ENGINEERING TRANSACTIONS • *Engng. Trans.* • **45**, 2, 305–325, 1997
Polish Academy of Sciences • Institute of Fundamental Technological Research

PROBLEMS OF PROGRAMMING THE EXCAVATOR WORK MOTIONS

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The present paper discusses the problems connected with programming the cutting tool trajectory in the automated earth mover, exemplified by the single-bucket hydraulic excavator. They are related mainly to the tool path planning within the configuration and Cartesian coordinates, and to trajectory parametrization in time. Furthermore, the author presented the examples of the trajectory design for the hydraulic excavator, and the results of experiments performed at the laboratory stand with K-111 excavator fixture.

1. INTRODUCTION

There are two fundamental approaches to the automation of the excavators. The first one, dominant in modern machines (especially in those of high power), deals with the automation of its power transmission system, and aims mainly at the energy saving. The other refers to the automation of the machine fixture work motions. In that case, the aim is to eliminate some operator's actions, to carry out tasks which are difficult to execute by means of manual control, or to obtain optimal motions, in accordance with the adopted criterions. Hence, the automation of machine motions increases the usability and precision of the device.

Excavation of the soil by the earth mover requires definite work motions of its fixture. These motions are intended to move the machine cutting tool along a proper path, at proper time from the start point to the final point. The location of such points is determined by the coordinate system related to Cartesian coordinates [1, 2]. In order to execute the programmed motions, the determination of the set points for the control systems of the hydraulic drive is required. Thus, it is necessary to plan the cutting tool trajectory within the machine configuration space, which is determined by generalised coordinates.

Hence, programming of the tool trajectory consists in searching a curve in the generalized coordinate space, which connects the initial point with the tool destination point, and which is parametrized in time.

2. PLANNING THE CUTTING TOOL TRAJECTORY WITHIN THE CONFIGURATION SPACE AND THE WORKING SPACE

Planning of the cutting tool trajectory of the excavator can be carried out within the generalized coordinates (in the configuration space) or in Cartesian coordinates (in the working space).

The task of the cutting tool trajectory design consists of two stages. In the first stage, the curve which connects the initial and final position of the tool is determined (as it was mentioned above, it can be done in the working or configuration space). In the second stage, the curve is parametrized in time in a determined manner, what defines the trajectory within the generalized coordinate space. On this basis, the time runs of the set points for the fixture automation systems are assigned [2].

Planning of the machine cutting tool trajectory within the configuration space is based on description of such a trajectory in the form of a function of generalized coordinates. The points of the trajectory (initial, final and nodal) are recalculated into the generalized coordinate value sets, through the solution of an inverse kinematics problem. Then, a displacement function passing through the nodal points is assigned for each drive. The motion time of all drives for a given trajectory segment is the same. It makes possible to reach the nodal points at the same time.

This way it is possible to reach the assumed tool Cartesian coordinates for all nodal points. It is worth to mention that the trajectory description between the nodal points in the generalized coordinates can be quite simple, but it becomes complex in the description in terms of Cartesian coordinates.

The methods of the interpolation of the trajectory points described in the configuration coordinates are different, but methods using linear interpolation, or the third-degree polynomials are most popular.

Application of the linear function means that the displacement of the drive (in the case of hydraulic machines – of a cylinder) from the present to the final position is performed along a straight line in time. Usually, in the case when the linear interpolation is used for all fixture drives, the resulting cutting tool trajectory can not be linear (due to geometrical nonlinearities). Additionally, linear interpolation can also cause velocity discontinuity at the beginning and the end of the tool path.

To create a smooth curve of tool displacement, and a continuos velocity curve for the linear function, the tool path should be modified and, for example, a parabolic segment can be added at the beginning and end of the path. The linear function and two parabolic functions are glued together to obtain the continuous curves of displacements and velocity [1].

Planning the smooth trajectory fragment, four limitations (at least) for each of the generalized coordinates should be assumed.

Two of them result from the choice of initial Q_p and final Q_k values (initial and final generalized coordinates), while the next two refer to the function derivative continuity.

These four limitations can be satisfied by the polynomial of the at least third degree (having four coefficients) in the form:

$$Q_i(l) = a_0 + a_1 l + a_2 l^2 + a_3 l^3 , \tag{1}$$

where l is a given parameter.

For one interval of the function (1) (between two nodal points), those limitations take the form:

$$\begin{aligned} Q_i(0) &= Q_{ip} , & Q_i(l_k) &= Q_{ik} , \\ \dot{Q}_i(0) &= \dot{Q}_{ip} , & \dot{Q}_i(l_k) &= \dot{Q}_{ik} . \end{aligned} \tag{2}$$

Substituting (2) to (1) and solving the obtained equation for coefficients a_i, the following relations are obtained:

$$\begin{aligned} a_0 &= Q_{ip} , \\ a_1 &= \dot{Q}_{ip} , \\ a_2 &= \frac{3}{l_k^2}(Q_{ik} - Q_{ip}) - \frac{2}{l_k}\dot{Q}_{ip} - \frac{1}{l_k}\dot{Q}_{ik} , \\ a_3 &= -\frac{2}{l_k^3}(Q_{ik} - Q_{ip}) + \frac{1}{l_k^2}(\dot{Q}_{ik} + \dot{Q}_{ip}). \end{aligned} \tag{3}$$

Having the trajectory nodal points, and knowing the values of derivatives corresponding to these points, it is possible to obtain the trajectory passing through the nodal points (with the derivative continuity being maintained) by gluing the third-degree polynomials.

Function $Q_i(l)$, specified by the formula (1), may be also normalized, assuming that parameter l for each trajectory segment changes in the range from 0 to 1.

Planning the fixture trajectory in Cartesian coordinates (disregarding the fixture rotation – plane fixture movement), the trajectory of the cutting tool, determined at the XOY plane, can be written in parametric form:

$$x = x(l), \qquad y = y(l), \tag{4}$$

where l is the length of the tool path referred to its initial position.

Solving the inverse problem of kinematics, the relation (4) takes the form:

$$Q_A = Q_A(l), \qquad Q_B = Q_B(l), \qquad Q_C = Q_C(l), \tag{5}$$

describing the motion for three drives (cylinders) of the machine fixture.

Planning the trajectory in Cartesian coordinates can cause some problems and the configuration space is usually used for trajectory planning in the case of the industrial manipulators (Cartesian coordinates are applied only if necessary [1]).

Realization of the programmed trajectory requires the tool to be taken to its initial position. It can be realised either by the fixture positioning procedure or automatically, when the trajectory reproduction procedure is activated. In the case of excavators, unlike that for industrial manipulators, repeated realization of the machine cutting tool trajectory for this same initial point is usually useless. However, it can be applied for all machine working cycles consisting of: tool trajectory realization - travelling - tool trajectory realization - travelling, and so on. Such a work cycle can be realized e.g. in cases of digging the trenches, or forming a slope.

3. FIXTURE TRAJECTORY TIME PARAMETRIZATION

Realization of the programmed work motion of the machine fixture is the result of the operation of control systems of the cylinders position. Proper set points should be given as an input for the systems. They are determined on the basis of known time change runs for the generalized coordinates, performing time parametrization of the earlier determined fixture trajectories.

The methods of time parametrization of the fixture trajectory are related to the trajectory representation forms. Time parametrization of trajectory based on the determination of time-dependence of parameter l in function (1), can be performed by assuming a proportional dependence between parameter l and the time t.

