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Research Paper

Exact Solution for Non-Newtonian Fluid Flow Beyond a Contaminated Fluid Sphere

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Stokes flow of non-Newtonian fluid beyond a partially contaminated non-Newtonian fluid sphere with interfacial slip condition is considered. An analytic solution for the flow fields indicated by the stream function and the drag force over the sphere was obtained. Special well-known cases are reduced. It was observed that with an increase in slip parameter values, there is a rise in drag coefficient values.

Key words: micropolar fluid; Gegenbauer function; modified Bessel's function; stagnant cap.

1. Introduction

Ever since the idea of micropolar fluids was introduced, multiple researchers have attempted to solve various fluid flow problems involving such fluids. The axisymmetric flow issues in micropolar fluids are an intriguing class of problems that have piqued the curiosity of researchers. Micropolar fluids have applications in blood flow, liquid crystals, lubricants, colloidal suspensions, bubble fluids, etc.

Basset [1] in his monograph presented slip condition, i.e., the tangent velocity relative to a solid boundary is proportional to the corresponding viscous stress. The slip parameter is given by $s = \frac{\beta a}{\mu}$. The no-slip case is obtained when $s \to \infty$. Happel and Brenner [2] in their monograph have also discussed the interfacial slip over a surface. Feng *et al.* [3] elaborated on the Newtonian flow over a Newtonian drop with finite Reynolds numbers and with interfacial slip condition. The outcome of their work concluded that the drag force over the surface was reduced with the presence of slip over the surface.

Eringen [4, 5] introduced the theory of micropolar fluids. The particles in such fluid can rotate with their own spins and micro rotations. According to the theory on the description of the fluid motion, two vectors, velocity and spin vectors, are used. The treatise by Łukaszewicz [6] also widely discussed the theory of micropolar fluids. Ramkissoon and Majumdar [7], in their study of flow over a micropolar fluid past an axisymmetric body, obtained a drag force formula in limiting form with a stream function. By using this drag force, a micropolar fluid flow past an impervious sphere was calculated. Jaiswal and Yadav [8] investigated the reaction of the thickness of the micropolar fluid layer through two layers of Newtonian fluid stresses at the interfaces, velocities of fluids, and on distinct Darcy number of porous layers. Dang *et al.* [9] analyzed the aspects of lubricants: power law fluids, couple stress lubricants and micropolar fluids and their influence on journal bearing effectiveness. Alouaoui *et al.* [10] considered a laminar boundary layer fluidity of a micropolar nano-fluid close to a vertical permeable plate, which is in motion. The effect of stability and magnetic fields on heat transfer are studied over vertical permeable plate. The above-mentioned authors have considered micropolar fluid in their investigations.

Rybczyński [11] and Hadamard [12] in their works studied the closed-form solution for the laminar flow past a fluid sphere. Clift *et al.* [13] and Michaelides [14] analysed Newtonian flow over a fluid sphere by applying no-slip condition in their research. Niefer and Kaloni [15] obtained an expression for stream function, spin and drag in their study of Newtonian fluid past a non-Newtonian fluid drop with the no-slip condition at the boundary. Ramkissoon [16], in his study on micropolar fluid flow past the moving fluid sphere, obtained an analytic solution for velocity and drag over the body. Ramkissoon and Majumdar [17] studied the creeping flow of non-Newtonian fluid over deformed viscous spheroid. The drag over it was obtained analytically. Hoffman *et al.* [18], in their investigation, considered the drag force exerted over a moving sphere placed in a micropolar fluid considered with a non-zero spin condition for the micro-rotation over the boundary. Deo and Shukla [19], in their study over a non-Newtonian fluid beyond a fluid drop with non-zero spin boundary condition, obtained an exact solution for the drag force. All the above investigations are related to works on fluid flow over a fluid sphere.

Sadhal and Johnson [20] examined the Stokes flow over a bubble or a liquid drop whose interface is partially contaminated and is placed in an unmixable fluid. The exact solution for the drag force was derived. Saboni *et al.* [21], in their study, concluded that the flow greatly affected the Reynolds number and stagnant cap segment, and also evaluated the drag coefficient inversely proportional to the Reynolds numbers in the state of stable stagnant cap and a given viscosity ratio κ. Ramana Murthy and Phani Kumar [22, 23] studied laminar Newtonian fluid flow past a partially contaminated fluid sphere with slip and no-slip conditions. Saboni *et al.* [24] developed the partially or fully contaminated drop by considering the model of mass transfer from a continuous phase. These are a few works in the literature on the contaminated fluid sphere.

