ENGINEERING TRANSACTIONS • *Engng. Trans.* • **44**, 1, 99–111, 1996
Polish Academy of Sciences • Institute of Fundamental Technological Research

THERMAL INSTABILITY OF AN OLDROYDIAN VISCOELASTIC FLUID IN POROUS MEDIUM

R. C. SHARMA and P. KUMAR (SHIMLA)

The thermal instability of an Oldroydian viscoelastic fluid in porous medium is considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For stationary convection, the medium permeability is found to have a destabilizing effect. A sufficient condition for non-existence of overstability is obtained. The thermal instability of a rotating Oldroydian viscolastic fluid in porous medium is also studied wherein the rotation is found to have a stabilizing effect on the stationary convection and the sufficient conditions for non-existence of overstability are obtained.

1. Introduction

The problem of thermal convection in a horizontal layer of fluid has been discussed in detail by CHANDRASEKHAR [1]. BHATIA and SEINER [2] have studied the thermal instability of a Maxwell fluid in the presence of rotation and have found that the rotation has a destabilizing influence, for a certain numerical range, in contrast to the stabilizing effect on Newtonian fluid. ELTAYEB [3] considered the convective instability in a rapidly rotating Oldroydian fluid. TOMS and STRAWBRIDGE [4] have demonstrated experimentally that a dilute solution of methyl methacrylate in n-butyl acetate is in a good agreement with the theoretical model of Oldroyd fluid. HAMABATA and NAMIKAWA [5] have studied the propagation of thermoconvective waves in Oldroyd fluid.

The medium has been considered to be non-porous in all the above studies. LAPWOOD [6] has studied the stability of convective flow in hydrodynamics in a porous medium using Rayleigh's procedure. WOODING [7] has considered the Rayleigh instability of a thermal boundary layer in flow through porous medium. The gross effect, when the fluid slowly percolates through the pores of the rock, is represented by the well known Darcy's law.

The present paper deals which the thermal instability of an Oldroydian viscoelastic fluid in porous medium. The effect of rotation on the above problem is also considered. The study may be relevant to the stability of some polymer solutions, such as a dilute solution of methyl methacrylate in n-butyl acetate and to the stability of Maxwellian viscoelastic fluids. The problem proves also to be useful in chemical technology and geophysics.

2. Perturbation equations

Consider an infinite layer of Oldroydian viscoelastic fluid confined between the planes $z = 0$ and $z = d$ in porous medium of porosity ε and permeability k_1, acted on by gravity force $\mathbf{g}(0,0,-g)$. This layer is heated from below and the surfaces $z = 0$ and $= d$ are maintained at constant temperature T_0 and T_1 ($T_1 > T_0$), so that a uniform temperature gradient is maintained. The fluid is described by the constitutive relations

$$
\begin{aligned}
&T_{ij} = -p\delta_{ij} + \tau_{ij}\,,\\
(2.1)\qquad &\left(1+\lambda\frac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_0\frac{d}{dt}\right)e_{ij}\,,\\
&e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),
\end{aligned}
$$

where T_{ij}, τ_{ij}, e_{ij}, μ, λ, λ_0 ($<\lambda$) denote the normal stress tensor, shear stress tensor, rate-of-strain tensor, viscosity, stress relaxation time, and strain retardation time, respectively. p is the isotropic pressure, δ_{ij} is the Kronecker delta, d/dt is the mobile operator, while u_i and x_i are velocity and position vectors, respectively. Relations of the type (2.1) were first proposed by Jeffreys for Earth and later studied by Oldroyd [8]. Oldroyd [8] also shows that many rheological equations of state, of general validity, reduce to (2.1) when linearized. If $\lambda_0 = 0$, the fluid is Maxwellian, while for $\lambda_0 \neq 0$ we shall refer to the fluid as Oldroydian. $\lambda = \lambda_0 = 0$ gives a Newtonian viscous fluid.

As a consequnce of Brinkman's equation, the resistance term $-(\mu/k_1)\mathbf{u}$ will also occur with the usual viscous term in the equations of motion. Here $\mathbf{u}$ denotes the filtration velocity of the fluid.

