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LOADING FREQUENCY EFFECTS ON CONCRETE FATIGUED IN COMPRESSION

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This papers reports the experimental results of tests performed on prismatic concrete specimens in compression. The specimens have been subjected to a large amount of fatigue cycles (High Cycle Fatigue, HCF). It is common opinion that, in the case of plain concrete in compression, fatigue does not depend on the loading frequency. On the contrary, the performed tests show that: a) the energy dissipation in the material depends on the loading frequency; b) the more fatigued is the material, the less its behaviour depends on the loading frequency.

1. INTRODUCTION

Experimental tests have been performed to study the constitutive law and the strength and deformation characteristics of concrete subjected to HCF loads. The stress range is kept within common service limits. According to the laws in force, common service is about 1/3 of the ultimate strength under usual loads. The performed tests consisted in subjecting prismatic concrete specimens to loading-unloading cycles at frequency ranging from 140 cycles per minute (cpm) to 450 cpm. The tested specimens were of two kinds: a) at ease, b) previously subjected to a large amount of cycles of pulsating load at a fixed frequency.

2. HYSTERESIS CYCLE AND DISSIPATED ENERGY

It is a common opinion that the dissipation due to material hysteresis depends on the loading frequency only when the stress approaches the ultimate strength. ACI rules [1] state that a loading frequency ranging from 70 to 900 cpm has only a slight effect on the fatigue strength, if the maximum stress is less than 75% of the static strength of the material. Nothing is said about the dissipative properties of the material, because dissipation due to hysteresis is relevant only when a cyclic load outside the elastic range is applied. Now, concrete exhibits a non-elastic behaviour even if the stress is low. So, if concrete is subjected to a pulsating load, there is an energy dissipation due not only to internal friction and viscosity but also to hysteresis. The area enclosed by the hysteresis cycle is a measure of the mechanical energy dissipated in each loading cycle per material volume unit (Fig. 1b):

$$W = \oint \sigma \, d\varepsilon .$$

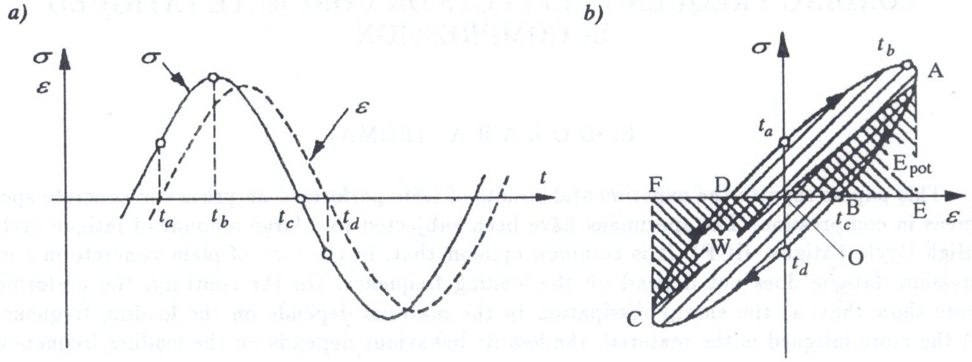

FIG. 1. Stress and strain phases (a) and hysteresis cycle (b).

Figure 1 shows a) stress and strain phases; b) the hysteresis cycle in terms of the related stress and strain. In Fig. 1 b loading is represented by the arcs DA and BC, unloading by the arcs AB and CD. The supplied energy is measured by the areas DAE and BCF and the energy given back by the areas AEB and CFD. The final result is that in each cycle, the dissipated energy is measured by means of the area ABCDA. This dissipation may be taken into account by performing the dynamical analysis of a damped elastic system equivalent to the tested one. Such equivalence follows from the energetic point of view, that is to say, the dissipated energy due to hysteresis is considered as produced by viscous damping. In this way the usual simple linear analysis may be performed; this technique has been proposed by JACOBSEN [5]. The basic idea of Jacobsen's technique is to replace a non-elastic material element, subjected to a steady sinusoidal load, with a linear elastic system subjected to the same load and to a viscous damping which dissipates the same amount of energy. In the equivalent system, the slope of the straight line AC is taken as elastic modulus and the equivalent viscous damping factor is given by

$$\xi_{\text{eq}} = \frac{1}{2\pi} \frac{\text{area of the cycle}}{\text{area OAE} + \text{area OCF}} = \frac{1}{2\pi} \frac{W}{E_{\text{pot}}},$$