In the above-mentioned works, most authors have considered Newtonian fluid flow over a contaminated fluid sphere with a no-slip condition on the boundary. In addition, the other studies' reviews reveal that, no study was conducted on the two-sided flow of non-Newtonian fluid past a partially contaminated fluid sphere with interfacial slip condition. This motivated the authors of the present study to consider micropolar fluid flow past a partially contaminated micropolar fluid sphere with interfacial slip over the boundary, continuity of shear stress on the clear part, and regularity condition far away from the body. The velocity components are expressed in terms of stream function and an exact solution is investigated. The drag coefficient is evaluated.

2. Formulation of the problem

2.1. Basic equations

We know the momentum equation for micropolar fluid from Eringen [4, 5] as:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0, \tag{2.1}$$

$$\rho \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \rho \mathbf{f} - \nabla p + k \nabla \times \mathbf{w} - (\mu + k) \nabla \times \nabla \times \mathbf{V} + (\lambda + 2\mu + k) \nabla (\operatorname{div} \mathbf{V}), \tag{2.2}$$

$$\rho J \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t} = \rho \mathbf{I} - 2k\mathbf{w} + k \nabla \times \mathbf{V} - \gamma \nabla \times \nabla \times \mathbf{w} + (\alpha + \beta + \gamma) \nabla (\operatorname{div} \mathbf{w}), \tag{2.3}$$

where ρ is the density, $\mathbf{V}$ is the velocity field, $\mathbf{w}$ is the microrotation field, J is the gyration parameter, $\mathbf{f}$ is body forces per unit mass, $\mathbf{I}$ is microrotation driving forces per unit mass, p is the pressure, μ is the classical viscosity coefficients. k, λ, μ are vortex viscosity coefficients, and α, β, γ are gyroviscosity coefficients satisfying the following inequalities:

$$\begin{gathered} 3\alpha + \beta + \gamma \geq 0, \qquad 2\mu + k \geq 0, \qquad 3\lambda + 2\mu + k \geq 0, \\ \gamma \geq |\beta|, \qquad k \geq 0, \qquad \gamma \geq 0. \end{gathered} \tag{2.4}$$

2.2. Formulation

Consider a stationary micropolar fluid sphere placed in a micropolar fluid region. The flow is considered to be steady, axisymmetric and uniform far away from the body. The micropolar fluid is assumed to flow from left to right. The surfactants are accumulated over the rear end, i.e., the interface is contaminated partially over a region. The contaminated portion is named the cap region, and the remaining portion with the clear interface is named the no-cap region. Consider for the no-cap zone as $-1 < x < x_0$ which is the clear segment, and for the cap zone $x_0 < x < 1$, which is the contaminated segment in the fluid where $x = \cos\theta$. Here $\mu = \frac{\mu_i}{\mu_e}$, $\rho = \frac{\rho_i}{\rho_e}$. The posture of the cap is the cosine angle of contamination x_0 (Ramana Murthy and Phani Kumar [23]). The geometry of the model is given in Fig. 1.

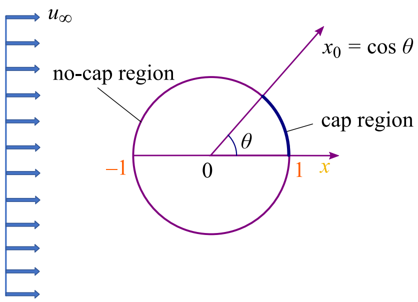

Fig. 1. Geometry of the model.

For convenience, spherical polar coordinates $(r,\ \theta,\ \phi)$ are taken with the $\theta = 0$ axis in the direction of the free stream flow. The velocity and angular rotation of the flow field are:

$$\mathbf{q} = (u_r, u_\theta, 0), \qquad \mathbf{V} = (0, 0, v_\phi)\,.$$

In the view of the axisymmetric flow, the velocity components expressed in terms of stream function ψ are:

$$u_r = \frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad u_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}.$$

To match uniform velocity at infinity, the solution for ψ can be assumed in the form:

$$\psi_e(r,x) = \begin{cases} \psi_{en}(r)G_2(x) & \text{for } -1 < x < x_0 \quad \text{(no-cap region)},\\ \psi_{ec}(r)G_2(x) & \text{for } x_0 < x < 1 \quad \text{(cap region)},\end{cases}$$