The equations of motion and continuity for the Oldroydian viscoelastic fluid, following the Boussinesq approximation, are

$$
\begin{aligned}
(2.2)\qquad &\frac{1}{\varepsilon}\left(1+\lambda\frac{d}{dt}\right)\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}(\mathbf{u}\cdot\nabla)\right]\mathbf{u}\\
&= \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\frac{1}{\varrho_0}\nabla p + \mathbf{g}\left(1+\frac{\delta\varrho}{\varrho_0}\right)\right] + \left(1+\lambda_0\frac{d}{dt}\right)\left[\frac{\nu}{\varepsilon}\nabla^2-\frac{\nu}{k_1}\right]\mathbf{u},\\
(2.3)\qquad &\nabla\cdot\mathbf{u} = 0.
\end{aligned}
$$

The equation of state

$$\varrho = \varrho_0 [1 - \alpha(T - T_0)], \tag{2.4}$$

contains a thermal coefficient of expansion α, and ϱ_0, T_0 are the density and the temperature at the surface $z = 0$. The equation of heat conduction (JOSEPH [9]) is

$$[\varrho_0 c\varepsilon + \varrho_s c_s(1-\varepsilon)] \frac{\partial T}{\partial t} + \varrho_0 c(\mathbf{u}\cdot\nabla)T = k\nabla^2 T, \tag{2.5}$$

where ϱ_0, c; ϱ_s, c_s denote the density and heat capacity of the fluid and the solid matrix, respectively. k is the thermal conductivity. Equation (2.5) can be rewritten as

$$E\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \xi\nabla^2 T, \tag{2.6}$$

where $E = \varepsilon + (1-\varepsilon)\dfrac{\varrho_s c_s}{\varrho_0 c}$. The kinematic viscosity $\nu(= \mu/\varrho_0)$ and the thermal diffusivity $\xi(= k/\varrho_0 c)$ are assumed to be constants.

The steady solution is

$$\mathbf{u} = (0,0,0), \qquad T = T_0 - \beta z, \qquad \varrho = \varrho_0(1 + \alpha\beta z), \tag{2.7}$$

where $\beta(= |dT/dz|)$ is the magnitude of the uniform temperature gradient.

Let $\delta\varrho$, δp, θ, and $\mathbf{v}(u, v, w)$ denote respectively the perturbations in density ϱ, pressure p, temperature T and velocity $\mathbf{u}$ (initially zero). The change in density $\delta\varrho$, caused by the perturbation θ in temperature, is given by

$$\varrho + \delta\varrho = \varrho_0[1 - \alpha(T + \theta - T_0)] = \varrho - \alpha\varrho_0\theta,$$

i.e.

$$\delta\varrho = -\alpha\varrho_0\theta\,. \tag{2.8}$$

Then the linearized perturbation equations for the Oldroydian viscoelastic fluid flow through porous medium are

$$\frac{1}{\varepsilon}\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial \mathbf{v}}{\partial t} = \left(1 + \lambda\frac{\partial}{\partial t}\right)\left[-\frac{1}{\varrho_0}\nabla\delta p - \mathbf{g}\alpha\theta\right] + \frac{\nu}{\varepsilon}\left(1 + \lambda_0\frac{\partial}{\partial t}\right)\left[\nabla^2 - \frac{\varepsilon}{k_1}\right]\mathbf{v}, \tag{2.9}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{2.10}$$

and

$$\left(E\frac{\partial}{\partial t} - \xi\nabla^2\right)\theta = \beta w\,. \tag{2.11}$$

The fluid is confined between the planes $z = 0$ and $z = d$ maintained at constant temperatures. Since no perturbation in temperature is allowed and since normal component of the velocity must vanish on these surfaces, we have

$$w = 0, \qquad \theta = 0 \qquad \text{at} \qquad z = 0 \qquad \text{and} \qquad z = d. \tag{2.12}$$

Here we consider both the boundaries to be free. The case of two free boundaries is slightly artificial, except in stellar atmospheres (Spiegel [10]) and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The vanishing of tangential stresses at free surfaces implies

$$\frac{\partial^2 w}{\partial z^2} = 0 \qquad \text{at} \qquad z = 0 \qquad \text{and} \qquad z = d. \tag{2.13}$$

Eliminating δp between the three component equations of (2.9) and using (2.10), we obtain

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\nabla^2 \frac{\partial w}{\partial t} - g\alpha \nabla_1^2 \theta\right] = \frac{\nu}{\varepsilon}\left(1 + \lambda_0 \frac{\partial}{\partial t}\right)\left(\nabla^2 - \frac{\varepsilon}{k_1}\right)\nabla^2 w, \tag{2.14}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \text{and} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

3. Dispersion relation and discussion

Decompose the disturbances into normal modes and assume that the perturbed quantities are of the form

$$[w, \theta] = [W(z), \Theta(z)] \exp(ik_x x + ik_y y + nt), \tag{3.1}$$

where k_x, k_y are the wave numbers along x- and y-directions, respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is a complex constant.

