ENGINEERING TRANSACTIONS • *Engng. Trans.* • **45**, 2, 199–213, 1997
Polish Academy of Sciences • Institute of Fundamental Technological Research

APPLICATION OF FLOQUET'S METHOD TO HIGH-SPEED TRAIN/TRACK DYNAMICS (*)

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The paper deals with the vertical dynamics of a railway track and a guideway for the magnetic high-speed system with contact-free levitation technology (Maglev). The conventional railway track is composed of rails mounted on the equally spaced sleepers which rest on the ballast, with a pad between the rail and sleeper. The guideway for Maglev system is composed of simply supported girders which are mounted on piers. Usually, the span of adjacent girders is equal and the guideway is composed of repetitive elements mounted with high positional accuracy. The track as well as the guideway form typical periodic systems which consist of a number of identical flexible elements coupled in an identical way. In the paper the free wave propagation problems and the steady-state system dynamic responses to a moving harmonic force are considered. In both cases the solution method proposed consists in the direct application of Floquet's theorem to the differential equations of motion of the periodic systems.

1. Introduction

The conventional track is composed of rails mounted on the sleepers which rest on the ballast, with a pad between the rail and sleeper. One of the most simple but reliable mechanical models of such a system is a continuous beam resting on spring-mass-damper elements which allow to model the elasticity of pad and ballast and the sleeper mass, Fig. 1. The beam can be modelled using either the Bernoulli - Euler or the Timoshenko theory. Assuming high positional accuracy, the system can be considered as one composed of repetitive elements. Actually, the track forms a typical periodic system which consists of a number of identical flexible elements (cells) coupled in an identical way (by means of the sleepers).

In recent years the magnetic high-speed system with contact-free levitation technology (Maglev) has been developed [1]. The guideways for Maglev systems are composed of simply supported girders which are mounted on piers. Usually, the span of adjacent girders is equal and the guideway is composed of repetitive elements mounted with high positional accuracy. Although designed as rigid and

(*) This paper has been presented at the 1995 ASME International Mechanical Engineering Congress and Exposition in San Francisco, California, USA.

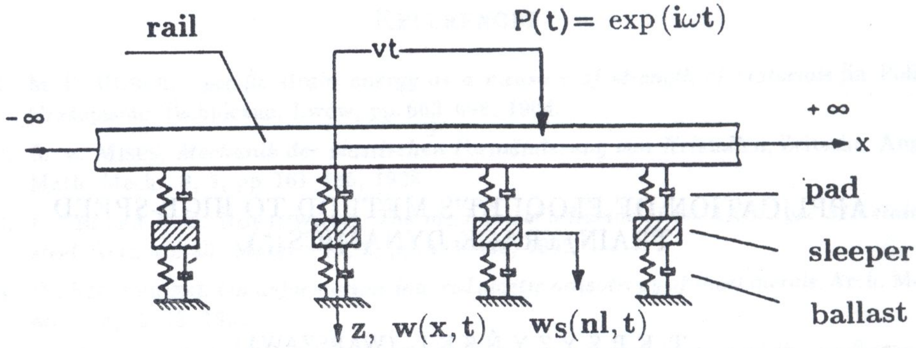

FIG. 1. Model of a periodic track.

insensitive to vibration, actually the guideway forms a typical periodic system which consists of a number of identical flexible elements (girders) coupled in an identical way (piers), Fig. 2.

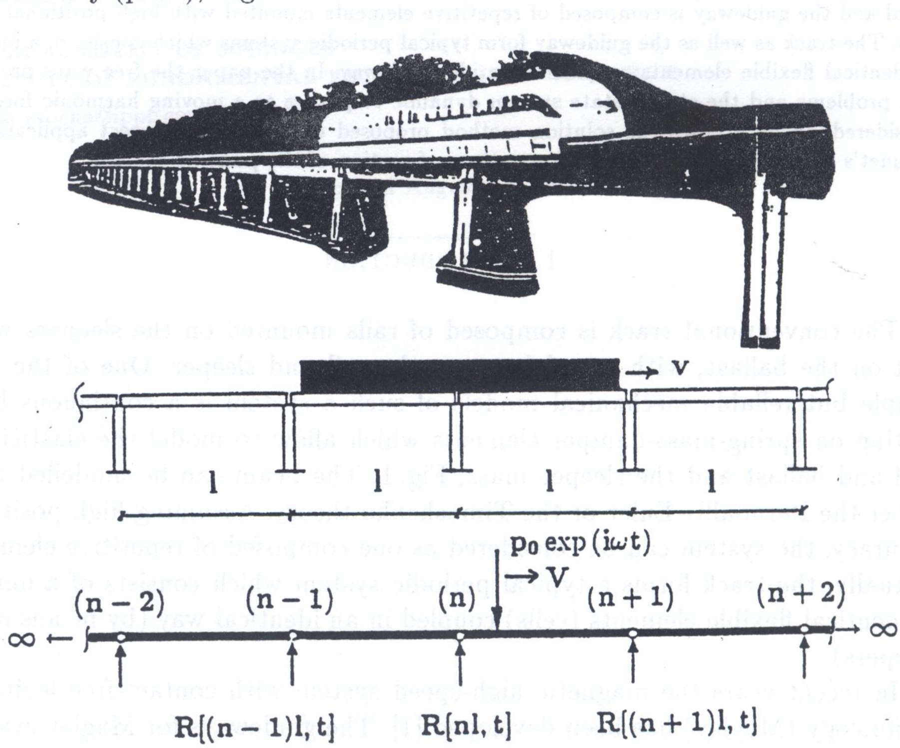

FIG. 2. Model of a periodic guideway for Maglev.

