic region will then be moving towards the free end of the specimen.

One can assume that for the small time increment Δt (around 10^{-7} s; notice that the duration of the deformation process is within the range of $10^{-6}\,\mathrm{s} \div 10^{-5}\,\mathrm{s}$, see [5]), the viscoplastic layer between the contact plane and the moving surface is a cylinder with the base identical with the initial cross-section of the specimen.

Let us also assume that the height of this viscoplastic layer x_{g_1} is of the order of 10^{-5} m, so we consider it to be small as compared with the base radius (Fig. 1).

We treat the above mentioned time increment as the first time step for the computational code. In order to give the initial velocity field profile, one has to determine the position x_{g_1} (Fig. 2) at the moving surface for $t = t_0 + \Delta t$. For this reason we have to formulate the global energy balance equation for the deforming specimen (see Fig. 1) for $t \in (t_0, t_0 + \Delta t)$.

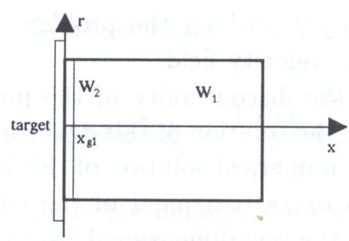

FIG. 2.

The general form of this equation is following (see [7])

$$\int_{B_t} \operatorname{tr} \mathbf{T}\,\mathbf{d}\,dv + \frac{dK}{dt} = 0, \tag{4.1}$$

where K is the kinetic energy of the specimen, while B_t is the specimen configuration for $t \in (t_0, t_0 + \Delta t)$.

Let us present this equation for the velocity field we are searching for. For this field the incompressibility and the kinematic boundary conditions have to be satisfied. Moreover, due to the geometry of the viscoplastic layer ($x_{g_1} \approx 10^{-5} \div 10^{-4}$, $R_0 = 10^{-2}$), the homogeneous boundary conditions at the contact plane and at the moving surface being

$$v_x(0,r,t) = 0, \qquad v_x(x_g(t),r,t) = v(t), \tag{4.2}$$

where v_x is the component of the velocity field $\mathbf{v}$, $x_g(t)$ is the position of the moving surface for $t \in (t_0, t_0+\Delta t)$ (see Fig. 2), one can assume the condition

$$\frac{\partial v_x}{\partial r} = 0 \quad \text{on} \quad \overline{W}_2 \quad \text{for} \quad t \in (t_0, t_0 + \Delta t) \tag{4.3}$$

to be satisfied.

For the incompressibility equation in the region W_2

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0, \tag{4.4}$$

one can give the following solution for the velocity component v_r:

$$v_r = \frac{r_0}{r} v_r(r_0) + \left(\frac{1}{r}\right) \int\limits_{r_0}^{r} -f(x,s,t)s\,ds. \tag{4.5}$$

Due to the symmetry condition on the x-axis $v_r(x,0,t) = 0$ and passing with r_0 to zero, in the last expression $f = \partial v_x/\partial x$, we get the final form of the solution (4.5).

Using the above relation one can express the velocity component v_r as follows:

$$v_r = -\frac{1}{r} \int\limits_0^r \frac{\partial v_x}{\partial x} s\,ds. \tag{4.6}$$

With regard to (4.3) we get the relation

$$v_r = -\frac{1}{2} \frac{\partial v_x}{\partial x} r. \tag{4.7}$$

In order to approximate the velocity field in the viscoplastic layer for $t \in (t_0, t_0 + \Delta t)$, the following form of v_x is proposed:

$$v_x(x,t) = \frac{1}{12} \frac{A(t)}{x_g^3(t)} x^4 - \frac{1}{6} \frac{A(t)}{x_g^2(t)} x^3 + B(t)x + C(t), \tag{4.8}$$

where functions $A(t)$, $B(t)$ and $C(t)$ are to be specified from the boundary conditions

$$\begin{gathered} \left.\frac{\partial}{\partial x} v_x(x,r,t)\right|_{x=x_g(t)} = 0, \qquad v_x(0,r,t) = 0, \\ v_x(x_g(t),r,t) = v(t). \end{gathered} \tag{4.9}$$

These conditions follow from the observations that the gradient jump $[\![\operatorname{grad}\mathbf{v}]\!]$ on the moving boundary S_4 vanishes (see [6]) and $\operatorname{grad}\mathbf{v} = 0$ on $\overline{W}_1$.

