ENGINEERING TRANSACTIONS • Engng. Trans. • 44, 2, 253–259, 1996
Polish Academy of Sciences • Institute of Fundamental Technological Research

DURABILITY OF BRITTLE MATERIAL SUBJECTED TO STATIC LOADS

K. M. MIANOWSKI (WARSZAWA)

In the paper the method of estimation the durability of brittle materials is presented. The durability of highly stressed materials is an experimental quantity, which may be determined in a laboratory. To find out the durability of less stressed materials it has been assumed that the fatigue is related to the thermal vibrations of the microstructural elements. As an example, the durability of the tendons in prestressed structures is calculated.

Durability means here a period of time since the application of a given load up to the failure of the element under that load. In the paper certain earlier concepts proposed by the author in the field of the strength of brittle materials are developed.

An element subjected to tensile load is considered. The load is quantified by the ratio between tensile stress σ and conventional strength f under tensile load.

The durability of highly stressed brittle materials (95 – 98 % of conventional strength) is the experimental quantity which is taken as a basis for the proposed generalization. That durability in the scale of months or hours may be easily determined in a laboratory. Expected durability of lower stressed materials is based on extrapolation which is derived by taking into account the damage mechanism of materials subjected to loads. According to the previously expressed opinion in the paper [1], this mechanism is related, as well as in the case of plastic yielding, to the phenomenon of thermal vibrations of molecules in the material structure (atoms, particles, ions).

In the case of plastic materials, thermal vibrations initiate slipping mechanisms. Consequently, the elements under tension become narrow and the cross-section area is reduced without damaging the internal bonds; destruction of these bonds appears only at the final stage of the process.

In brittle materials the slipping mechanisms do not exist. Influence of thermal vibrations consists in the fact that beyond certain stresses caused by external loads, the energy of thermal vibrations may exceed the activation energy and certain bonds may fail.

The mechanism described above is analyzed on a model composed of two single structural elements. The influence of the activation energy is represented by an elastic bond between these elements with strength equal to f_1.

Strength f_1 is reduced by forces S_1, produced by a constant external load. Therefore, the activation energy W for the assumed model is:

$$W = \frac{2r}{2E_1}(f_1 - S_1)(f_1 + S_1)$$

or

$$W = \frac{r f_1^2}{E_1}\left(1 - \frac{S_1^2}{f_1^2}\right). \tag{1}$$

Here r – radius of the structural element,

$$f_1 = \alpha\frac{f}{L}, \qquad S_1 = \frac{\sigma_t}{L}, \qquad E_1 = \frac{E}{L}, \tag{2}$$

L – number of structural elements in a unit of the cross-section area,
E – Young modulus,
σ_t – macro-stress caused by the external load,
f – macro-strength of the material, determined in a conventional way,
α – coefficient representing increase of strength of the bonds when the influence of thermal vibrations is eliminated.

The equation (1) represents a limit which has to be exceeded by the energy of thermal vibrations if the bonds fail. The appearance of vibrations of such energy may be defined in a random way. The number n of vibrations corresponding to that energy is given by the relation:

$$np(W) = 1; \tag{3}$$

here $p(W)$ – probability that the energy of vibration will exceed the value W.

The vibrations which exceed the value of W may appear during any vibrations within the limits $(1, n)$. Therefore, average number of vibrations necessary to impose the vibrations of the needed energy is $0.5n$, and average time τ_i corresponding to the vibrations of energy exceeding W is equal to

$$\tau_i = 0.5\frac{1}{p(W)}\nu^{-1}(T); \tag{4}$$

here $\nu(T)$ is the frequency of thermal vibrations at temperature T.

The form of equation $p(W)$ is taken after Maxwell - Boltzmann:

$$p(W) = Be^{-W/(kT)};$$

here B is a material constant.

Then, taking into account Eq. (1) and superposing (2), the relation (4) takes the form

$$\tau_i = 0.5\nu^{-1}(T)e^{\frac{r\alpha^2 f^2}{ELkT}\left[1-\left(\frac{\sigma_t}{\alpha f}\right)^2\right]}; \tag{5}$$

here k – Boltzmann constant, T – temperature at which the process is realized.

Up to now the analyzed model has been composed of two structural elements and one bond. When the process is considered over the total fracture surface, then the number of couples of elements and their bonds is high. It is assumed that all bonds have the same strength (f_1) and at a given moment of the process any difference betweeen them may concern only the amplitude of thermal vibrations. The damage is caused by thermal vibrations. At the beginning of the process the breaking of the bonds occurs at high values of thermal vibrations. At the extension of damage, when the forces in bonds increase, they fail at lower values of the energy of vibrations. Finally, the damage process reaches the spontaneous phase and one can suppose that it is advancing without any influence of the thermal vibrations. Therefore, the basic factor in the process is the damage degree, which may be expressed by the factor $(1-u)^{-1}$, where u means relative damage, i.e. the ratio between the broken bonds and the initial number of bonds.