where E_{pot} is the elastic potential energy stored in the equivalent system. This relationship is obtained by equating the area enclosed by the hysteresis cycle and the dissipated energy due to a viscous damping factor ξ_{eq}. If the path AC is not a part of a straight line, many suggestions for calculating E_{pot} have been proposed (Rea, Clough *et al.*, [16]). Unfortunately, each suggestion provides a different value for ξ_{eq}, so a choice among them should be careful. Moreover, an equivalent system subjected to a viscous damping is correct only if ξ_{eq} has a low value, otherwise a relevant error is made (Jennings [7, 8]). Fortunately enough, in the performed tests ξ_{eq} had low values, so that Jacobsen's technique was properly applied. A global equivalent viscous damping factor $\xi_{\text{eq},g}$ is obtained by integrating ξ_{eq} over the volume, that is to say, by means of a weighted mean in which the weight is the volume of the material elements:

$$\xi_{\text{eq},g} = \frac{\int \xi_{\text{eq}}\, dV}{V} = \frac{W_g}{E_{\text{pot},g}}; \qquad \text{in case of homogeneity,} \qquad \xi_{\text{eq},g} = \xi_{\text{eq}}\,.$$

In the relationship given above, the letter g denotes the global value of the indicated physical quantity. It is to be observed that, in a system subjected to dynamical loads, the energy dissipation is due to many causes, of which the hysteresis is one. Figure 2 shows the most important causes of energy dissipation.

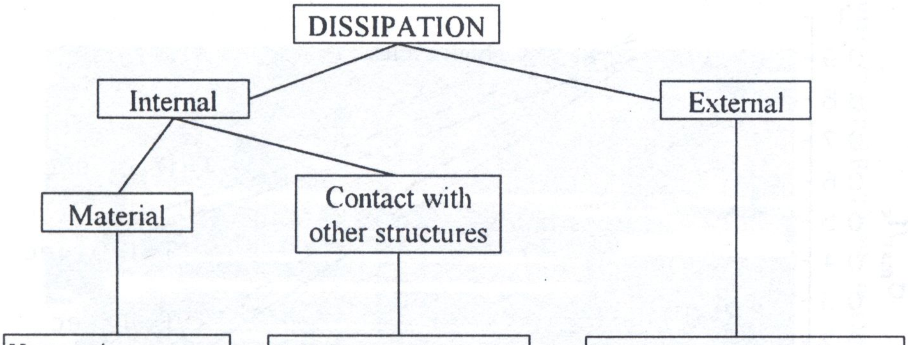

Fig. 2. Different causes of energy dissipation.

3. Loading frequency

The effect of the loading frequency on concrete specimens is usually studied to find a relationship between the accelerated fatigue tests and the actual life of the fatigued material. Graf and Brenner [9], Holmen [10], Van Leeuwen [11], Siemes [12] pointed out that an increase of the loading frequency ($\Delta\omega$) causes an increase of the number of cycles (N_F) required for the failure of the material, so that $(dN_F/d\omega) < 1$ if $0.06\,\text{Hz} < \omega < 6\,\text{Hz}$ (Cornelissen [13]). These observations agree with the experimental results of SPARKS and MENZIES [14]: once the loading cycle is given, if the loading speed goes from 0.5 to 50 MPa/sec and the stress is more than 75% of the static strength in compression, the average fatigue life of prismatic concrete specimens grows 10 times higher. This implies that the accelerated especially HCF, fatigue tests on concrete structures may overestimate their actual fatigue life under common service loading speed. HSU [15] studied the influence of the loading speed on fatigue and completed the S - N relationship given by TEPPERS and KUTTI [16]. Hsu suggests the following S - N - T relationship for high cycle stress:

$$\frac{\sigma_{\max}}{f_c} = 1.0 - 0.0662 \cdot (1 - 0.556 \cdot R) \cdot \log(N) - 0.0294 \cdot \log(T),$$

where f_c is the ultimate stress of the material and $R = \sigma_{\min}/\sigma_{\max}$. In Hsu formula it is put into evidence that, if $\omega > 60\,\mathrm{Hz}$ ($T < 1\,\mathrm{sec}$), there is a positive contribution to the fatigue strength (Fig. 3).