Taking into account the above boundary conditions, the velocity component $v_x(x, r, t)$ is given by the polynomial

$$v_x(x,t) = \frac{v(t)}{x_g^4(t)}x^4 - 2\frac{v(t)}{x_g^3(t)}x^3 + 2\frac{v(t)}{x_g(t)}x, \tag{4.10}$$

and the gradient component by

$$\frac{\partial}{\partial x}v_x(x,t) = 4\frac{v(t)}{x_g^4(t)}x^3 - 6\frac{v(t)}{x_g^3}x^2 + 2\frac{v(t)}{x_g(t)}. \tag{4.11}$$

Taking into account the incompressibility condition (4.6), the v_r component is given by

$$v_r(x,r,t) = -\left[2\frac{v(t)}{x_g^4(t)}x^3 - 3\frac{v(t)}{x_g^3(t)}x^2 + \frac{v(t)}{x_g(t)}\right]r \tag{4.12}$$

and satisfies the condition $v_r[x_g(t), r, t] = 0$.

Recall that the stretching tensor component d_{xr} vanishes on S_2 (due to (2.1), $(2.5)_2$, cf. Fig. 1), hence taking into account

$$d_{xr} = \frac{1}{2}\left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x}\right) \tag{4.13}$$

and the assumption for the thin viscoplastic layer $\partial v_x/\partial r = 0$, we get for $t \in (t_0, t_0 + \Delta t]$ the following condition

$$\left.\frac{\partial}{\partial x}v_r(x,r,t)\right|_{x=0} = 0 \tag{4.14}$$

for $t \in (t_0, t_0 + \Delta t]$.

As all the gradient components disappear on the moving surface S_4, the condition

$$\left.\frac{\partial}{\partial x}v_r(x,r,t)\right|_{x=x_g(t)} = 0 \tag{4.15}$$

has to be satisfied.

After differentiation of (4.12), the gradient component $\dfrac{\partial v_r}{\partial x}(x, r, t)$ for the thin viscoplastic layer takes the form

$$\frac{\partial}{\partial x}v_r(x,r,t) = -\left(6\frac{v(t)}{x_g^4(t)}x^2 - 6\frac{v(t)}{x_g^3(t)}x\right)r. \tag{4.16}$$

Note that now the conditions (4.14) and (4.15) are satisfied.

The above considerations lead to the conclusion that the velocity field on $\overline{W}_2$ given by polynomials (4.10) and (4.12) satisfies the kinematic boundary conditions.

Let us assume the functions $v(t)$ and $x_g(t)$ to be affine

$$(4.17) \qquad v(t) = \frac{v_1 - v_0}{\Delta t} t + v_0 \, , \qquad x_g(t) = \frac{x_{g1}}{\Delta t} t$$

for $t \in (t_0, t_0 + \Delta t)$ and for the small time increment $\Delta t \approx 10^{-7}$ s, where $v_1 = v(t_0 + \Delta t)$.

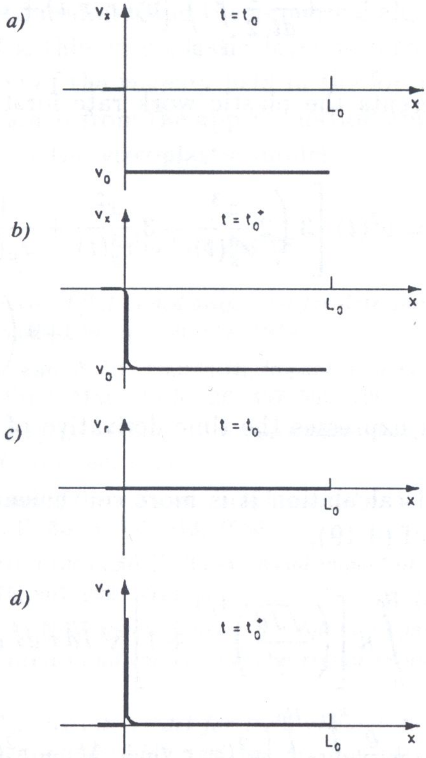

FIG. 3.