The non-dimensional form of Eqs. (2.14) and (2.11), with the help of expression (3.1), becomes

$$(1 + F\sigma)\left[\sigma(D^2 - a^2)W + \frac{g\alpha d^2 \varepsilon}{\nu} a^2 \Theta\right] = (1 + F^* \sigma)\left(D^2 - a^2 - \frac{\varepsilon}{P_l}\right)(D^2 - a^2)W, \tag{3.2}$$

and

$$(3.3) \qquad (D^2 - a^2 - Ep_1\sigma)\Theta = -\frac{\beta d^2}{\xi} W,$$

where we have introduced new coordinates $(x', y', z') = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right)$ in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have put $a = kd$, $\sigma = nd^2/\nu$, $F = \lambda\nu/d^2$ and $F^* = \lambda_0\nu/d^2$. $p_1 = \nu/\xi$ is the Prandtl number and $P_l = k_1/d^2$ is the dimensionless medium permeability.

Eliminating Θ between Eqs. (3.2) and (3.3), we get

$$(3.4) \qquad (1 + F\sigma)\left[\sigma(D^2 - a^2)(D^2 - a^2 - Ep_1\sigma) - Ra^2\right] W$$
$$= (1 + F_0\sigma)\left(D^2 - a^2 - \frac{\varepsilon}{P_l}\right)(D^2 - a^2)(D^2 - a^2 - Ep_1\sigma)W,$$

where $R = \dfrac{g\alpha\beta d^4\varepsilon}{\nu\xi}$ is the modified Rayleigh number for porous medium.

The boundary conditions (2.12) and (2.13) transform to

$$(3.5) \qquad W = 0, \qquad D^2W = 0, \qquad \Theta = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1.$$

Using (3.5), it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence the proper solution of Eq. (3.4) characterizing the lowest mode is

$$(3.6) \qquad W = A \sin \pi z,$$

where A is a constant. Subsituting (3.6) in Eq. (3.4), we obtain the dispersion relation

$$(3.7) \qquad R_1 = \frac{i\sigma_1(1 + x)(1 + x + i\sigma_1 p_1 E)}{x} + \frac{(1 + x)\left(1 + x + \frac{\varepsilon}{P}\right)(1 + x + i\sigma_1 p_1 E)(1 + i\pi^2 F^* \sigma_1)}{x(1 + i\pi^2 F\sigma_1)},$$

where we have put

$$x = \frac{a^2}{\pi^2}, \qquad R_1 = \frac{R}{\pi^4}, \qquad i\sigma_1 = \frac{\sigma}{\pi^2}, \qquad P = \pi^2 P_l,$$

and $i = \sqrt{-1}$.

4. The stationary convection

For stationary convection, $\sigma = 0$ and Eq. (3.7) reduces to

$$R_1 = \frac{(1+x)^2\left(1+x+\frac{\varepsilon}{P}\right)}{x}. \tag{4.1}$$

Thus for stationary convection, the stress relaxation time parameter F and the strain retardation time parameter F^* vanish with σ and the Oldroydian fluid behaves like an ordinary Newtonian fluid. Equation (4.1) yields

$$\frac{dR_1}{dP} = -\frac{(1+x)^2\varepsilon}{xP^2}, \tag{4.2}$$

which is always negative meaning thereby that permeability of the medium has a destabilizing effect on the viscoelastic Oldroydian fluid, for stationary convection.

The dispersion relation (4.1) is analysed numerically. In Fig. 1, R_1 is plotted against x for $\varepsilon = 0.5$ and $P = 10, 100$. The destabilizing role of the medium permeability is clear from the decrease of Rayleigh number with the increase in permeability parameter P.