The present paper deals with the vertical dynamics of railway tracks and guideways for Maglev modelled as a periodic Bernoulli-Euler beam. The free wave propagation problem and the steady-state system dynamic response to a moving harmonic force are considered. In both cases the solution method proposed consists in a direct application of Floquet's theorem to the differential equation of motion of the periodic system.

2. Formulation of the problem

The equation of motion of the beam resting on a viscoelastic foundation, which is subjected to the load $\overline{p}(x,t) = p_0\delta(x - vt)\exp(i\omega t)$, is taken in the following form:

$$EI\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial^2 w}{\partial t^2} + \eta\frac{\partial w}{\partial t} + qw = p_0\delta(x - vt)\exp(i\omega t), \tag{2.1}$$

where $w = w(x,t)$ is the displacement function of spatial variable x and time t.

In Eq. (2.1) EI, μ, q and η denote the beam flexural stiffness, the beam mass per unit length, the elasticity and viscosity coefficient, respectively. The right-hand side in Eq. (2.1) represents the load in the form of a force travelling with constant velocity v and oscillating harmonically in time with frequency ω, whereby the term $\delta(x - vt)$ denotes the Dirac-delta function. It is more convenient to analyze the problem by means of non-dimensional quantities. The non-dimensional equation of motion reads

$$\frac{\partial^4 W}{\partial X^4} + \frac{\partial^2 W}{\partial \tau^2} + N\frac{\partial W}{\partial \tau} + QW = \delta(X - V\tau)\exp(i\Omega\tau), \tag{2.2}$$

where the notations are given in Table 1.

The periodicity of the railway track results in the following boundary conditions for the function $W = W(X,\tau)$:

$$\begin{aligned}
&W(nL_-,\tau) = W(nL_+,\tau),\\
&\frac{\partial W}{\partial X}(nL_-,\tau) = \frac{\partial W}{\partial X}(nL_+,\tau),\\
&\frac{\partial^2 W}{\partial X^2}(nL_-,\tau) = \frac{\partial^2 W}{\partial X^2}(nL_+,\tau),\\
&\frac{\partial^3 W}{\partial X^3}(nL_-,\tau) - \frac{\partial^3 W}{\partial X^3}(nL_+,\tau) - R(nL,\tau) = 0,
\end{aligned} \tag{2.3}$$

where $L = la_0$ is the non-dimensional spacing of the sleepers, n is a subsequent support number ($n \in \{-\infty, .., -1, 0, 1, ..., +\infty\}$). Equations (2.3) represent the conditions of continuity of displacement, rotation, bending moment and the equilibrium of shearing forces, respectively, for the n-th periodic support. In case of

Table 1. Notations.

$X = xa_0$	$\tau = t\omega_0$	$W(X,\tau) = w/w_0$
$\Omega = \omega/\omega_0$	$V = v/v_0$	$N = \eta/\eta_0$
$Q = q/E$	$\eta_0 = \sqrt{E\mu}$	$a_0 = \sqrt[4]{1/I}$
$\omega_0 = \sqrt{E/\mu}$	$w_0 = p_0 a_0/E$	$v_0 = \omega_0/a_0$

a steady-state motion with a frequency ω_0, the reaction force $R = R(nL,\tau)$ determined for the n-th sleeper reads

$$R(nL,\omega_0) = \Delta(\omega_0)\cdot W(nL,\omega_0), \tag{2.4}$$

where a generalized stiffness $\Delta = \Delta(\omega_0)$ of the flexible support depends on the elasticity coefficient of the pad q_P and ballast q_B and the pad and ballast viscosity coefficient η_P, η_B, respectively, and reads

$$\Delta(\omega_0) = \frac{a_0}{E}\cdot\frac{(q_P + i\omega_0\eta_P)\cdot(-\omega_0^2 m_S + q_B + i\omega_0\eta_B)}{-\omega_0^2 m_S + q_P + q_B + i\omega_0(\eta_P + \eta_B)}. \tag{2.5}$$

In Table 2 the system parameters used in numerical examples are presented.

Table 2. Periodic track.