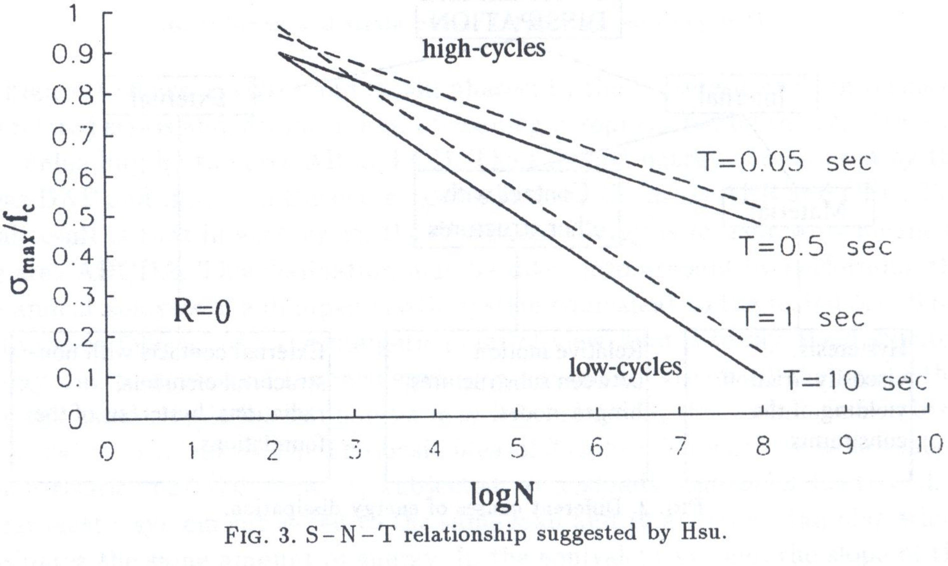

FIG. 3. S - N - T relationship suggested by Hsu.

It has been pointed out that high loading frequencies make the tests on concrete adiabatic because of low heat conductivity and difficult heat exchange with the environment. This implies a temperature increase that is supposed to cause a new hydration in the concrete, so that fatigue microcracks are filled and the material behaviour is slightly improved. On the contrary, global effects cause a degradation: The dissipated energy turns partly into heat (new hydration), but also initiates certain negative phenomena which are permanent even after the load is removed, while the positive effects due to new hydration vanish as the temperature goes back to its initial value.