Time parametrization of the fixture trajectory, planned in the generalized coordinates, may be also a result of some functional optimization, e.g., maximum permissible velocities for the individual fixture cylinders. These velocities can be determined experimentally, by testing the machine fixture, or by using the mathematical model.

In the case of work machines with hydraulic drives, velocity limitations result from the dynamic properties of the mechanical and hydraulic parts of the system, as well as from the hydraulic pump characteristics. They also depend on the properties of the excavated material. It is possible to select the fixture cylinder velocities according to the characteristics of the hydraulic feeding system, taking into the account the fixture dynamics and soil properties.

When the trajectory planning in Cartesian coordinates space is considered, the time parametrization consists in the determination of the dependence between parameter l (4) and the time. In such a case a typical motion description, taking a rectangular time run of acceleration and a trapezoidal velocity run, can be used. It is also possible to determine the description of the $l(t)$ change by some functional minimization. Substituting $l(t)$ to $Q_i(l)$, time runs of the generalized coordinates change take the form:

$$Q_A = Q_A(t), \qquad Q_B = Q_B(t), \qquad Q_C = Q_C(t). \tag{6}$$

Function $l(t)$ can be also determined on the basis of the assumed time of the complete movement of the cutting tool, from the initial to the final point of the trajectory, using conventional velocities at the nodal points. They determine time of the motion between the individual nodal points. Time of the motion and conventional velocities must be matched to ensure the trajectory realization.

4. EXAMPLES OF THE PROGRAMMING METHODS FOR THE HYDRAULIC EXCAVATOR FIXTURE WORK MOTIONS

A special numerically controlled stand, making use of the hydraulic excavator K-111 fixture, was constructed [3]. The control system of the fixture motions, based on PC 486, utilized the control system of the cylinder positions, or the fixture position angle. The fixture was controlled by the proportional hydraulic valves fed by the variable output multi-piston pump.

The control system enables us to plan the cutting tool trajectory, both in the configural and working space. When the trajectory of the exactly defined shape is necessary, it is designed in Cartesian coordinates, namely in the excavator work space. If it is only important that the trajectory should pass through the defined points (initial, nodal and final), then Cartesian or generalized coordinates can be used, but planning in the generalized coordinates is preferred (as it was mentioned before). Examples of the methods of the excavator work motion programming and the results of investigations carried out on the stand are shown below.

4.1. "Point to Point" method

The method called "Point to Point", PTP [3, 5] enables to program the work motions in the excavator work space, or in its configuration space, by determining the coordinates of the initial and final points, and a sufficient number of the nodal points.

The method, later called PTP1, enables us to plan the tool trajectory in Cartesian coordinates. The nodal points can be determined using the control desk and tracing the fixture current position on the display screen, or using the fixture picture on the display screen, and initiating the fixture position from the keyboard.

The nodal point is determined in the excavator work space by 5 quantities: two coordinates of the tool (x, y), the module and angle of the conventional velocity vector against the level (r, g), and the cutting angle as referred to the velocity vector (f) – Fig. 1. For each point, the values of these 5 quantities are stored in the memory, and the point is marked on the screen. After filling the tables with point parameters, the remaining points (on the path) are determined by interpolation. Linear interpolation of coordinates for the path points, cutting velocity and angle is applied. The problem of time parametrization of the tool path is solved by setting total time of the motion, and then by dividing that time into individual segments of the path, in the inversely proportional manner to the adopted values of the conventional velocities. By analogy, the time is assigned to the individual path points, assuming the change of length of the velocity vector between the consecutive nodal points.

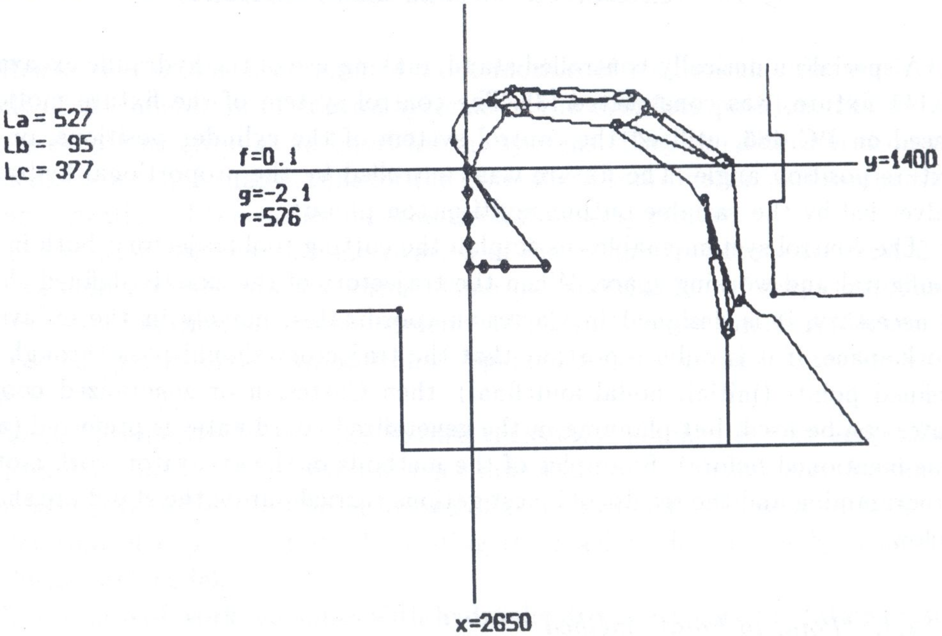


FIG. 1. Example of programming by the PTP1 method.

Having planned the tool trajectory within Cartesian coordinates (by solving the inverse problem of kinematics), time runs of the cylinder length changes or the angles of the fixture position are determined as the set points for the drive control systems.

An example of programming the fixture motions is shown in Figs. 1 and 2, for a tool moving along an assumed triangular trajectory. The number of nodal points comes from the necessity of reducing the tool velocity, when the direction of motion is changed from vertical to the horizontal one. Figure 2 shows the assigned runs of the set points for the control systems of the cylinder position.

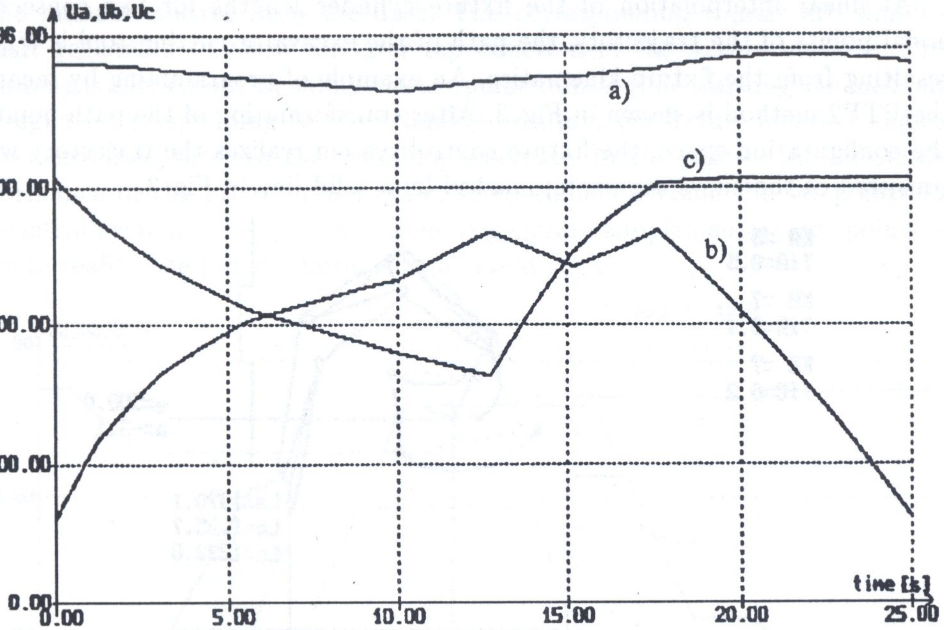


FIG. 2. Runs of the set points for the path shown in Fig. 1.