$E = 2.1\ 10^{11}$ N/m^2	$I = 3.052\ 10^{-5}$ m^4	$\mu = 60.31$ kg/m
$q_P = 2.6\ 10^8$ N/m	$\eta_P = 6.3\ 10^4$ Ns/m	$m_S = 145$ kg
$q_B = 1.8\ 10^8$ N/m	$\eta_B = 8.2\ 10^4$ Ns/m	$l = 0.6$ m

According to the periodicity of the guideway for Maglev, Fig. 2, the boundary conditions for the function $W = W(X,\tau)$ read

$$\begin{aligned} &W(nL_-,\tau) = W(nL_+,\tau), \qquad \frac{\partial^2 W}{\partial X^2}(nL,\tau) = 0,\\ &\frac{\partial^3 W}{\partial X^3}(nL_-,\tau) - \frac{\partial^3 W}{\partial X^3}(nL_+,\tau) - R(nL,\tau) = 0, \end{aligned} \tag{2.6}$$

which represent the continuity of displacements, zero bending moment and the equilibrium of shearing forces, respectively, for the n-th periodic support. The reaction force $R = R(nL,\tau)$ is to be determined separately for the vertical and lateral case of the system dynamics. In case of vertical system dynamics it is assumed that $(EA)_{\text{pillar}} \gg (EI/l^2)_{\text{bay}}$ i.e. the pillar can be modelled as a rigid mass. According to the design procedure for dynamically loaded foundations proposed by RICHART *et al.* [2], an equivalent model of the foundation resting on an elastic half-space is applied in the present paper, see also [3]. The equation of motion of the equivalent system is the equation for a simple spring-damper-mass-system and we have

$$\begin{aligned} R(nL,\omega_0) &= \Delta(\omega_0)\cdot W(nL,\omega_0),\\ \Delta(\omega_0) &= q_E + 2i\eta_E\omega_0 - m_E\omega_0^2, \end{aligned} \tag{2.7}$$

where $W(nL,\omega_0)$ – vertical displacement of the pillar, m_E – equivalent mass (pillar + foundation + vibrating ground), η_E – equivalent damping ("radiation" damping + viscous damping), q_E – elasticity coefficient (strong soil). In Table 3 the system parameters used in numerical examples are presented. The first eigenfrequency of the bay reads $\omega_1 = 6.03$ Hz (the experiment shows 6.6 Hz) and the corresponding logarithmic decrement – $\Lambda = 0.05$. The logarithmic decrement Λ_E for the equivalent system equals 3.14.

Table 3. Periodic guideway.

$E = 4.6\ 10^{10}$ N/m^2	$I = 0.5$ m^4	$\mu = 4\ 10^3$ kg
$q_E = 1.5\ 10^9$ N/m	$\eta_E = 1.6\ 10^7$ Ns/m	$m_E = 1.8\ 10^5$ kg
$q \to 0$	$\eta = 2.4\ 10^3$ Ns/m	$l = 25$ m

There are many methods used in the dynamical analysis of mechanical periodic systems. For example, the transfer matrix method [4], receptance method [5], space harmonic analysis [6], energetic methods [7], Fourier series method [8], travelling wave method [9]. In the field of solid state physics and electrical engineering, the approach using Floquet's theorem in solving Hill's equation is applied, which is widely discussed in the classical book of Brillouin [10]. In the present paper the above named theorem is used to solve the equation of motion of a periodic track and a periodic guideway. Also periodic guideways have been investigated in papers [11, 12].

3. Free wave propagation in periodic structures

In case of free wave propagation ($p_0 = 0$) the solution can be written in the following form

$$W(X,\tau) = A(X,\lambda)\cdot \exp[i(\lambda X + \Omega_0\tau)]. \tag{3.1}$$

We assume that the function $A = A(X,\lambda)$ describes the dynamically admissible displacement field and, according to Floquet's theorem, is a periodic function, i.e. is a function which is independent of the choice of a cell of the periodic structure,

$A(X+L,\lambda) = A(X,\lambda)$. Introducing the relation (3.1) into Eq. (2.2) in which the right-hand side equals zero, yields the equation for the function $A = A(X,\lambda)$,

$$[D_A^4(X,\lambda) - S^4] \cdot A(X,\lambda) = 0, \tag{3.2}$$

where

$$D_A(X,\lambda) = i\lambda + \frac{\partial}{\partial X}, \qquad S^4 = \Omega_0^2 - iN\Omega_0 - Q. \tag{3.3}$$

3.1. The case of a periodic railway track

The conditions for the function $A = A(X,\lambda)$ which follow from the boundary conditions of the displacement $W = W(X,\tau)$, Eqs. (2.3), read

$$\begin{aligned} A(nL) &= A[(n+1)L], \\ D_A(X)\cdot A(nL) &= D_A(X)\cdot A[(n+1)L], \\ D_A^2(X)\cdot A(nL) &= D_A^2(X)\cdot A[(n+1)L], \\ D_A^3(X)\cdot\{A[(n+1)]L &- A(nL)\} - \Delta\cdot A(nL) = 0. \end{aligned} \tag{3.4}$$