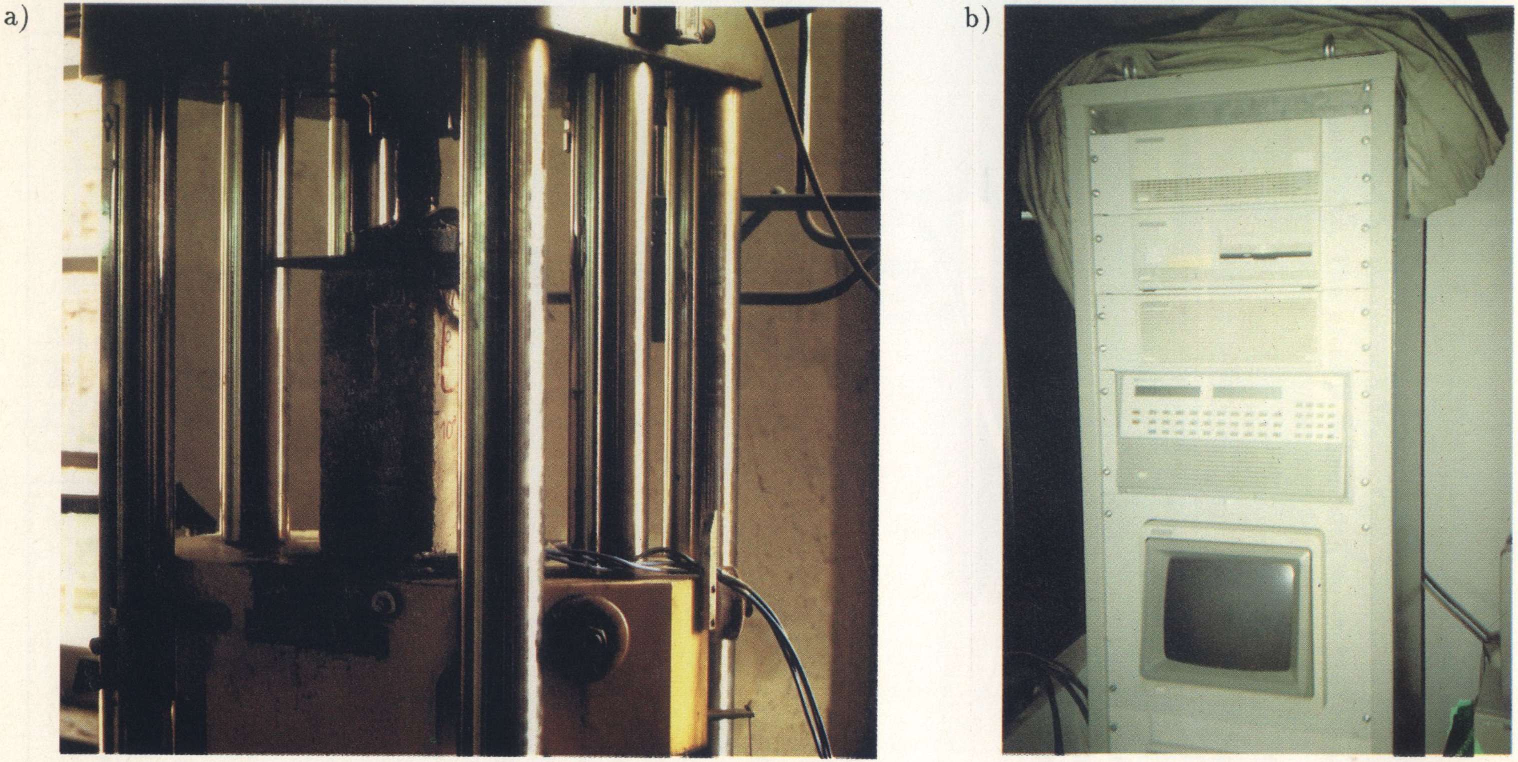

FIG. 4. a) Metro-Com press; b) HP 3852A Data acquisition unit.

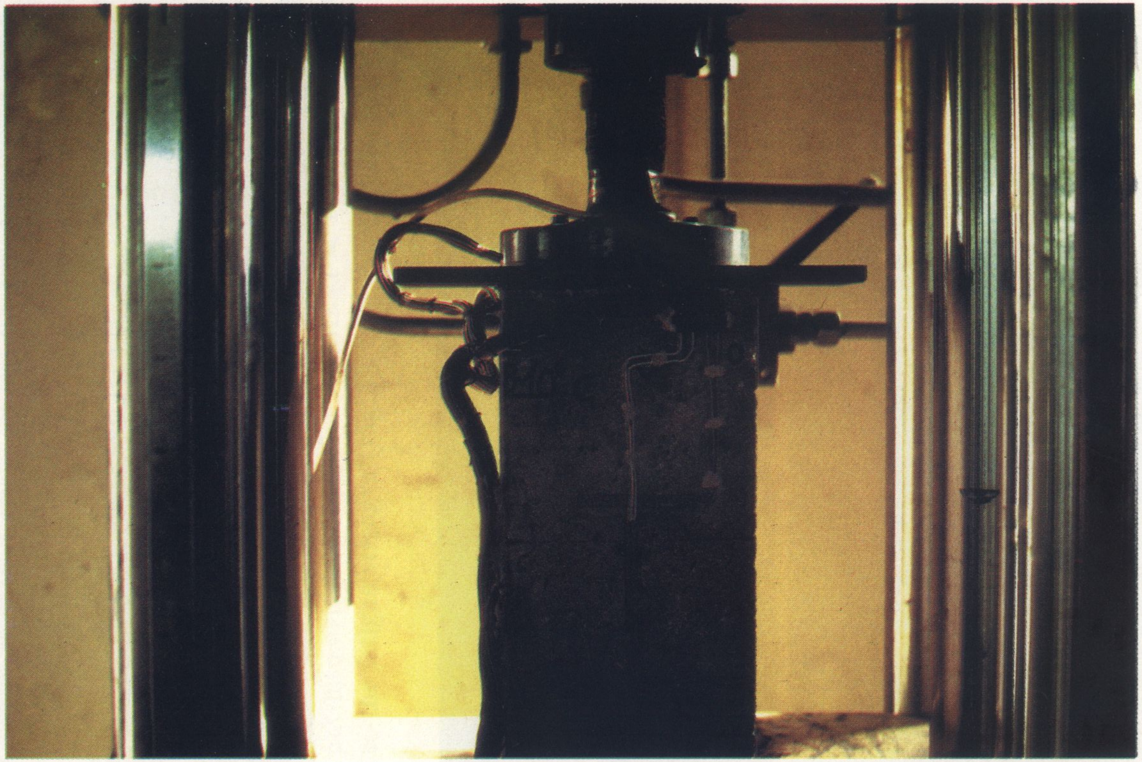

FIG. 5. Concrete specimen.

4. TEST EQUIPMENT

Tests are performed by means of a Metro-Com press with 100 tons full-scale and ±0.2 tons sensitivity. The press is linked with an electrohydraulic axial fatigue machine J. Amsler & Co. 131 – 130 with 600 cpm full-scale. The applied load is measured by means of a resistive pressure transducer calibrated for a peak load of 100 tons at the Istituto Colanetti, Torino. The strains are measured by means of PL-30-11 resistive strain-gauges 60 mm long with a $120\,\Omega \pm 0.3\,\Omega$ resistor. The signals coming from the strain-gauges and the transducer are properly calibrated and amplified by means of a Huggenberger signal calibrating device and of a HBM KWS Darmstadt model 3071 signal amplifer. At the end of the chain, there is a data acquisition and control unit HP 3852A performing 100 000 samples/s on 72 channels (Fig. 4).