Programming the excavator fixture work motions by the PTP method in the configuration space (later called the PTP2 method) also requires bringing the cutting tool to the chosen path points, and storing the respective data in the control system. In this case every nodal point is determined by three values of the cylinders length, or three angles of the fixture position.

It is necessary to solve the inverse problem of kinematics for every nodal point. The linear interpolation of cylinder length between the nodal points is assumed. Time parametrization of the path is carried out by the system in real time, on the basis of the assumed value of the feeder (hydraulic pump) output. For the consecutive sampling periods, permissible forward movements of individual cylinders are calculated, which are limited by the feeder output. A clear distinction is made here between planning the path which consists of straight segments and its time parametrization performed on the basis of characteristics of the hydraulic feed station.

The applied method of determining the nodal points as well as the time of motion has two advantages. The first one results from the necessity of memorizing only the cylinder lengths corresponding to the initial, final and nodal points. The control signal values are calculated in real time for each stroke of the system.

The second advantage of the method is due to the easy approach of the initial point of the trajectory by the fixture. This point is then treated as one of the trajectory nodal points.

At linear interpolation of the fixture cylinder lengths for two consecutive nodal points of the trajectory, the path of the excavator cutting tool is a curve resulting from the fixture kinematics. An example of programming by means of the PTP2 method is shown in Fig. 3. After transformation of the path points to the configuration space, the fixture control system realizes the trajectory which consisted of the curve segments, marked by a solid line in Fig. 3.

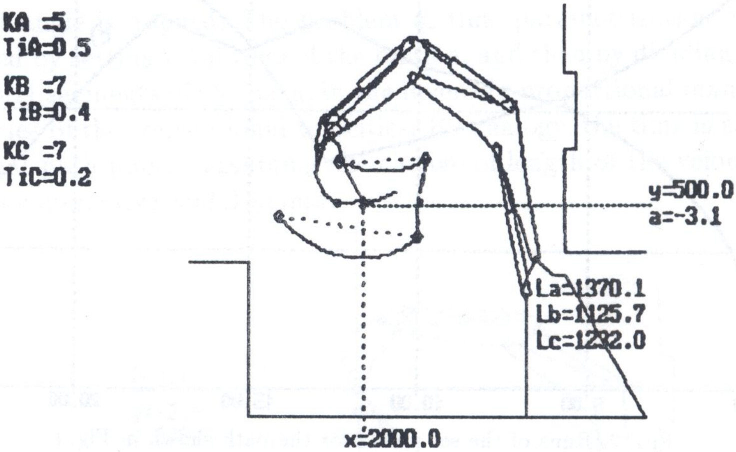


FIG. 3. An example of programming by the PTP2 method.

The tool path is closer to a linear function if the number of nodal points increases. An example of trajectory from Fig. 3, completed by 4 nodal points, is shown in Fig. 4.

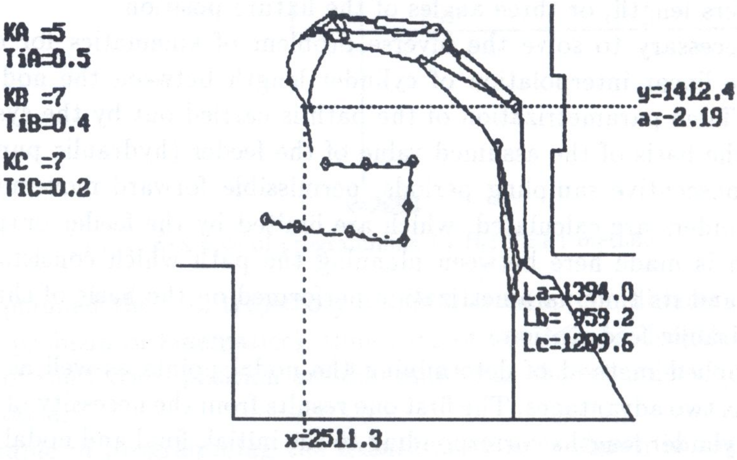


FIG. 4. The cutting tool path for a higher number of nodal points.

4.2. "Teach in" method

In this method the complete work cycle of the fixture is carried out by means of the manual control from the desk. The corresponding signals are simultaneously registered to allow for later reproduction of the fixture motions. Programming is carried out by filling the set point tables while learning the motions. The signals corresponding to the cylinders positions (or fixture location angles) are synchronised through the sampling moments. In the case of the automatic performance of the programmed cycle, those signals become the set points for the control system. The system is able to generate supplementary set points in order to realize the fixture motions at a closed cycle.

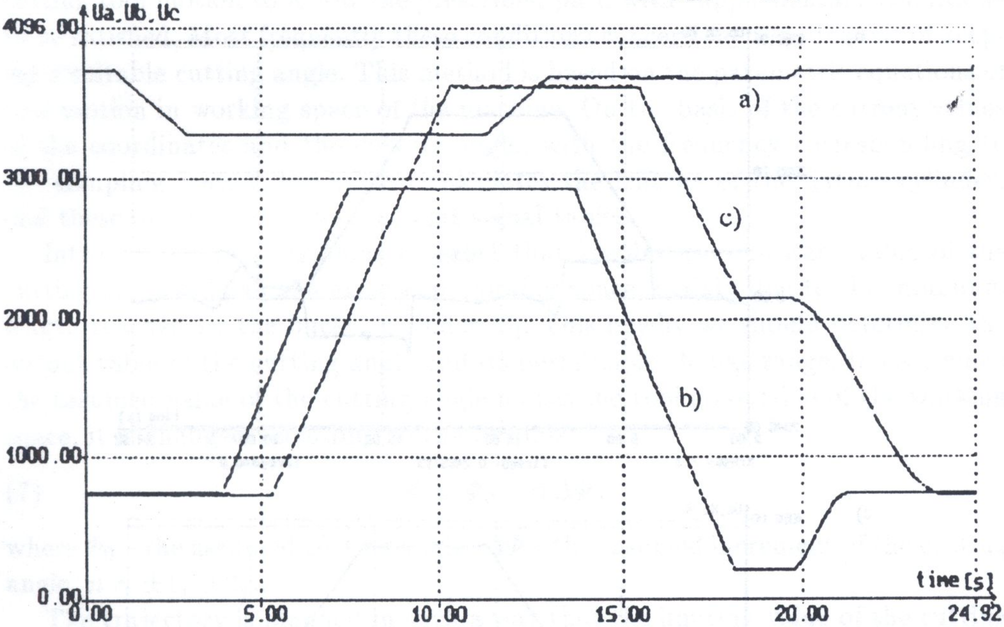


FIG. 5. An example of programming by the "Teach-in" method – runs of the set points.