The solution of the ordinary differential equation (3.2) satisfying the conditions given by Eq. (3.4) reads

$$W(X,\tau) = \overline{A}(\xi,\lambda)\cdot\exp[i(\lambda nL + \Omega_0\tau)], \tag{3.5}$$

where

$$\begin{aligned} \overline{A}(\xi,\lambda) = {} & [\sin S\xi e^{i\lambda L} + \sin S(L-\xi)](\cos\lambda L - \cosh SL) \\ & - [\sinh S\xi e^{i\lambda L} + \sinh S(L-\xi)](\cos\lambda L - \cos SL), \end{aligned} \tag{3.6}$$

for

$$X \in \langle nL,\ (n+1)L\rangle, \qquad \xi = X - nL, \qquad \xi \in \langle 0, L\rangle.$$

Equation (3.5) represents a travelling wave in the periodic system, its "shape" being given by Eq. (3.6). The dispersion relation, i.e. the relation "frequency Ω_0 – wavenumber λ" can be written in the following way:

$$\begin{aligned} f(\lambda,\Omega_0) = {} & \cos^2\lambda L - \cos\lambda L\Big[\cos SL + \cosh SL \\ & + \frac{\Delta}{4S^3}(\sin SL - \sinh SL)\Big] + \cos SL\cosh SL \\ & + \frac{\Delta}{4S^3}(\sin SL\cosh SL - \sinh SL\cos SL) = 0. \end{aligned} \tag{3.7}$$

Equation (3.7), which can also be written in the following form

$$f(\lambda, \Omega_0) = [\cos \lambda L - f_1(\Omega_0)] \cdot [\cos \lambda L - f_2(\Omega_0)] = 0, \tag{3.8}$$

yields two values of the wavenumber λ for a given frequency Ω_0. In the general viscoelastic case the dispersion relation (3.7) is satisfied by a complex wavenumber $\lambda = \lambda_R + i\lambda_I$, with $\lambda_R = \mathrm{Re}(\lambda)$ – the wavenumber, $\lambda_I = \mathrm{Im}(\lambda)$ – the attenuation number. The solution given by Eq. (3.5) takes the following form:

$$W(X, \tau) = \overline{A}(\xi, \lambda) \cdot \exp[-\lambda_I nL + i(\lambda_R nL + \Omega_0 \tau)]. \tag{3.9}$$

In the pure elastic case ($N = 0$, $\Delta(\Omega_0) = \text{const}$) one can distinguish two characteristic cases: When $\lambda_I = 0$, $\lambda_R \neq 0$ – a travelling wave given by Eq. (3.9) can propagate in the whole infinite structure, which corresponds to so-called "passing bands" in the (Ω_0, λ)-plane [10], and when $\lambda_I \neq 0$ – a wave cannot propagate and its attenuation in space is determined by the term $\exp(-\lambda_I nL)$, which corresponds to "stopping bands" in the (Ω_0, λ)-plane.

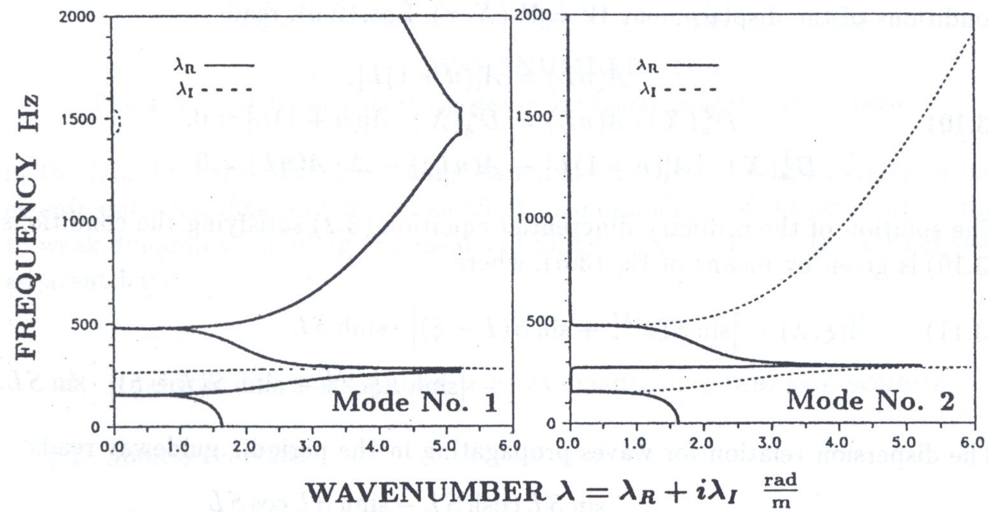

FIG. 3. Railway track: Dispersion relations in the pure elastic case.