5. TEST SPECIMENS

The tested concrete specimens are square-based (15 cm×15 cm) prisms 30.5 cm high (Fig. 5). They are prepared so as to obtain the best material homogeneity and density: for this purpose, severe controls are made on the temperature, the mix slump, the setting of consecutive layers in the mould, the demoulding procedure against quick superficial evaporation. The curing of the specimens takes place under the following conditions: during the first 24 hours, the environment is kept at 20° C, with no less than 95% relative humidity; the specimens are then demoulded and kept in water at 20° C for 28 days. Two hours before the beginning of the tests, the specimens are taken out of the curing environment.

The mix proportions and the characteristic strength are listed in Tables 1 and 2.

Table 1. Concrete mix proportions.

Concrete 425 Ptl	3.500 Kg	Slump = 2.3 cm
Fine sand	5.150 Kg	$T_c = 15°$ C
Sand	4.400 Kg	$T_{\mathrm{env}} = 19.5°$ C
Small gravel	3.020 Kg	rh = 64%
Gravel	5.150 Kg	
Water ($W/C = 0.6$)	2.100 Kg	weights for 1 cubic meter
Total	23.360 Kg	

Table 2. **Concrete and strength during the curing.**

Strength (MPa)			
1 day	3 days	7 days	28 days
7.2/7.5	15.9/15.9	22.1/22.2	32.8/32.5

6. TEST PERFORMING

The specimens are fatigued by means of a pulsating load ranging from 100 kN to 400 kN at a frequency of 450 cpm, so that

$$\sigma_{\min} = \frac{10^5\,\mathrm{N}}{2.25 \cdot 10^{-2}\,\mathrm{m}^2} \approx 4.5\,\mathrm{MPa}, \qquad \sigma_{\max} = \frac{4 \cdot 10^5\,\mathrm{N}}{2.25 \cdot 10^{-2}\,\mathrm{m}^2} \approx 17.8\,\mathrm{MPa}.$$

After about a million loading cycles, two cyclic loading tests are performed without stopping the fatigue machine, at loading frequencies of 450, 400, 300, 200, 140 cpm. In each test (Fig. 6) the load is applied along the direction of the major axis of the specimen and is measured by means of the pressure transducer, which is placed between the specimen and the top plate of the press. The horizontal and vertical strains, ε_h and ε_v respectively, are measured by means of couples of strain gauges; the mean value of these measures is calculated. The couples of strain gauges are placed on the four free faces of the specimen, along the mutually orthogonal directions. The errors of the equipment due to the unbalanced temperature gradients are calculated and taken into account.

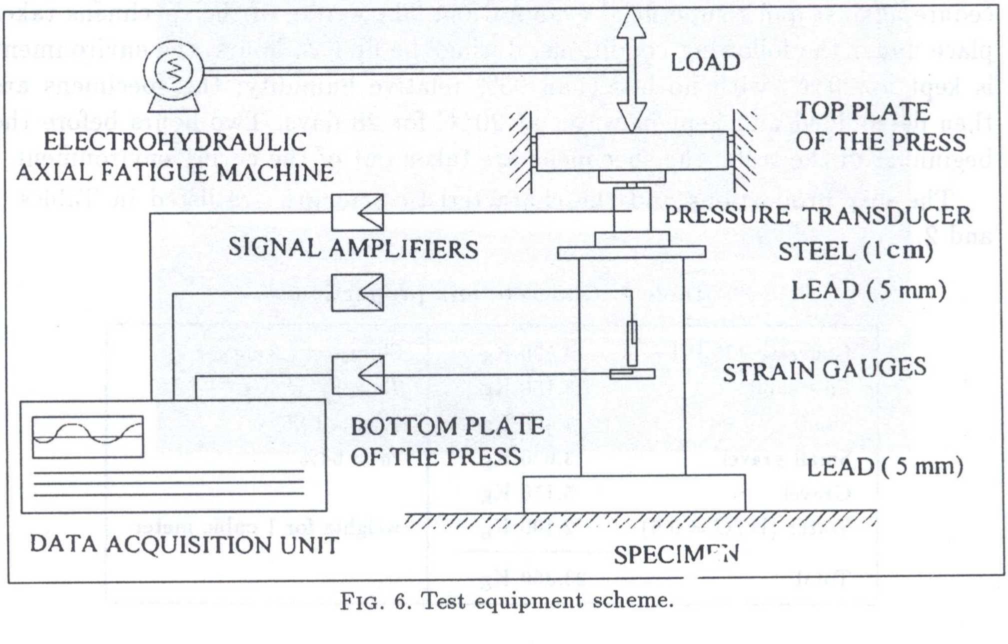

FIG. 6. Test equipment scheme.