Such method of programming the work motions is relatively simple to perform, but the quality of the machine work depends on operator's skill. The set point runs for one cycle of fixture motions, programmed by the discussed method, are shown in Fig. 5 and 6. Figure 5 shows the set points runs for the cylinder position control systems, with the supplement of signals for cycle closing, whereas Fig. 6 shows the runs of the set points, the real length cylinder signals and output signals from the controller to the outrigger, arm and bucket, successively. The investigation was conducted for the bucket without loading. PI controllers with parameters indicated in the Fig. 6 were used.

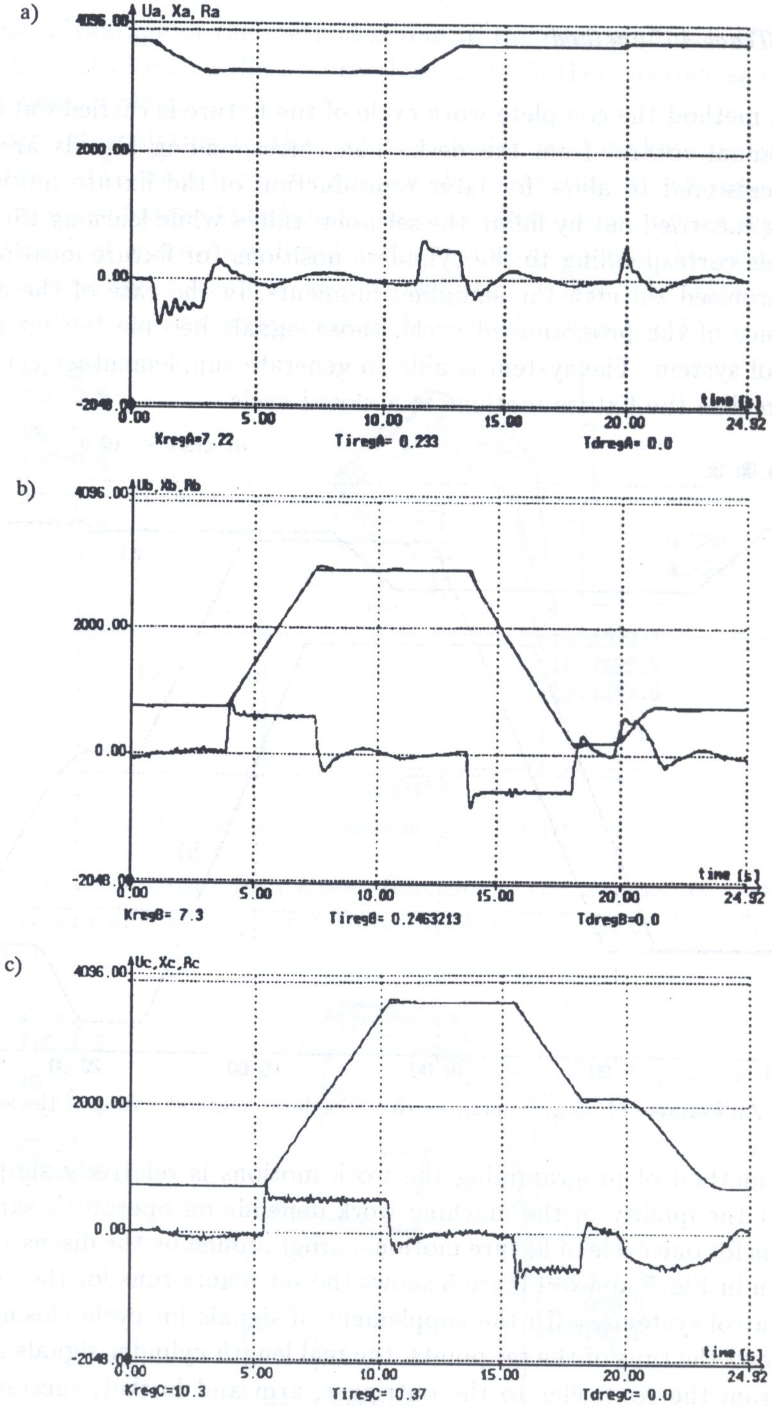


FIG. 6. Runs of signals at trajectory reproduction shown in Fig. 5, a) outrigger, b) arm, c) bucket.

The reduction of the table size is possible due to the procedures applied in the PTP2 method. They enable to eliminate in the table the set points of the supplementary motions and recording the cylinder lengths at frequencies lower than the sampling frequency. For the sampling period of the order of 10 ms, even drastic reduction (by several times) of the recording frequency should not cause considerable deviations of the reproduced trajectory from the planned one.

4.3. "Continuous Path Control" method

The method of the fixture motion programming by the computer continuous control, called CPC, consists in the calculations of set points which assure the cutting tool motion to follow the prescribed path with supplementary conditions to be satisfied. Most frequently these conditions concern the requirement of keeping a suitable cutting angle. This method is based on the parametric equations of tool motion in working space of the machine. On the basis of the current values of the coordinates and the cutting angle, with the frequency corresponding to the sampling period, the system calculates the lengths of the fixture cylinder, and these are filed into the set point signal tables.

Introductory investigations revealed that keeping the constant value of the cutting angle reduces the excavator working space, mainly due to the minimum length reached by the bucket cylinder [3]. This is why we should determine the output value of the cutting angle and its permissible change range. In case when the assumed value of the cutting angle makes the tool go outside of the working space, it is changed according to the formula:

$$\Phi = \Phi_0 - n\Delta\Phi\,, \tag{7}$$

where Φ_0 – the assumed cutting angle, $\Delta\Phi$ – the assumed increment of the cutting angle, $n = \pm 1,\ \pm 2, \ldots$.

The trajectory is planned in such a way that the limiting value of the cutting angle is reached.

To "close" the trajectory, the set points for the additional trajectory segment should be generated. In the system under discussion, length change for each of the fixture cylinders (from the values corresponding to the final tool position to the initial value) is carried out along the cosine curve at the assumed, allowable velocity of the cylinder. It is one of many possible methods.

To elaborate the control signals in real time, calculations should be repeated when it is no longer possible to preserve the assumed value of the cutting angle. That requires a fast computer, or the preparation of the set points before the motion start-up. Programming the cutting tool motion along a horizontal straight segment $y = 0$, for cutting angle $\Phi = 0.3$, is shown in Figs. 7 and 8.