In Fig. 3 the dispersion relations (3.7) calculated for the pure elastic case are illustrated. The curves representing the attenuation number λ_I (dashed lines) and the wavenumber λ_R (continuous lines) are symmetrical with respect to both Ω_0 and λ axis. The first Brillouin zone (or propagation zone, [10]), for $l = 0.6$ m reads $\lambda_R \in <-\pi/l, \pi/l> = <-5.24\,\text{rad/m}, 5.24\,\text{rad/m}>$. The case shown in Fig. 3 illustrates negative-going waves corresponding to the first Brillouin zone. In case of the first mode we have passing bands in the following frequency ranges (in Hertz): (156.2, 263.9), (481.4, 1422.4), (1547.1, 5690.0), and stopping bands for

(0, 156.2), (263.9, 481.4), (1422.4, 1547.1). For the frequency equal to 277.0 Hz, which is an eigenfrequency of the system pad-sleeper-ballast, the contact force in a pad tends to infinity and flexible supports become rigid ones which results in uncoupling the adjacent cells of the periodic system. In the frequency range (277.0, 481.4) Hz in both wavemotion modes we have so-called "propagating and attenuating wave" for which $\lambda_I = \lambda_R$, Fig. 3. The second mode of wavemotion is always attenuated ($\lambda_I > 0$ for any value of Ω). The frequencies 1422.4 Hz and 5690.0 Hz are eigenfrequencies of a simply supported beam of the same length and the same other parameters as the periodic structure cell. The corresponding eigenforms are so-called "pinned-pinned" modes [13], with nodes at the periodic supports. A discussion of eigenforms corresponding to the cases $\lambda_I = 0$, $\lambda_R = 0$ or $\lambda_I = 0$, $\lambda_R = \pi/L$, can be found in the paper [14].

3.2. The case of a periodic guideway for Maglev

The conditions for the function $A = A(X, \lambda)$ which follow from the boundary conditions of the displacement $W = W(X, \tau)$, Eqs. (2.6), read

$$
\begin{aligned}
A(nL) &= A[(n+1)L], \\
D_A^2(X) \cdot A(nL) &= D_A^2(X) \cdot A[(n+1)L] = 0, \\
D_A^3(X) \cdot \{A[(n+1)L] &- A(nL)\} - \Delta \cdot A(nL) = 0.
\end{aligned}
\tag{3.10}
$$

The solution of the ordinary differential equation (3.2) satisfying the conditions (3.10) is given by means of Eq. (3.5), where

$$
\begin{aligned}
\overline{A}(\xi, \lambda) = {} & \Big[\sin S\xi e^{i\lambda L} + \sin S(L-\xi)\Big] \cdot \sinh SL \\
& - \Big[\sinh S\xi e^{i\lambda L} + \sinh S(L-\xi)\Big] \cdot \sin SL.
\end{aligned}
\tag{3.11}
$$

The dispersion relation for waves propagating in the periodic guideway reads

$$
\begin{aligned}
f(\lambda, \Omega_0) = {} & \cos \lambda L - \frac{\sin SL \cosh SL - \sinh SL \cos SL}{\sin SL - \sinh SL} \\
& - \frac{\Delta(\Omega_0)}{S^3} \cdot \frac{\sin SL \sinh SL}{\sin SL - \sinh SL} = 0,
\end{aligned}
\tag{3.12}
$$

which yields one mode of the wavemotion.

The dispersion relation in case of vertical system dynamics is illustrated in Fig. 4, where continuous lines correspond to the wavenumber λ_R, dashed lines to the attenuation number λ_I and the shaded areas represent passing bands in the (Ω_0, λ)-plane. As follows from Fig. 4, the stopping bands, i.e. the frequency ranges where the propagation of free harmonic waves is not possible, dominate

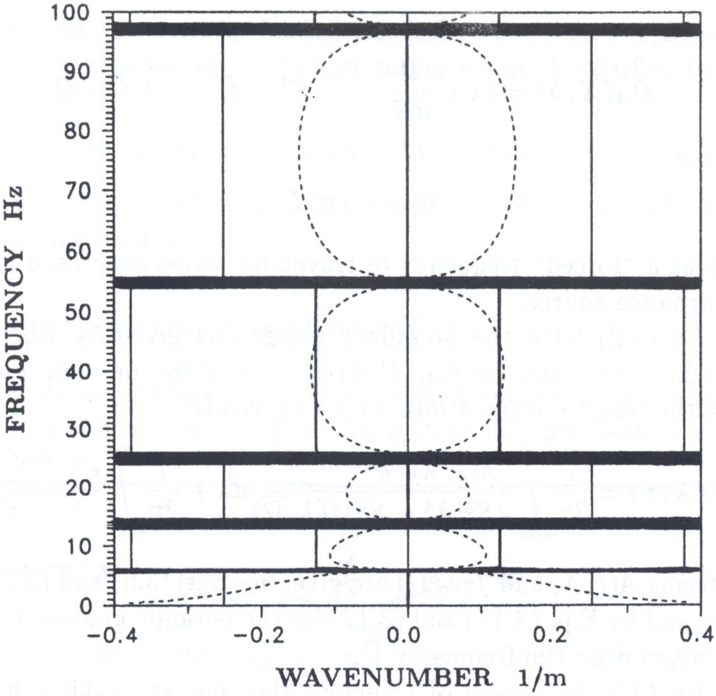

FIG. 4. Vertical dynamics of the guideway: configuration of the (Ω_0, λ)-plane.

in the (Ω_0, λ)-plane. Narrow passing bands can be found in the vicinity of bay eigenfrequencies. The property of small "transmissibility" of the structure is due to weak dynamical coupling (by means of pillars resting on strong soil) between adjacent bays.