A lead plate 5 mm thick is placed between the specimen and both plates of the press, in order to:

a) limit the stress concentration at the roughest points of the surface of the specimen;

b) reduce friction between the steel plate and the specimen;

c) limit the shocks.

The test without stopping the fatigue machine lasts for about 0.25 sec and the scanning step of the data acquisition unit is 10^{-3} sec. In this way it is possible to get 250 records for each channel and to plot 125 points of the hysteresis cycle.

In each test data, acquisition unit measures and stores:

a) the number, type and magnification scale of each channel;

b) the acquisition duration;

c) the scanning step;

d) the numerical data on each channel.

All data are moved from the data acquisition unit, working under "Basic system" HP, to a PC, which decodes the data and gives the possibility to work under DOS.

7. RESULTS

The hysteresis cycle is difficult to plot when the loading frequency is high, but this difficulty has been overcome by means of the digital data acquisition unit. This device can get a large amount of values, so that the hysteresis cycle can be plotted, as in Figs. 7 and 8; Ac is the arc of the cycle and m is the slope of the dashed line (axis of the cycle).

Figure 7 shows in sequence, for a specimen subjected to 8.8 million fatigue cycles, that the hysteresis cycle reduces in size as the loading frequency decreases. The same phenomenon is observed for a specimen subjected to 10.2 million fatigue cycles (Fig. 8). The axis of the hysteresis cycle is defined as the part of a straight line that joins the two reversal points of the cycle. Figure 8 shows that the axis of the hysteresis cycle of the more fatigued specimen has smaller slope than that of the less fatigued one. Moreover, the loading frequency variation modifies not only the size of the hysteresis cycle, but also its shape. Actually, the hysteresis cycle becomes concave as the loading frequency decreases and the axis of the cycle moves away from its interior, becoming partly external. As a consequence, the tangent elastic moduli at the origin and at the top reversal point of the cycle are modified. A smaller change of the shape of the hysteresis cycle occurs in the case of the more fatigued specimen.