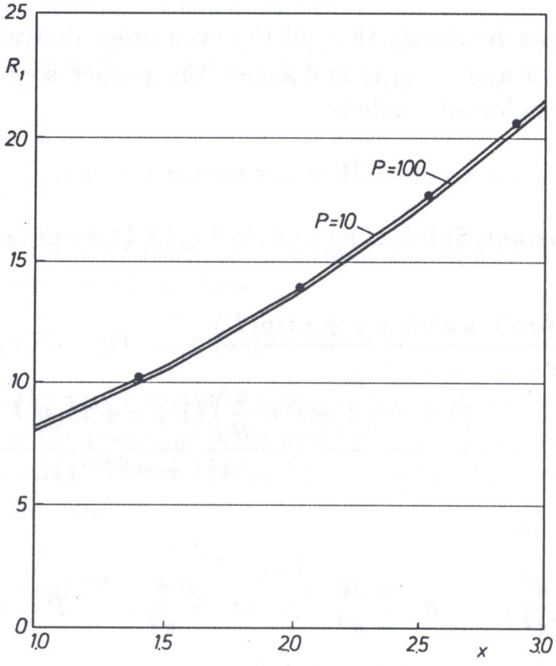

FIG. 1. The variation of Rayleigh number R_1 with x for $\varepsilon = 0.5$, $P = 10$ and 100.

5. The case of overstability

Here we examine the possibility of whether instability may occur as overstability. Since for overstability we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find the condition under which Eq. (3.7) will admit solutions with real values of σ_1. Puting $b = 1 + x$ and equating real and imaginary parts of Eq. (3.7), we get

$$R_1 \frac{b-1}{b} = b\left(b + \frac{\varepsilon}{P}\right) - \sigma_1^2 \left(Ep_1 + \pi^2 Fb + \pi^2 EF^* p_1 b + \frac{\pi^2 EF^* p_1 \varepsilon}{P}\right), \tag{5.1}$$

and

$$R_1 \frac{b-1}{b} \pi^2 F = b\left(1 + Ep_1 + \frac{\pi^2 F^* \varepsilon}{P} + \pi^2 F^* b\right) + \frac{Ep_1 \varepsilon}{P} - \pi^2 EFp_1 \sigma_1^2. \tag{5.2}$$

Eliminating R_1 between Eqs. (5.1) and (5.2), we obtain

$$\sigma_1^2 = -\frac{b\left[1 - \pi^2 F\left(b + \frac{\varepsilon}{P}\right)\right] + \left(b + \frac{\varepsilon}{P}\right)(p_1 E + \pi^2 F^* b)}{\pi^4 F^2 b + \pi^4 EFF^* p_1 \left(b + \frac{\varepsilon}{P}\right)}. \tag{5.3}$$

Since σ_1 is real in case of overstability, σ_1^2 should always be positive. Equation (5.3) shows that this is clearly impossible, i.e. σ_1^2 is always negative if

$$1 > \pi^2 F\left(b + \frac{\varepsilon}{P}\right),$$

which implies that

$$k^2 < \frac{1}{\lambda\nu} - \frac{\varepsilon}{k_1} - \frac{\pi^2}{d^2}. \tag{5.4}$$

Thus if

$$k^2 < \frac{1}{\lambda\nu} - \frac{\varepsilon}{k_1} - \frac{\pi^2}{d^2},$$

overstability is not possible. Inequality (5.4) is, therefore, the sufficient condition for the non-existence of overstability.

6. Rotating configuration

Here we consider the same problem as that described above, except that the system is in a state of uniform rotation $\mathbf{\Omega}(0,0,\Omega)$. The linearized perturbation equation are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \mathbf{v}}{\partial t}=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\frac{1}{\varrho_0}\nabla\delta p+\mathbf{g}\frac{\delta\varrho}{\varrho_0}\right] + \frac{2}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)(\mathbf{v}\times\mathbf{\Omega})+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\frac{\nu}{\varepsilon}\nabla^2-\frac{\nu}{k_1}\right]\mathbf{v}, \tag{6.1}$$

$$\nabla\cdot\mathbf{v}=0, \tag{6.2}$$

$$\left(E\frac{\partial}{\partial t}-\xi\nabla^2\right)\theta=\beta w. \tag{6.3}$$

Equations (6.1) and (6.2), using (2.8), yield

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}\nabla^2 w-g\alpha\varepsilon\nabla_1^2\theta+2\Omega\frac{\partial\zeta}{\partial z}\right] =\nu\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\varepsilon}{k_1}\right)\nabla^2 w, \tag{6.4}$$

and

$$\left[\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}-\nu\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\varepsilon}{k_1}\right)\right]\zeta =2\Omega\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}, \tag{6.5}$$

where

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

denotes the z-component of vorticity.