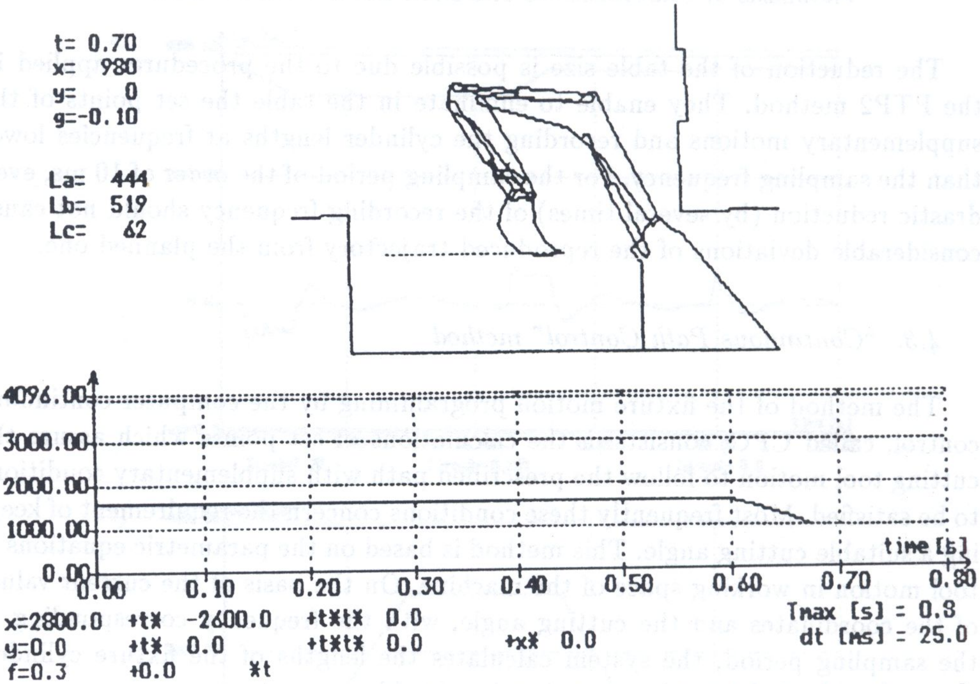


FIG. 7. An example of programming by the CPC method – tool path and soil cutting angle changes.

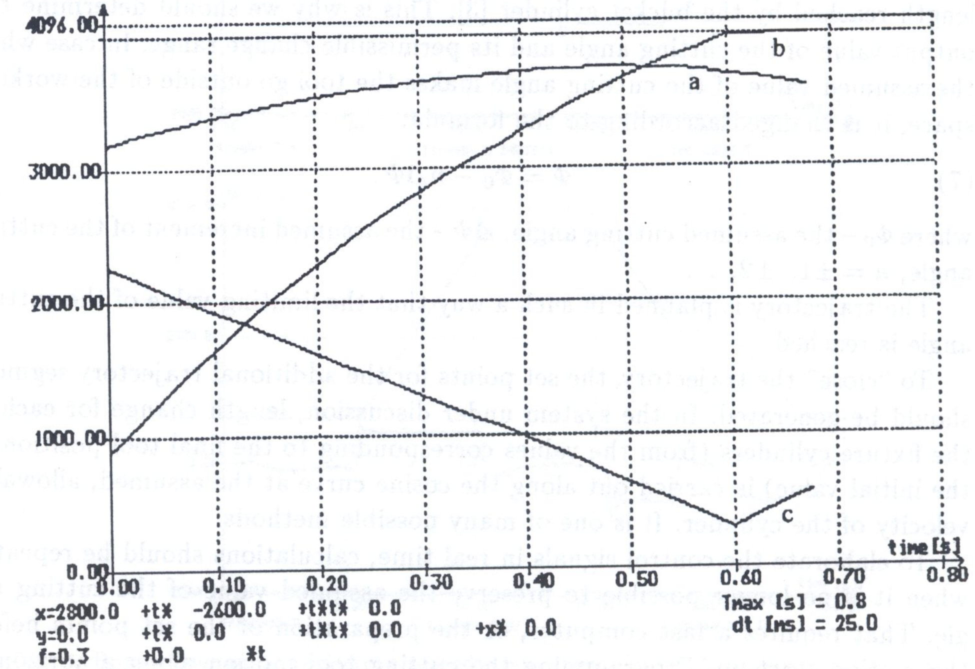


FIG. 8. Runs of the set points for path as in Fig. 7.

4.4. Method using Bezier's curves

It is one of the methods where third-degree curves are applied for programming the tool trajectory. The curve consists of cubic segments. Each segment is determined by four points V_0, V_1, V_2, V_3 (Fig. 9). The four points could be also interpreted in a different way:

$$(8)\qquad \begin{aligned} V_0 &= P(0), & V_1 &= P(0) + \frac{1}{3}P'(0),\\ V_2 &= P(1) - \frac{1}{3}P'(1), & V_3 &= P(1), \end{aligned}$$

where $P'(0)$ and $P'(1)$ are vectors tangent to the curve at points $P(0)$ and $P(1)$.

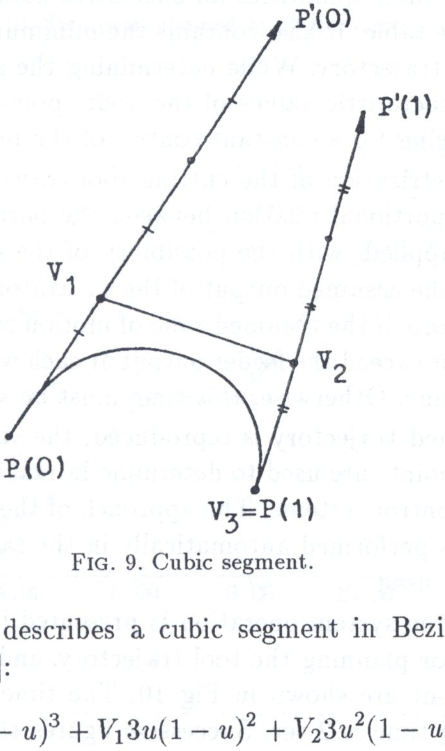


FIG. 9. Cubic segment.

The mapping which describes a cubic segment in Bezier representation is determined as follows [4]:

$$(9)\qquad P(u) = V_0(1-u)^3 + V_1 3u(1-u)^2 + V_2 3u^2(1-u) + V_3 u^3,$$

where u is changeable parameter, different for each of the cubic segments, varying from 0 to 1, or from 0 to u_0.

Bezier curves are formed by joining successive polynomial segments. Determining the segments by means of their points and tangential vectors, a smooth curve is obtained which passes through all these points. The lengths of its tangent vectors are the measures of velocities at individual points. Curves defined in this manner are used for programming the trajectory of the hydraulic excavator cutting tool.

The trajectory is designed in Cartesian coordinates, that is in the machine working space, while operator's task consists in determining the shape of the tool path and the minimum time of motion on the assumed path. Planning the path is carried out on the display screen. The operator is able to use the excavator fixture picture, and by means of the keyboard, he can bring the cutting tool to the successive nodal points of the path, determining also characteristic values at all the points. These characteristic values are: two coordinates in the rectangular coordinate system, with origin located at the outrigger rotation axis; absolute value and slope of the velocity vector, the cutting angle determined in relation to the path and the value of the parameter u referred to the preceding segment of the path. The values of these quantities for successive nodal points of the trajectory are registered in the table. It also contains the minimum time of the tool motion along the assumed trajectory. While determining the successive nodal points or changes of the characteristic values of the nodal points, the tool path is visible on the screen, allowing for a constant control of the process.

For time parametrization of the cutting tool trajectory in the system under discussion, the proportional relation between the parameter u of Bezier curves and the time was applied, with the possibility of the automatic modification of this relation when the assumed output of the excavator hydraulic feeding system is exceeded. Therefore, if the assumed time of motion on the planned path is long enough and does not exceed the feeder output at each work stroke, it is considered to be the motion time. Otherwise, this time must be suitably extended.

When the planned trajectory is reproduced, the values of quantities characterising the nodal points are used to determine in real time the set points for the cylinder position control systems. The approach of the cutting tool to the initial point of the path is performed automatically in the same but simplified manner where line segment used.