Influence of the damage is taken into the account when factor $(1-u)$ is introduced to the exponent in Eq. (5). Also the influence of the stress concentration around the damaged regions is considered. According to the assumed model, maximum value of the forces in the bonds cannot be higher than the strength of a bond after it is released from the thermal vibrations. This means that the value of the concentration factor m should be higher than 1 and lower than α; the value of α is discussed below. It is finally assumed that the value of m may be represented by the relation

$$m = (1-u)^{-1/2}. \tag{5'}$$

Combined influence of both these factors can be represented by their product $(1-u)^{-3/2}$.

Therefore, the general relation expressing the damaged state of an arbitrary bond is expressed by the relation

$$\tau_i = 0.5\nu^{-1}(T)e^{\frac{r\alpha^2 f^2}{ELkT}\left[1-\left(\frac{\sigma_t}{\alpha f(1-u)^{3/2}}\right)^2\right]}. \tag{6}$$

Finally, the material durability can be expressed by the integral:

$$\tau_i = 0.5\nu^{-1}(T) \int_{u=0}^{u=1-\left(\frac{\sigma_t}{\alpha f}\right)^{2/3}} e^{\frac{r\alpha^2 f^2}{ELkT}\left(1-\left[\frac{\sigma_t}{\alpha f(1-u)^{3/2}}\right]^2\right)} du. \tag{7}$$

The upper limit of integration corresponds to such a state of damage in which the damage process is developing spontaneously. Its value is determined by assuming, that the exponent in Eq. (7) is equal to zero.

The values of all quantities appearing in the exponent of Eq. (7) in front of the braces are constant for the given material and temperature. That factor denoted by A is equal

$$A = \frac{r\alpha^2 f^2}{ELkT}.$$

Relation (7) takes the form:

$$\tau_i = 0.5\nu^{-1}(T) \int_{u=0}^{u=1-\left(\frac{\sigma_t}{\alpha f}\right)^{2/3}} e^{A\left(1-\left[\frac{\sigma_t}{\alpha f(1-u)^{3/2}}\right]^2\right)} du. \tag{8}$$

There is no doubt that further research is needed to determine the theoretical value of A, but at present it is assumed that this value can be determined experimentally from the measurements of durability of a highly stressed brittle material. Then the appropriate values of the durability τ and of the ratio σ_t/f should be introduced to the formula (8). When A is determined and introduced to (8), then the durability for an arbitrary degree of loading may be calculated.

Up to now, the value of the coefficient α has not been determined. It has to express the increase of strength of the bonds caused by elimination of the influence of temperature. The value of α can be determined from the strength of the respective materials at cryogenic temperatures as close as possible to absolute zero.

A different method to determine the value of α at room temperature is based on testing of the strength of materials under very short loadings such as of a Hopkinson bar, in which a bar is loaded by a falling mass. In such conditions, the time needed for the highest value of stress to travel between two neighbouring material particles is

$$t = \frac{2r}{c}. \tag{9}$$

Here $2r$ – distance between particles, c – velocity of elastic wave.

When time t is smaller or equal to the period of vibration of the particles, then any appearance of thermal vibrations of high energy is improbable in that time.

In both the above experiments there is an appreciable enhancement of the strength. For instance, in the case of steel at temperature $-200°$ C the strength is doubled, [2]. Also a similar increase of strength was obtained for various brittle materials in Hopkinson bar tests, [3].

In general, the influence of both the low temperatures and short time of loading consists in the elimination of the effect of thermal vibrations of high energy. Consequently, it can be assumed in this model that all bonds have the same strength. Their failure after the external load reaches its critical value is simultaneous. Besides, in both the described experiments the mechanisms of slipping and destruction are blocked and the fracture is brittle. Therefore, the value of coefficient α can be calculated from the relation:

$$\frac{f_K}{f} = \left(1 + \frac{\Delta F}{F}\right)\alpha . \tag{10}$$

Here f_K – macroscopic strength of the material at cryogenic temperature, or in a test similar to the Hopkinson bar (nominal stress), f – macroscopic strength in an ordinary test, $\Delta F/F$ – relative increment of the cross-sectional area caused by blocking of the slipping mechanisms.