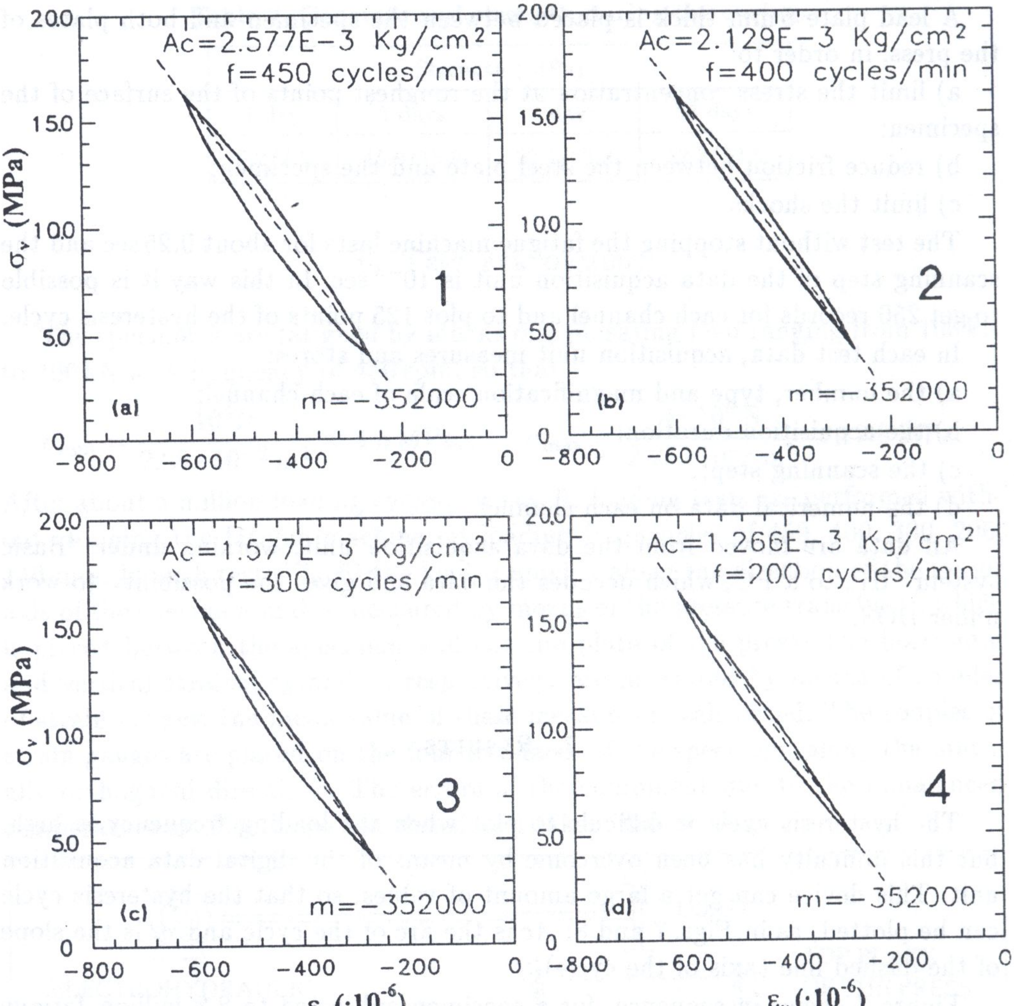

FIG. 7. Hysteresis cycles for a specimen subjected to 8.8 million fatigue cycles.

Figure 9 shows the relationship between the dissipated energy per material volume unit (area enclosed by the hysteresis cycle) and the loading frequency. If the loading frequency ranges from 100 to 500 cpm, a good interpolation of the numerical data is obtained by means of arcs of parabolae. It is:

$$\frac{W_{(1)}}{W_0} = 3.286 - 5.08 \cdot 10^{-3} \cdot \omega + 3.16 \cdot 10^{-5} \cdot \omega^2,$$

$$\frac{W_{(2)}}{W_0} = 3.261 - 6.47 \cdot 10^{-3} \cdot \omega + 2.79 \cdot 10^{-5} \cdot \omega^2,$$

where $W_0 = E_{\text{pot}} = 3.4425 \cdot 10^{-3}$ MPa is the potential elastic energy per unit