An example of the system operation is presented in Figs. 10 to 12. A screen used by the operator planning the tool trajectory, and an example of trajectory with one nodal point are shown in Fig. 10. The time of the motion is 10 sec., while the approach time is 2 sec. Successive figures show the set point runs for control systems of the cylinder positions, and output runs determined during the reproduction of the planned trajectory for two values of the feeder output. For the assumed output of $2 \cdot 10^{-3}\mathrm{m}^3/\mathrm{sec.}$ (Fig. 11), the motion time was not extended in comparison to the assumed one. In the other case (Fig. 12), the assumed value of output did not allow to realize the motion within the assumed time period. Due to that, time was extended by the system so that instantaneous output values did not exceed the assumed ones.

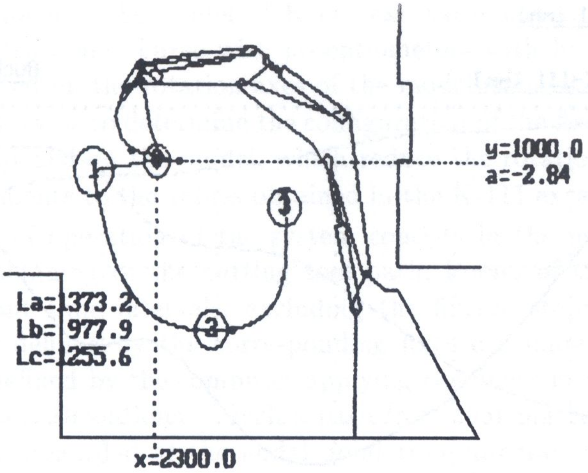


FIG. 10. Tool path planned by Bezier curves.

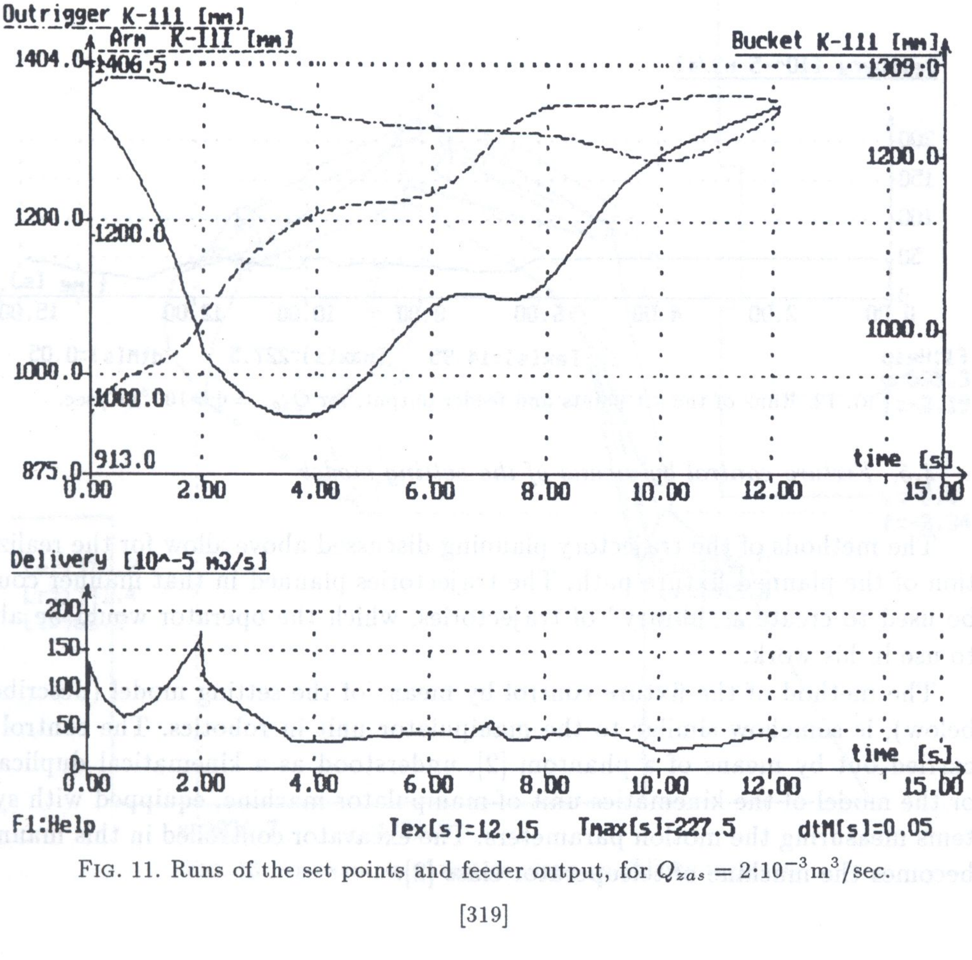


FIG. 11. Runs of the set points and feeder output, for $Q_{zas} = 2 \cdot 10^{-3} \mathrm{m}^3/\mathrm{sec}$.

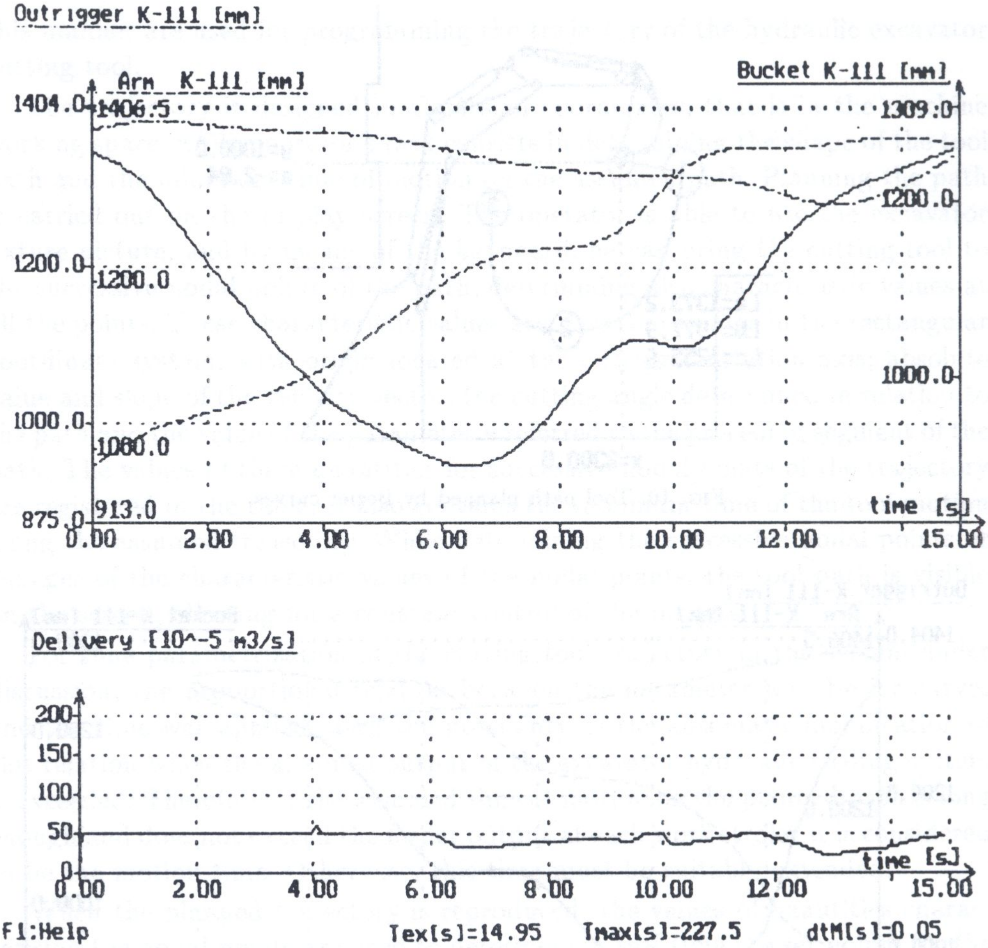


FIG. 12. Runs of the set points and feeder output, for $Q_{zas} = 0.5 \cdot 10^{-3} \mathrm{m}^3/\mathrm{sec}$.