(2.5)

$$\psi_i(r,x) = \begin{cases} \psi_{in}(r)G_2(x) & \text{for } -1 < x < x_0 \quad \text{(no-cap region)},\\ \psi_{ic}(r)G_2(x) & \text{for } x_0 < x < 1 \quad \text{(cap region)}.\end{cases}$$

Here ψ_{en}, ψ_{ec} are external stream functions for no-cap and cap regions, respectively. Also, ψ_{in}, ψ_{ic} represent internal stream function with no-cap and cap regions, respectively, and $G_2(x)$ is the Gegenbauer function of order 2.

The moment equation and angular velocity for a micropolar fluid flow past a contaminated fluid sphere are given by:

$$-\nabla p + k\nabla \times \mathbf{w} - (\mu + k)\nabla \times \nabla \times \mathbf{v} = 0, \tag{2.6}$$

$$-2k\mathbf{w} + k\nabla \times \mathbf{v} - \gamma\nabla \times \nabla \times \mathbf{v} + (\alpha + \beta + \gamma)\,\nabla(\nabla \cdot \mathbf{w}) = 0. \tag{2.7}$$

The momentum equation on simplification reduces to:

$$E^4 \left(E^2 - \frac{\lambda^2}{a^2}\right)\psi = 0, \tag{2.8}$$

and angular velocity C is given as:

$$C = -\frac{1}{2}\left(\frac{\Gamma\,(\mu + k)}{k^2} E^4\psi + E^2\,\psi\right), \tag{2.9}$$

where

$$\frac{\lambda^2}{a^2} = \frac{k(2\mu + k)}{\Gamma(\mu + k)}, \qquad c = \frac{k}{\mu + k},$$

and

$$E^2 = \frac{\partial^2\psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right)$$

is the axi-symmetrical potential operator.

When $k = 0 \Longrightarrow c = 0$, micropolar fluid reduces to a viscous fluid.

The general stream function of (2.8) from Ramkissoon [16] is:

$$\psi = \left(\frac{A}{r} + Br + Cr^2 + Dr^4 + C_e\sqrt{r}k_{3/2}\,(\lambda_e r) + C_i\sqrt{r}I_{3/2}\,(\lambda_i r)\right)\frac{\sin^2\theta}{2}. \tag{2.10}$$

The stream function for the two regions is given as $\psi = \psi_i + \psi_e$ to satisfy Eq. (2.8).

The solutions for external, internal, no cap and cap regions are:

$$\psi_{en} = \left[r^2 + \frac{A_1}{r} + B_1 r^4 + C_1\sqrt{r}k_{3/2}(\lambda_{en} r)\right] G_2(x), \tag{2.11}$$

$$\psi_{ec} = \left[r^2 + \frac{A_2}{r} + B_2 r^4 + C_2\sqrt{r}k_{3/2}(\lambda_{ec} r)\right] G_2(x), \tag{2.12}$$

$$\psi_{in} = \left[A_3 r^2 + B_3 r^4 + C_3\sqrt{r}I_{3/2}(\lambda_{in} r)\right] G_2(x), \tag{2.13}$$

$$\psi_{ic} = \left[A_4 r^2 + B_4 r^4 + C_4\sqrt{r}I_{3/2}(\lambda_{ic} r)\right] G_2(x), \tag{2.14}$$

where $I_{3/2}$, $k_{3/2}$ are modified Bessel's functions (Abramowitz, Stegun [25]).

The parameters A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3, A_4, B_4, C_4 in (2.11)–(2.14) are evaluated using the following boundary conditions:

(i) Regularity conditions:

$$\lim_{r\to\infty}\psi_e = \frac{1}{2}Ur^2\sin^2\theta \quad \text{(outside the region) and} \qquad \psi_i = \text{finite (inside the region).} \tag{2.15}$$

(ii) Tangential velocity is zero on the boundary condition:

$$\lim_{r\to 0}\psi_{ec} = \psi_{ic} = \psi_{en} = \psi_{in} = 0 \quad \text{on} \quad r = 1. \tag{2.16}$$

(iii) Slip condition: tangential velocity is proportional to the tangential shear along the clear surface, see Happel and Brenner [2]:

$$\tau_{r\theta} = \beta(q_\theta - v_\theta) \quad \text{on} \quad r = 1. \tag{2.17}$$