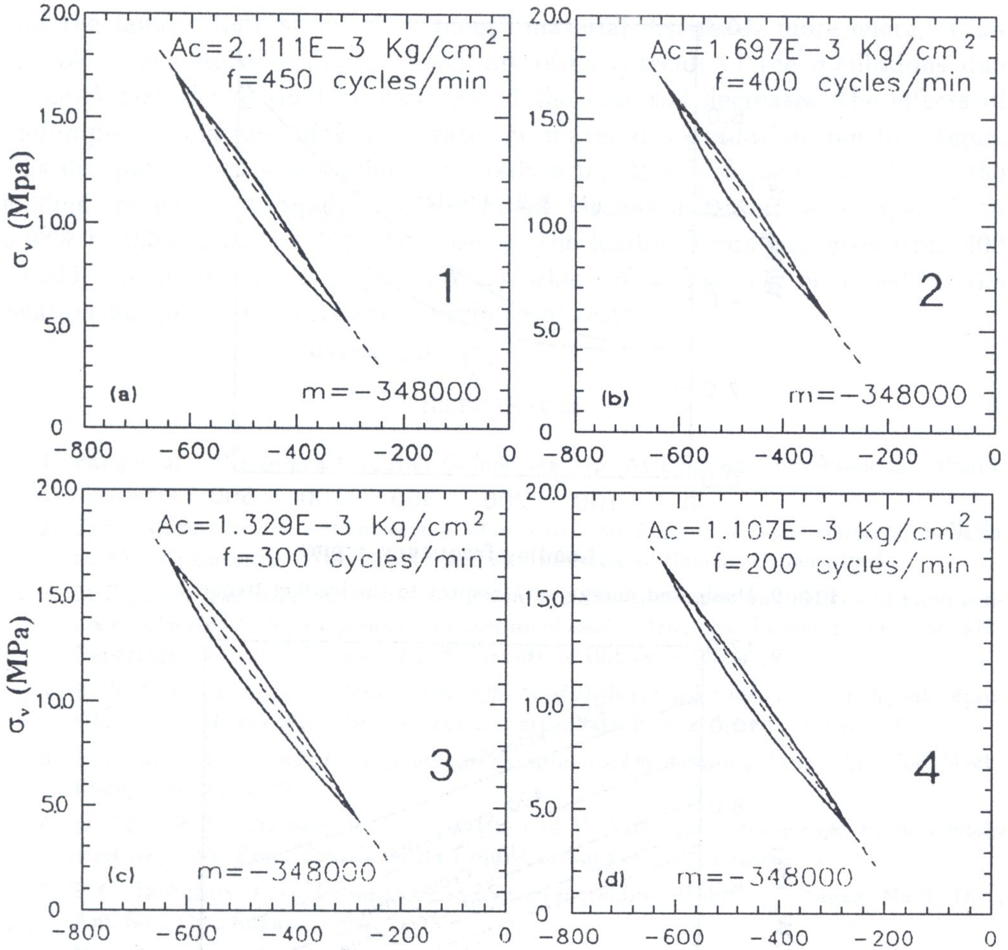

FIG. 8. Hysteresis cycles for a specimen subjected to 10.2 million fatigue cycles.

volume in a loading cycle. Analysing the interpolated curves it can be seen that:

a) as fatigue increases, the dissipation decreases,

b) as the loading frequency increases, the dissipation increases according to a nonlinear relationship;

c) as fatigue increases, the material sensitivity to the frequency variation decreases (the slope of the curve (1) is greater than that of (2)).

These observations agree with Fig. 10, that shows how the dissipated energy decreases as fatigue increases and the loading frequency decreases. The curves have been plotted extrapolating the data at the loading frequency of 450 cpm (only the data concerning specimens subjected to 8.8 and 10.2 million fatigue cycles are at disposal).

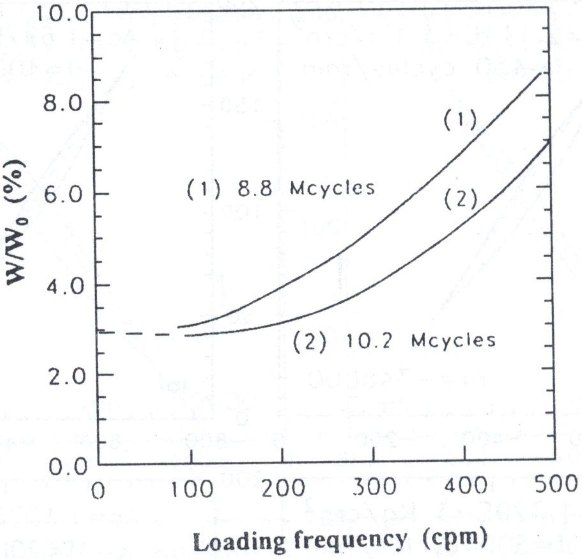

FIG. 9. Dissipated energy with respect to the loading frequency.

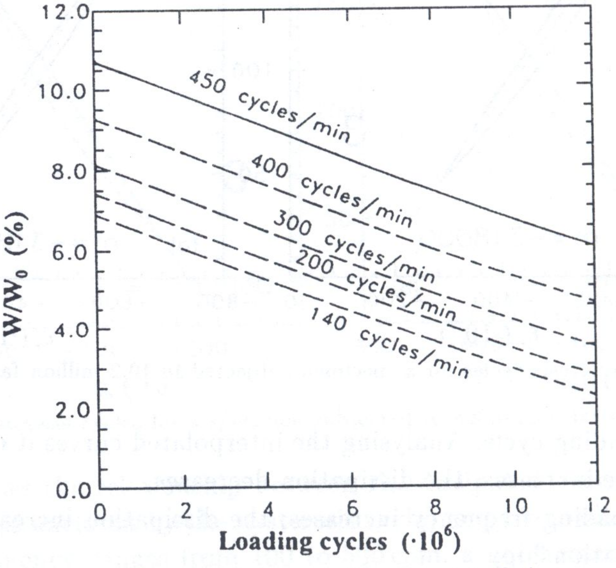

FIG. 10. Dissipated energy with respect to fatigue.

8. CONCLUSIONS

The degradation of the material subjected to a large amount of loading cycles is not negligible, even under common service stress. As fatigue increases, the material capacity of damping cyclic loads decreases in time, as well as the stiffness and the failure limit stress. A "young" material dissipates more energy than an "old" one. Moreover, cyclic loads are often a result of the oscillations due to shocks; as the dissipation capacity of the material decreases, the effects of undumped oscillations may accelerate the material degradation due to fatigue. The dissipation in the material is not only a function of fatigue but also of the loading frequency. Actually, the equivalent viscous damping factor goes from 0.98% to 0.58%, with a 40% decrease, as the loading frequency goes from 400 to 200 cpm. It seems that $\xi_{\mathrm{eq}} = f(N, \omega)$, where N and ω may be at will, is the relationship pointed out by the experimental tests.

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