4.5. Fixture control by means of the setting model

The methods of the trajectory planning discussed above allow for the realization of the planned fixture path. The trajectories planned in that manner could be used to create a "library" of trajectories, which the operator would be able to use in his work.

The method of the fixture control by means of the setting model (described below), is somehow similar to the manipulator unit in robotics. The control is carried out by means of a phantom [2], understood as a kinematical duplicate or the model of the kinematics unit of manipulator machine, equipped with systems measuring the motion parameters. The excavator controlled in this manner becomes the machine of teleoperator class [2].

The setting model is the model of K-111 excavator fixture situated on the plate, made in 1:10 scale. Three wire potentiometers with linearity not worse than 2% are located on the rotation axes of the model element. Signals of these potentiometers allow us to determine the configuration of the fixture. Mechanical end stops are provided on the model, which reduce the rotation angles of individual fixture elements to the values obtained in the K-111 excavator fixture [6].

The principle of operation of the system consists in the use of the setting model only for planning of the cutting tool path. Points of the path are registered at constant time intervals, excluding the fixture stoppages. The path nodal points are defined by the corresponding fixture cylinder lengths. Other path points are defined by the computer applying the linear interpolation in the configuration space. Deviations of such a path from that marked by the setting model could be disregarded at the nodal point time intervals corresponding to several sampling periods. Parametrization of such a path is realized on the basis of the assumed output of the hydraulic feeder. The system operates through

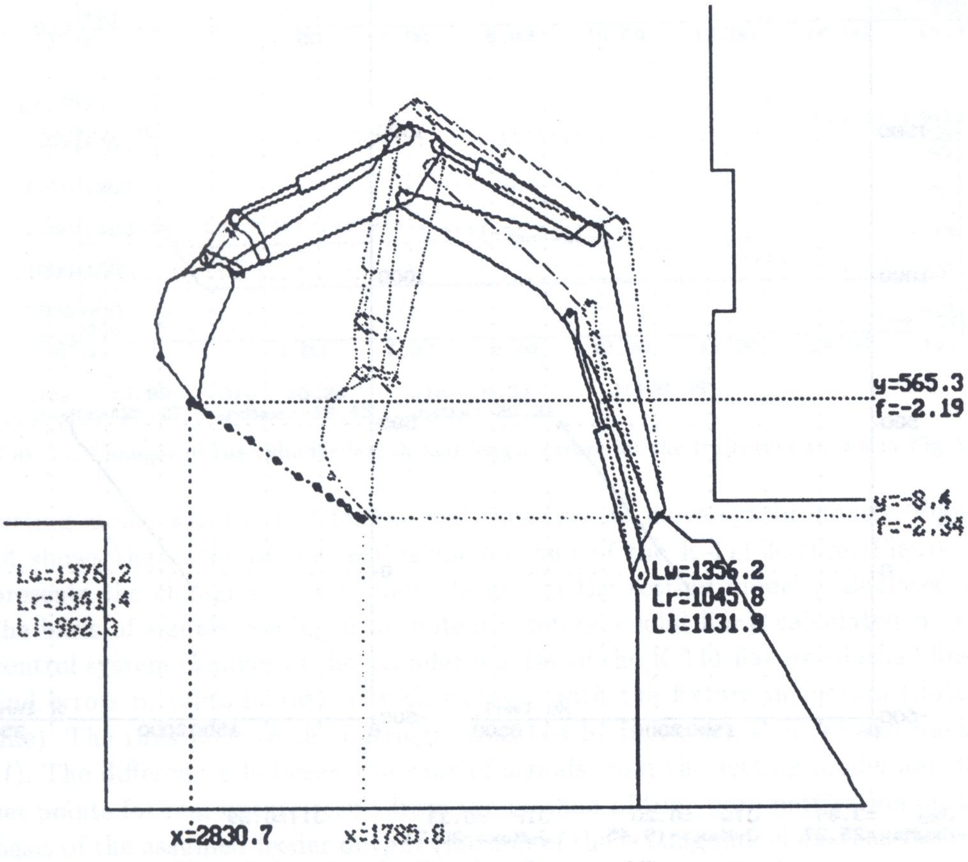
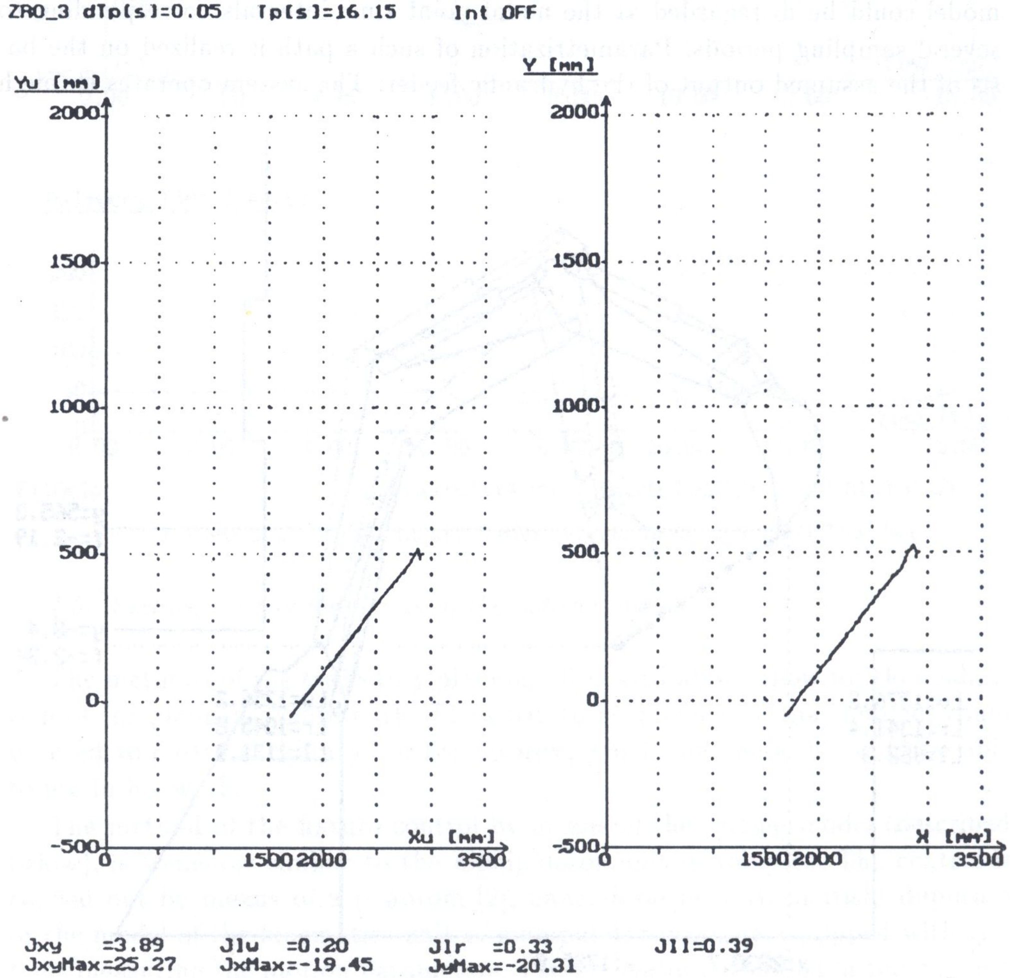


FIG. 13. An example of motion along an oblique segment.

nodal point registration and by determining, on the basis of the already described nodal points and the assumed output of the feeder, the set points for the control systems of the fixture cylinder positions.