(iv) Shear stress is continuous across the surface, i.e.:

$$\tau_{r\theta e} = \tau_{r\theta i} \quad \text{on} \quad r = 1. \tag{2.18}$$

(v) Angular velocity is zero on the boundary condition, i.e., the micro-rotation vector $C = 0$ on $r = 1$:

$$C_{en} = C_{ec} = C_{in} = C_{ic} = 0 \quad \text{on} \quad r = 1. \tag{2.19}$$

3. Solution of the problem

Using the boundary condition (2.15)–(2.19) in (2.11)–(2.14), we obtain the system of equations as:

$$A_1+B_1 + C_1' = -1, \qquad A_2+B_2 + C_2' = 0, \qquad A_3+B_3 + C_3' = 0,$$

$$B_1 = -1 - A_1 - C_1', \qquad B_2 = -1 - A_2 - C_2', \qquad B_3 = -1 - A_3 - C_3',$$

$$(6+s)A_1 - sB_1 + C_1'\left(4 + (2+s)\Delta_1(\lambda_{en})\right) + 2sA_3 + 4sB_3 - C_3'\Delta_3(\lambda_{in}) = 2s,$$

$$(6+s)A_2 - B_2 s + C_2'\left(4 + (2+s)\Delta_2(\lambda_{ec})\right) + 2sA_4 + 4sB_4 - C_4'\Delta_4(\lambda_{ic})s = 2s,$$

$$-6A_1 - C_1'\left(4 + 2\Delta_1(\lambda_e)\right) + 6\mu B_3 + \mu C_3'(4 + 2\Delta_3(\lambda_{in})) = 0,$$

$$A_4 = 0, \qquad B_4 = 0, \qquad \text{and} \qquad C_4' = 0,$$

where

$$C_1' = C_1 k_{3/2}(\lambda_{en}), \qquad C_2' = C_2 k_{3/2}(\lambda_{ec}),$$

$$C_3' = C_3 I_{3/2}(\lambda_{in}), \qquad C_4' = C_4 I_{3/2}(\lambda_{ic}),$$

$$\Delta_1(\lambda_{en}) = 1 + \frac{\lambda_{en} k_{1/2}(\lambda_{en})}{k_{3/2}(\lambda_{en})}, \qquad \Delta_2(\lambda_{ec}) = 1 + \frac{\lambda_{ec} k_{1/2}(\lambda_{ec})}{k_{3/2}(\lambda_{ec})},$$

$$\Delta_3(\lambda_{in}) = 1 + \frac{\lambda_{in} I_{1/2}(\lambda_{in})}{I_{3/2}(\lambda_{in})}, \qquad \Delta_4(\lambda_{ic}) = 1 + \frac{\lambda_{in} I_{1/2}(\lambda_{ic})}{I_{3/2}(\lambda_{ic})},$$

and slip parameter $(s) = \frac{\beta a}{\mu}$.

Solving the above system of equations, we obtain the values of parameters as:

$$A_1 = -1 - C_1' \left(\frac{\lambda_{en}^2}{c} + 1 \right), \qquad A_2 = -1 - C_2' \left(\frac{\lambda_{ec}^2}{c} + 1 \right),$$

$$A_3 = -\left(1 - \frac{1}{5} \frac{\lambda_{in}^2}{c}\right) C_3',$$

$$B_1 = \frac{\lambda_{en}^2}{c} C_1', \qquad B_2 = \frac{\lambda_{ec}^2}{c} C_2',$$

$$B_3 = -\frac{1}{5} \frac{\lambda_{in}^2}{c} C_3', \tag{3.1}$$

$$C_1' = \frac{(3s+6) f_2 + 3t_2}{\Lambda}, \qquad C_2' = \frac{(3s+6)\, f_2 - 3t_3}{t_3 f_1},$$

$$C_3' = \frac{-\,(3s+6)\, f_1 - 3t_1}{\Lambda},$$

where

$$\Lambda = t_1 f_2 - t_2 f_1,$$

$$f_1 = -\frac{3\lambda_{en}^2}{c} + 1 - (\Delta_1(\lambda_e)), \qquad f_2 = \mu \left(-\frac{3}{5} \frac{\lambda_{in}^2}{c} + 2 + (\Delta_2(\lambda_{in})) \right),$$

$$t_1 = -2 \left(\frac{\lambda_{en}^2}{c} \right) (s+3) + ((2+s)(\Delta_1(\lambda_{en}) - 1)),$$

$$t_2 = -s \left(\frac{\lambda_{in}^2}{c} \right) \left(\frac{2}{5} \right) + (2 + (\Delta_2(\lambda_{in}))),$$

$$t_3 = -2 \left(\frac{\lambda_{ec}^2}{c} \right) (s+3) + ((2+s)(\Delta_3(\lambda_{ec}) - 1)).$$

Thus, the stream functions for external and internal flows are obtained.