Let us decompose the disturbances into normal modes and assume that the perturbed quantities are of the form

$$[w,\theta,\zeta]=[W(z),\Theta(z),Z(z)]\exp(ik_x x+k_y y+nt), \tag{6.6}$$

where n, k_x, k_y have usual meanings.

Using expression (6.6), Eqs. (6.4) and (6.5) in nondimensional form become

$$(1+F\sigma)\left[\sigma(D^2-a^2)W+\frac{g\alpha d^2\varepsilon}{\nu}a^2\Theta+\frac{2\Omega d^3}{\nu}Dz\right] =(1+F^*\sigma)\left(D^2-a^2-\frac{\varepsilon}{P_l}\right)(D^2-a^2)W, \tag{6.7}$$

and

$$\left[\sigma(1+F\sigma)-(1+F^*\sigma)\left(D^2-a^2-\frac{\varepsilon}{P_l}\right)\right]Z=\frac{2\Omega d}{\nu}(1+F\sigma)DW. \tag{6.8}$$

Eliminating Θ and Z between Eqs. (6.7), (6.8) and (3.3), we obtain

$$\begin{aligned}(D^2-a^2)(D^2-a^2-Ep_1\sigma)\left[\sigma(1+F\sigma)-(1+F^*\sigma)\left(D^2-a^2-\frac{\varepsilon}{P_l}\right)\right]^2W\\ +T_A(1+F\sigma)^2(D^2-a^2-Ep_1\sigma)D^2W\\ =Ra^2(1+F\sigma)\left[\sigma(1+F\sigma)-(1+F^*\sigma)\left(D^2-a^2-\frac{\varepsilon}{P_l}\right)\right]W,\end{aligned} \tag{6.9}$$

where $T_A=\dfrac{4\Omega^2d^4}{\nu^2}$ is the Taylor number.

The boundary conditions (3.5) remain the same here for free boundaries and the proper solution of Eq. (6.9) characterizing the lowest mode is

$$W=A\sin\pi z. \tag{6.10}$$

Substituting (6.10) in Eq. (6.9), we obtain the dispersion relation

$$\begin{aligned}R_1=&\frac{(1+x)(1+x+ip_1E\sigma_1)}{x(1+i\pi^2F\sigma_1)}\\ &\cdot\left[\left(1+x+\frac{\varepsilon}{P}\right)(1+i\pi^2F^*\sigma_1)+i\sigma_1(1+i\pi^2F\sigma_1)\right]\\ &+T_{A_1}\frac{(1+x+ip_1E\sigma_1)(1+i\pi^2F\sigma_1)}{x\left[\left(1+x+\frac{\varepsilon}{P}\right)(1+i\pi^2F^*\sigma_1)+i\sigma_1(1+i\pi^2F\sigma_1)\right]},\end{aligned} \tag{6.11}$$

where

$$T_{A_1}=\frac{T_A}{\pi^4}\,.$$

For stationary convection $\sigma=0$ and the Eq. (6.11) reduces to

$$R_1=\frac{(1+x)^2\left(1+x+\frac{\varepsilon}{P}\right)}{x}+T_{A_1}\frac{(1+x)}{x\left(1+x+\frac{\varepsilon}{P}\right)}\,. \tag{6.12}$$

Thus for stationary convection, the stress relaxation time parameter F and strain retardation time F^* vanish with σ and the Oldroydian fluid behaves like an ordinary Newtonian fluid.

Equation (6.12) yields

$$(6.13)\qquad \frac{dR_1}{dP} = -\frac{(1+x)\varepsilon}{xP^2}\Bigg[x(1+x+P)^2 + x^2 + 2x\left(1+\frac{\varepsilon}{P}\right) + \left(1+\frac{\varepsilon}{P}\right)^2 - T_{A_1}\Bigg]\left(1+x+\frac{\varepsilon}{P}\right)^{-2}.$$

When $T_{A_1} < \left(1+\frac{\varepsilon}{P}\right)^2$, the medium permeability has a destabilizing effect on the system for Oldroydian viscoelastic fluid.