4. Periodic systems under a travelling disturbance source

4.1. *Solution method*

The solution for a periodic system subjected to a moving harmonic force can be written in the following form:

$$W(X,\tau) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} A(X,\lambda)\exp[i\lambda(X - V\tau) + i\Omega\tau]d\lambda. \tag{4.1}$$

Introducing the relation (4.1) into the Eq. (2.2), yields the following non-homogeneous equation for the function $A = A(X, \lambda)$:

$$[D_A^4(X,\lambda) - S^4]\cdot A(X,\lambda) = 1, \tag{4.2}$$

where

$$D_A(X,\lambda) = i\lambda + \frac{\partial}{\partial X}, \qquad S^4 = \overline{\Omega}^2 - iN\overline{\Omega} - Q. \tag{4.3}$$

The frequency

$$\overline{\Omega} = -\lambda V + \Omega \tag{4.4}$$

can be seen as a "forced" frequency of travelling waves generated in the system by the disturbance source.

Solving Eq. (4.2) with the boundary conditions given by Eq. (3.4) in case of the periodic track, and by Eq. (3.10) in case of the periodic guideway, and introducing the result obtained into Eq. (4.1), yields

$$W(X,\tau) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \frac{\overline{A}(\xi,\lambda)\cdot e^{i(\lambda nL+\overline{\Omega}\tau)}}{4S^3(\lambda^4 - S^4)f(\lambda,\overline{\Omega})}d\lambda + \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \frac{e^{i(\lambda X+\overline{\Omega}\tau)}}{\lambda^4 - S^4}d\lambda, \tag{4.5}$$

where the terms $\overline{A}(\xi,\lambda)$ and $f(\lambda,\overline{\Omega})$ are given by Eqs. (3.6) and (3.7), for the periodic track, and by Eqs. (3.11) and (3.12) for the periodic guideway, respectively, which now depend on the frequency $\overline{\Omega}$.

Solving Eq. (4.5) by means of Cauchy's theorem and taking into account a certain finite number of poles of the integrand, yields the solution for cells ahead of or behind the load which can be written in the following form:

$$W(X,\tau) = \sum_{M=1}^{N_M}\sum_{k=1}^{K_M} Q_{Mk}(V,\Omega)\cdot \overline{A}_{Mk}(\xi,\lambda_{Mk}) \cdot \exp i(\lambda_{Mk}\cdot nL + \overline{\Omega}_{Mk}\cdot\tau), \tag{4.6}$$

for

$$X \in \langle nL,\ (n+1)L\rangle, \qquad \xi = X - nL, \qquad \xi \in \langle 0, L\rangle .$$

The solution obtained is a superposition of waves travelling in the periodic system. The "shape" of the waves which correspond to two modes M, ($N_M = 2$) in case of the periodic track is given by Eq. (3.6) and one mode M, ($N_M = 1$) in case of the periodic guideway is given by Eq. (3.11). In Eq. (4.6) we have $Q_{Mk}(V,\Omega)$ – the wave amplitude, $K_M = K_{M(A)}$ – number of waves ahead of the load $[nL \ge V\tau]$, $K_M = K_{M(B)}$ – number of waves behind the load $[(n+1)L \le V\tau]$, for a given mode M.

The wavenumber λ_{Mk} is determined by means of the following relation:

$$f(\lambda_{Mk},\overline{\Omega}_{Mk}) = 0, \qquad \overline{\Omega}_{Mk} = -\lambda_{Mk}V + \Omega, \tag{4.7}$$

where $f = 0$ is the dispersion relation given by Eq. (3.7) for the periodic track and by Eq. (3.12) for the periodic guideway.

The solution for the actually loaded cell $[nL \le V\tau \le (n+1)L]$ is composed of terms where λ satisfies Eq. (4.7) and terms where λ satisfies the following relation:

$$\lambda^4 - (-\lambda V + \Omega)^2 + iN(-\lambda V + \Omega) + Q = 0, \tag{4.8}$$

which is the characteristic equation of a continuous and non-periodic beam under a moving harmonic load [15].