The value of α for steel is calculated from formula (10) and from the experimental data [2], where it was assumed that $\Delta F/F = 0.25$. Then $\alpha = 1.6$.

When the relation between the durability and stress is known, then it is possible to determine the admissible stress, according to the requested durability of the structure.

Other application of the proposed theory is the estimation of durability of existing structures, according to the actual stresses. An example is calculated for a high-carbon steel of conventional strength $f = 1500\,\mathrm{MPa}$ at room temperature, and for the assumed frequency of thermal vibrations $\nu = 10^{13}\,\mathrm{s}^{-1}$. The experiments [4] have proved, that the material stressed up to $0.96f$ exhibited durability $\tau \approx 5000\,\mathrm{s}$. In the calculations, according to the above analysis, it has been assumed that $\alpha = 1.6$, hence $A = 67.955$.

The results of the analysis are shown in Table 1 and in Fig. 1. It can be seen that the durability is a continuous function of stress. When stress is increased up to the value αf, then the durability is reduced and is approaching the duration of a single vibration $\tau = 10^{-13}\,\mathrm{s}$.

The high carbon steel subjected to testing is applied for prestressed structures. For many prestressed structures built in Western Europe, long term

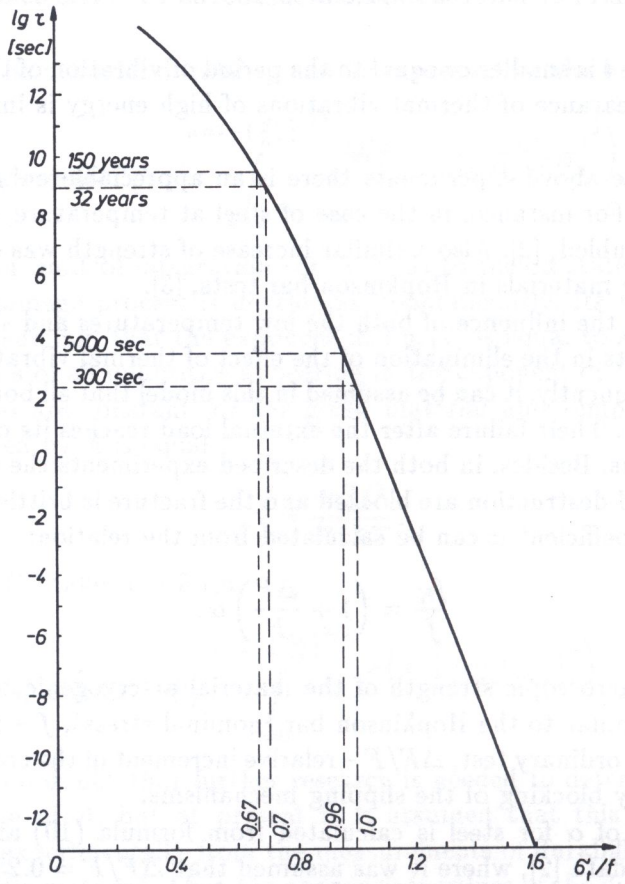

FIG. 1. Durability as a function of σ_t/f for a high-carbon steel.

Table 1. $\alpha = 1.6$; $A = 67.955$.

σ_t/f	τ
0.16	
0.32	2985172 years
0.48	52722 years
0.64	273 years
0.72	12.3 years
0.80	5 months
0.88	81.5 hours
0.96	5000 sec
1.12	0.5 sec
1.28	1.5×10^{-5} sec
1.60	10^{13} sec

stresses in the steel are equal to 0.7 of the conventional strength. In such conditions, as it may be deduced from the diagram in Fig. 1, the durability of prestressing steel equals approximately 32 years. For comparison, in prestressed structures built in Poland the long term stresses are limited to $0.55f$, and therefore their durability is practically unlimited.

When the requested durability for prestressed structures is between 150 and 200 years, then it can be determined that the value of admissible stress lies between 0.67 and $0.65f$.

References

1. K.M. Mianowski, *Tensile strength of brittle materials in high strain rate processes*, Engng. Trans., **36**, 4, 611–627, 1988.
2. J. Nemec, *Stiffness and safety of structural steel elements* [in Czech], Ceskoslovenske Akademie Ved, Praha 1963.
3. A.J. Zieliński, *Fracture of concrete and mortar under uniaxial impact tensile loading*, Delft University Press, 1982.
4. *Creep of prestressing steel subjected to high stresses* [in Polish], Instytut Techniki Budowlanej, Praca Naukowo-Badawcza NW-6/94, Warszawa 1995.

INSTITUTE FOR BUILDING ENGINEERING, WARSZAWA.

Received January 15, 1996.