In the absence of rotation, the medium permeability has a destabilizing effect.

Also Eq. (6.12) gives

$$(6.14)\qquad \frac{dR_1}{dT_{A_1}} = \frac{(1+x)}{x\left(1+x+\frac{\varepsilon}{P}\right)},$$

which shows that rotation has a stabilizing effect on the system for Oldroydian fluid in a porous medium, for stationary convection.

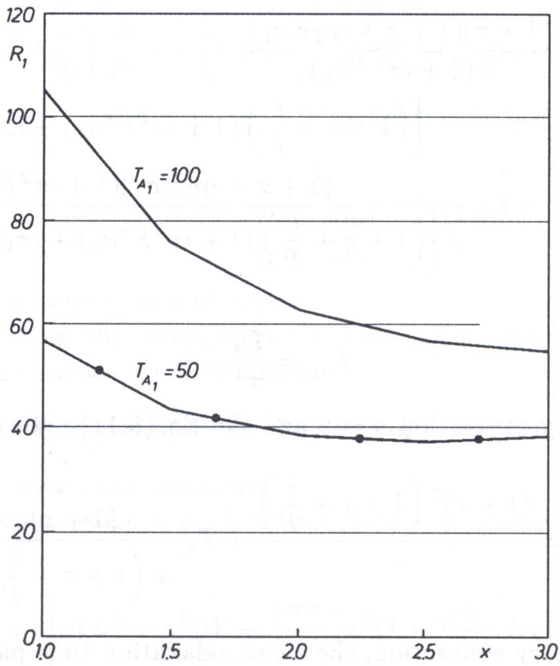

FIG. 2. The variaton of Rayleigh number R_1 with x for $\varepsilon = 0.5$, $P = 10$, $T_{A_1} = 50$ and 100.

The dispersion relation (6.12) is also analysed numerically. In Fig. 2, R_1 is ploted against x for $\varepsilon = 0.5$, $P = 10$ and $T_{A_1} = 50, 100$. The stabilizing role of the rotation is clear from the increase of the Rayleigh number with increasing rotation parameter T_{A_1}.

We now consider the possibility of whether instability may occur as an overstability. Since for overstability we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions under which (6.11) will admit solutions with real-valued σ_1.

Separating the real and imaginary parts of Eq. (6.11), we obtain

$$R_1(b-1)\left[\left(b+\frac{\varepsilon}{P}\right)-c_1\left\{2\pi^2F+\pi^4FF^*\left(b+\frac{\varepsilon}{P}\right)\right\}\right] \tag{6.15}$$
$$=c_1^2\left[2Ep_1\pi^2Fb+\pi^4F^2b^2+2Ep_1\pi^4FF^*b\left(b+\frac{\varepsilon}{P}\right)\right]$$
$$-c_1\left[\pi^4F^{*2}b^2\left(b+\frac{\varepsilon}{P}\right)^2+2\pi^2F^*b^2\left(b+\frac{\varepsilon}{P}\right)+b^2\right.$$
$$+2\pi^2Fb^2\left(b+\frac{\varepsilon}{P}\right)+2Ep_1b\left(b+\frac{\varepsilon}{P}\right)+2Ep_1\pi^2F^*b\left(b+\frac{\varepsilon}{P}\right)$$
$$\left.+\pi^2FT_{A_1}(2Ep_1+\pi^2Fb)\right]+b\left[T_{A_1}+b\left(b+\frac{\varepsilon}{P}\right)^2\right],$$

and

$$R_1(b-1)\left[1+\pi^2(F+F^*)\left(b+\frac{\varepsilon}{P}\right)-\pi^4F^2c_1\right]=c_1^2\left[\pi^4F^2Ep_1b\right] \tag{6.16}$$
$$-c_1\left[Ep_1b+\pi^4F^{*2}Ep_1b\left(b+\frac{\varepsilon}{P}\right)^2+2\pi^2Ep_1b(F+F^*)\left(b+\frac{\varepsilon}{P}\right)\right.$$
$$\left.+2\pi^2Fb^2+2\pi^4FF^*b^2\left(b+\frac{\varepsilon}{P}\right)+\pi^4F^2Ep_1T_{A_1}\right]$$
$$+\left[b\left(b+\frac{\varepsilon}{P}\right)^2\left\{Ep_1+2\pi^2F^*b\right\}+2b^2\left(b+\frac{\varepsilon}{P}\right)\right.$$
$$\left.+T_{A_1}(Ep_1+2\pi^2Fb)\right],$$