4.2. Periodic track under a travelling load

In Fig. 5 the dynamic track response to a load moving with the velocity 50 m/s is illustrated. As follows from Fig. 5, the system displacement is a wave with

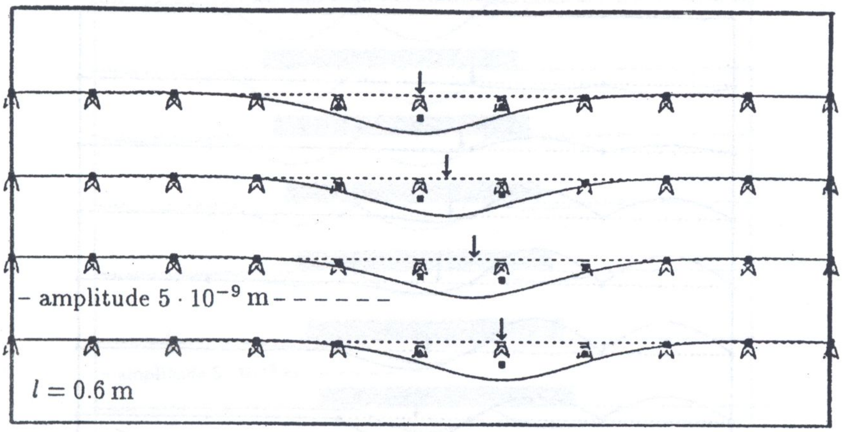

FIG. 5. Track displacements for selected times and $V = 50$ m/s and $\Omega = 0$.

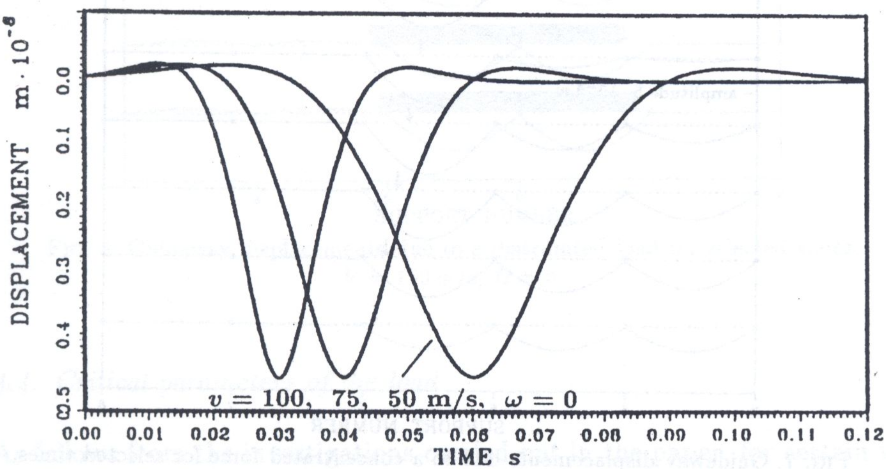

FIG. 6. Track deflection at $X = 5 \cdot l$ for selected load velocities and $\Omega = 0$.

wavelength of ≈ 3.8 m which travels with the load velocity yielding the frequency of ≈ 13 Hz at the fixed rail point. In Fig. 6 the rail deflection at a fixed point $X = 51$ is shown in case when the moving force "starts" at the point $X = 0$ and travels during the time $t = 0.12$ s. As follows from Fig. 6, for the considered load velocities the dynamic system responses are qualitatively similar.

4.3. Periodic guideway under a travelling load

The dynamic responses of the guideway to a load in the form of a concentrated force $p_0 = 3.3 \cdot 10^5$ N and a load distributed over a section of the length of 48 m are illustrated in Figs. 7 and 8, respectively. Actually, the load distributed over a

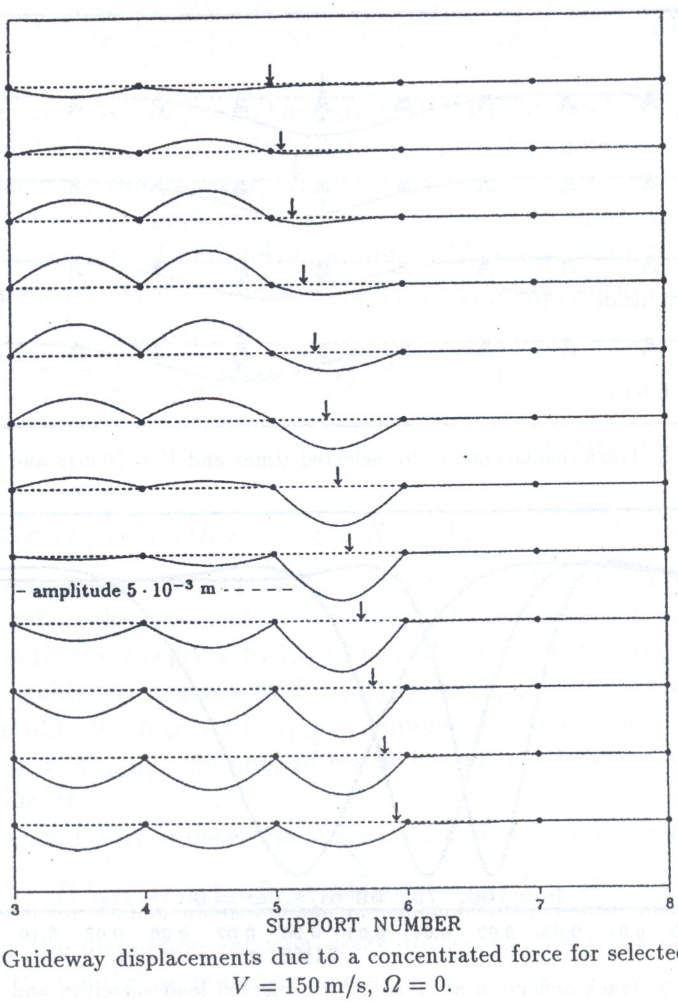

FIG. 7. Guideway displacements due to a concentrated force for selected times, $V = 150$ m/s, $\Omega = 0$.