If the motion of the setting model is slow, for the properly assumed feeder output, the real excavator fixture moves like its model. For faster motions, the path planning precedes its realization by the real excavator fixture.

An example of the fixture control system operation is shown in Figs. 13 – 15. Figure 13 shows the path of the cutting tool moving along an oblique segment. This figure also shows the display screen. The fixture drawn in dashed lines refers to the setting model, whereas the one drawn in solid line refers to the real excavator. In that case, with the assumed feeder output, the motion of the



```
ZRO_3 dTp[s]=0.05   Tp[s]=16.15   Int OFF
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Jxy    =3.89     Jlu   =0.20      Jlr   =0.33       Jll=0.39
JxyMax=25.27     JxMax=-19.45     JyMax=-20.31
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FIG. 14. Assumed (X_u, Y_u markings) and real (X, Y) path of the cutting tool.

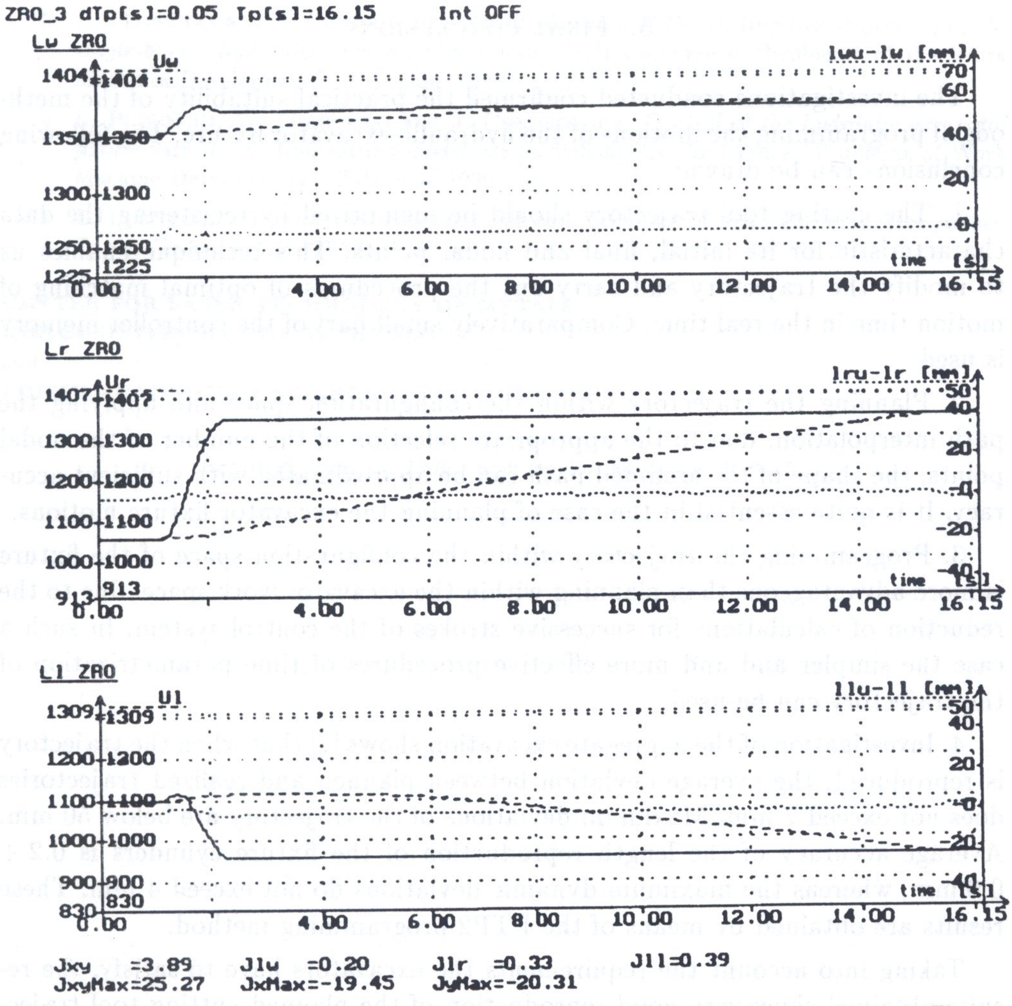

FIG. 15. Changes of the cylinder length and length errors for the trajectory shown in Fig. 14.

setting model was too fast for the real excavator to move synchronously. Figure 14 shows the paths of the setting model, and of the K-111 fixture. Figure 15 presents the change of the cylinder length in the setting model (calculated on the basis of signals coming from potentiometers – solid line), calculated by the control system changes of the cylinder lengths of the K-111 fixture (dashed line), and errors in reproduction of such changes with the fixture in motion (dotted line). The runs refer to the outrigger (marked by index w), arm (r) and bucket (l). The differences between the runs of signals from the setting model and the set points for real fixture result from the method of time parametrization on the basis of the assumed feeder output (motion of the setting model exceeds the real fixture possibilities).

5. FINAL CONCLUSIONS

The investigations conducted confirmed the practical suitability of the methods of programming the motions of the hydraulic excavator fixture. The following conclusions can be drawn:

1. The cutting tool trajectory should be memorized by registering the data characteristic for its initial, final and nodal points. This technique enables us to modify the trajectory and carry out the procedures of optimal matching of motion time in the real time. Comparatively small part of the controller memory is used.

2. Planning the trajectory within the configuration space and applying the path interpolation, due to the appropriate selection of the number of the nodal points, the shape of the assumed path can be approximated with sufficient accuracy. It is quite essential in the case of planning the excavator fixture motions.

3. Programming the trajectory within the configuration space of the fixture is more advantageous than planning within the excavator work space, due to the reduction of calculations for successive strokes of the control system. In such a case the simpler and and more effective procedures of time parametrization of the trajectory can be used.

4. Investigation of the aggregate excavation shows [7] that when the trajectory is reproduced, the average deviation between planned and realized trajectories does not exceed 7 mm. Maximum deviations of the trajectory are below 50 mm. Average accuracy of the length reproduction of the fixture cylinders is $0.2 \div 0.6$ mm, whereas the maximum dynamic deviations do not exceed 4 mm. These results are obtained by means of the PTP2 programming method.

Taking into account the requirements the excavators have to satisfy, the results obtained show very good reproduction of the planned cutting tool trajectory.

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Received June 19, 1996; new version February 2, 1997.