The plots of the calculated velocity components are given on the graphs above (Fig. 3). Their limiting values are as follows:

$$(4.19) \qquad \begin{aligned} \lim_{t \to t_0^+} v_x(x_g(t), r, t) &= v_0, \\ \lim_{t \to t_0^+} v_x(0, r, t) &= 0, \\ \lim_{t \to t_0^+} v_r(x_g(t), r, t) &= 0, \\ \lim_{t \to t_0^+} v_x(0, r, t) &= \infty. \end{aligned}$$

Introducing the constitutive equations (2.1) and (2.2) into the equation (4.1) we obtain the global energy balance equation for a rigid-viscoplastic rod striking a rigid target in the following form:

$$\tag{4.19} 4\int\limits_0^{R_0}\int\limits_0^{x_g(t)} \kappa\left[\left(\frac{\sqrt{J_d}}{\gamma}\right)^{1/3}+1\right]\sqrt{J_d}\,r\,dx\,dr + \frac{d}{dt}\frac{\varrho}{2}\int\limits_0^{R_0}\int\limits_0^{L_0}\left[v_x^2(x,r,t)+v_r^2(x,r,t)\right]r\,dx\,dr = 0.$$

The first term represents the plastic work rate for the viscoplastic layer, where

$$\tag{4.20} J_d(x,r,t) = v^2(t)\left[3\left(2\frac{x^3}{x_g^4(t)} - 3\frac{x^2}{x_g^3(t)} + \frac{1}{x_g(t)}\right)^2 + 9\left(\frac{x^2}{x_g^4(t)} - \frac{x}{x_g^3(t)}\right)^2 r^2\right],$$

while the second one expresses the time derivative of the kinetic energy of the specimen.

For the numerical calculation it is more convenient to use the equation given below instead of (4.19),

$$\tag{4.21} \int\limits_0^{t_0+\Delta t}\int\limits_0^{x_g(t)}\int\limits_0^{R_0} \kappa\left[\left(\frac{\sqrt{Jd}}{\gamma}\right)^{1/3}+1\right]\sqrt{Jd}\,r\,dx\,dr\,dt + \frac{\varrho}{2}\int\limits_0^{x_{g1}}\int\limits_0^{R_0}\left[v_x^2(x,r,t_0+\Delta t)+v_r^2(x,r,t_0+\Delta t)\right]r\,dx\,dr + \frac{1}{2}\varrho R_0^2(L_0-x_{g1})v_1^2 - \frac{1}{2}\varrho R_0^2 L_0 v_0^2 = 0,$$

where the second and the third terms express the kinetic energy after the impact, i.e., for $t = t_0 + \Delta t$, and the last one represents the initial energy of the specimen; v_x and v_r are given by (4.10) and (4.12).

The solution of the system of equations composed of the last relation for the unknown quantity x_{g1} together with the basic system of equations with the initial-boundary conditions (2.1)–(2.11) formulated for the recurrent iteration (see [7]), gives the procedure for the first time step.

5. Conclusions

To conclude the above considerations, one should notice that the initial condition for velocity given by the jump of the v_x component does not belong to the problem solution since the equations describing the problem do not permit the first order discontinuity of the velocity field. This viscoplastic process has to be considered in a left-sided open interval $(t, t_k]$ and the component v_x for $t \to t_0$ is a function with a high gradient.

To create the procedure initiating the numerical algorithm for this impact problem, the idea of a thin viscoplastic layer is introduced, and the parametric approximation of the velocity field in the form of (4.8) is proposed. The velocity field obtained from the approximation approches for $t \to t_0$ the profile characteristic for the viscoplastic model.

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