guideway length of $L_V = 48$ N is modelled as a superposition of 49 concentrated forces yielding the total force of 10^6 N, which approximately corresponds to the Maglev weight. When the travelling forces leave the bay, this vibrates with its first eigenfrequency. As follows from the investigations carried out in the paper, the dynamic response of a single bay is very similar to the case of a simply supported beam with zero initial conditions.

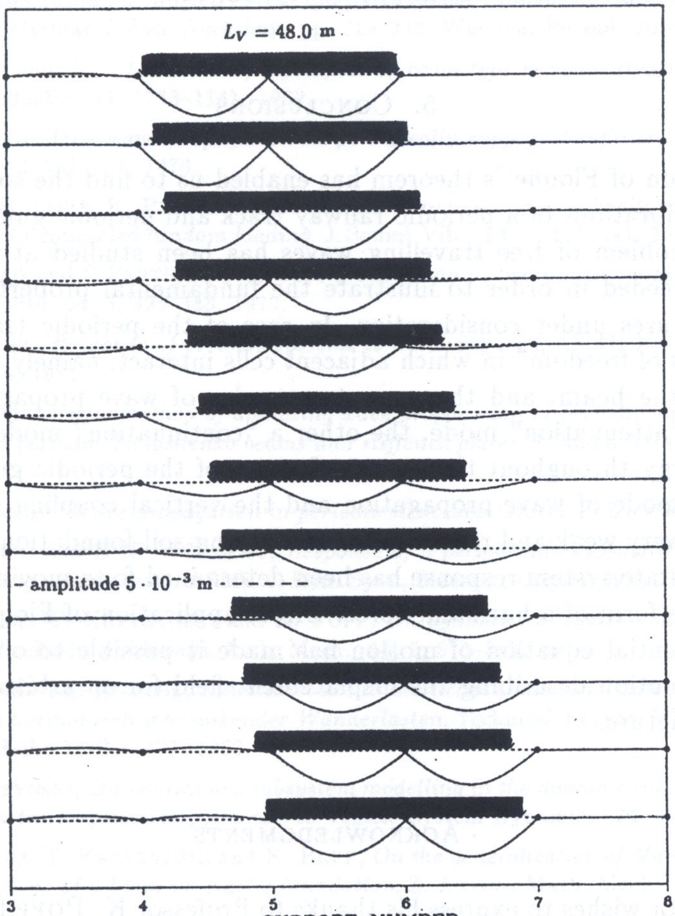

FIG. 8. Guideway displacements due to a distributed load for selected times, $V = 150$ m/s, $\Omega = 0$.

4.4. Critical parameters of the load

As follows from the investigations carried out in the paper, for certain values of load velocity and frequency, the dynamic response of a viscoelastic system takes maximum values, and displacements of the pure elastic system increase infinitely in time. This phenomenon takes place when the load moves with the group velocity of the travelling wave which is generated. In the pure elastic case, the energy supplied to the system cannot be radiated which results in an infinite increase of wave amplitudes. For the same load parameters the amplitudes of waves propagating in a viscoelastic system can reach their local maxima. This problem is discussed in details in papers [15, 12, 14].

5. Conclusions

Application of Floquet's theorem has enabled us to find the solution for free and forced vibrations of a periodic railway track and periodic guideway for Maglev. The problem of free travelling waves has been studied at first. Such an analysis is needed in order to illustrate the fundamental properties of the periodic structures under consideration. In case of the periodic track, there are two "degrees of freedom" in which adjacent cells interact, namely deflection and rotation of the beam, and there are two modes of wave propagation. One of them is an "attenuation" mode, the other a "continuation" mode which transfers the energy throughout the system. In case of the periodic guideway, there is only one mode of wave propagation and the vertical coupling between adjacent bays is very weak and results from the "strong-soil-foundation" assumption. The steady-state system response has been determined for a moving disturbance source in the form of a harmonic force. Direct application of Floquet's theorem to the differential equation of motion has made it possible to obtain an exact analytical solution describing the displacement field for an arbitrary cell of the periodic structure.

Acknowledgments

The author wishes to express his thanks to Professor K. Popp from the Institute of Mechanics, University of Hannover, and Professor R. Bogacz from the Institute of Fundamental Technological Research, Polish Academy of Sciences, for their support and for many discussions.

The paper contains results of investigation sponsored by the Polish State Committee for Scientific Research (KBN) grant T07B01410 and the cooperative research project "Dynamics of Periodic Structures with Mistuning" granted by the Volkswagen Foundation, Hannover, Germany. The support of these institutions is highly appreciated.

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Received October 3, 1996.