where

$$c_1=\sigma_1^2.$$

Eliminating R_1 between Eqs. (6.15) and (6.16), we obtain

$$A'c_1^3+B'c_1^2+C'c_1+D'=0, \tag{6.17}$$

where

$$A' = \pi^8 F^3 b \left[Fb + F^* E p_1 \left(b + \frac{\varepsilon}{P} \right) \right], \tag{6.18}$$

and

$$D' = b\left(b+\frac{\varepsilon}{P}\right)^2 \left[b\left(1-\pi^2 F \overline{b+\frac{\varepsilon}{P}}\right) + \left(b+\frac{\varepsilon}{P}\right)\left(\pi^2 F^* b + E p_1\right)\right] + T_{A_1}\left[E p_1 \frac{\varepsilon}{P} + \pi^2 (F - F^*)\left(b+\frac{\varepsilon}{P}\right) b + b(E p_1 - 1)\right]. \tag{6.19}$$

The values of B' and C', involving large numbers of terms, have not been written here. $F > F^*$ is true since the stress relaxation time parameter λ is always greater than the strain retardation time parameter λ_0 (by definition). Since σ_1 is real for overstability, the three values of c_1 are positive. The product of the roots of Eq. (6.17) is $-(D'/A')$ and if this is to be positive, then $D' < 0$, since from (6.18) $A' > 0$.

Equation (6.19) shows that this is clearly impossible if

$$i > \pi^2 F \left(b + \frac{\varepsilon}{P} \right) \quad \text{and} \quad p_1 E > 1, \tag{6.20}$$

which further implies that

$$k^2 < \frac{1}{\lambda \nu} - \frac{\varepsilon}{k_1} - \frac{\pi^2}{d^2} \quad \text{and} \quad \frac{\nu}{\xi}\left[\varepsilon + (1-\varepsilon)\frac{\varrho_s c_s}{\varrho c}\right] > 1. \tag{6.21}$$

Thus the inequalities (6.21) are the sufficient conditions for the non-existence of overstability.

7. Conclusion

A dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd's viscoelastic fluid. The stability of such polymer solutions and of Maxwellian viscoelastic fluids may prove to be useful in chemical technology and geophysics.

A layer of Oldroydian viscoelastic fluid heated from below in a porous medium is considered. For stationary convection, the medium permeability is found to speed up the onset of thermal convection. A sufficient condition for the non-existence of overstability is obtained. For a rotating Oldroydian fluid in porous medium, the rotation is found to have a stabilizing effect on stationary convection and the sufficient conditions for non-existence of overstability are obtained.

References

1. S. Chandrasekhar, *Hydrodynamic and hydromagnetic stability*, Dover Publication, New York 1981.
2. P.K. Bhatia and J.M. Steiner, Z. Angew. Math. Mech., **52**, 321, 1972.
3. I.A. Eltayeb, Z. Angew. Math. Mech., **55**, 559, 1975.
4. B.A. Toms and D.J. Strawbridge, Trans. Faraday Soc., **49**, 1225, 1953.
5. H. Hamabata and T. Namikawa, J. Phys. Soc. Japan, **52**, 90, 1983.
6. E.R. Lapwood, Proc. Camb. Phil. Soc., **44**, 508, 1948.
7. R.A. Wooding, J. Fluid Mech., **9**, 183, 1960.
8. J.G. Oldroyd, Proc. Roy. Soc. (London), **A245**, 278, 1958.
9. D.D. Joseph, *Stability of fluid motions II*, Springer Verlag, New York 1976.
10. E.A. Spiegel, Astrophysical J., **141**, 1068, 1965.

DEPARTMENT OF MATHEMATICS
HIMACHAL PRADESH UNIVERSITY, SUMMER HILL, SHIMLA, INDIA.

Received January 18, 1995; new version August 18, 1995.