Special cases:

(1) External stream function:

(a) For the no-cap region:
A micropolar fluid reduces to a viscous fluid when $\lambda_{in}^2 \to \infty$, $\lambda_{en}^2 \to \infty$. Then, we obtain:

$$C_1 = 0, \qquad B_1 = -\frac{(3s+6)\mu + 2s}{(6+2s)\mu + 2s}, \qquad A_1 = \frac{s\mu}{(6+2s)\mu + 2s}.$$

The external stream function for the no-cap region is:

$$\psi_{en} = \left(r^2 - \left(\frac{2s+3s\mu+6\mu}{2s\mu+6\mu+2s}\right) r + \left(\frac{s\mu}{2s\mu+6\mu+2s}\right)\frac{1}{r}\right) G_2(x), \tag{3.2}$$

which matches the results of Ramana Murthy and Phani Kumar [22]. In addition, when $s \to \infty$, we obtain a no-slip condition, and with this, we get external velocity for a fluid sphere with the no-slip condition as:

$$\psi_{en} = \left(r^2 - \left(\frac{3\mu+2}{2\mu+2}\right) r + \left(\frac{\mu}{2\mu+2}\right)\frac{1}{r}\right) G_2(x). \tag{3.3}$$

In addition, when $\mu \to \infty$, the viscous fluid sphere reduces to a solid sphere, and hence, we obtain external velocity for a solid sphere with the no-slip condition as:

$$\psi_{en} = \left(r^2 - \left(\frac{3}{2}\right) r + \left(\frac{1}{2}\right)\frac{1}{r}\right) G_2(x). \tag{3.4}$$

Equations (3.3) and (3.4) match the results of Happel and Brenner [2].

(b) For the cap region:
A micropolar fluid reduces to a viscous fluid when $\lambda_{ic}^2 \to \infty$, $\lambda_{ec}^2 \to \infty$. With this, we obtain:

$$C_2 = 0, \qquad B_2 = -\frac{3s+6}{2s+6}, \qquad A_2 = \frac{s}{2s+6}.$$

Hence, the external stream function ψ_{ec} for the Newtonian fluid sphere with the slip condition is:

$$\psi_{ec} = \left(r^2 - \left(\frac{3s+6}{2s+6}\right) r + \left(\frac{s}{2s+6}\right) \frac{1}{r}\right) G_2(x), \tag{3.5}$$

which matches RAMANA MURTHY and PHANI KUMAR'S results [22]. In addition, when $s \to \infty$, we obtain external velocity for a fluid sphere with the no-slip condition:

$$\psi_{ec} = \left(r^2 + \left(\frac{1}{2}\right) \frac{1}{r} - \left(\frac{3}{2}\right) r\right) G_2(x), \tag{3.6}$$

which matches HAPPEL and BRENNER' result [2].

(2) Internal stream function:

(a) For the no-cap region:
We know that micropolar fluid tends to be viscous fluid when $\lambda_{in}^2 \to \infty$, $\lambda_{en}^2 \to \infty$, then we obtain:

$$C_3 = 0, \qquad B_3 = \frac{s}{2s + 2s\mu + 6\mu}, \qquad A_3 = \frac{-s}{2s + 2s\mu + 6\mu}.$$

Hence, the internal stream function ψ_{in} for the Newtonian fluid sphere with the slip condition is obtained as:

$$\psi_{in} = \left(\left(\frac{-s}{2s + 2s\mu + 6\mu}\right) r^2 + \left(\frac{s}{2s + 2s\mu + 6\mu}\right) r^4\right) G_2(x), \tag{3.7}$$

which matches RAMANA MURTHY and PHANI KUMAR'S result [22]. In addition, when $s \to \infty$, we obtain internal velocity for a fluid sphere with the no-slip condition as:

$$\psi_{in} = \left(-\left(\frac{s}{(2\mu+2)}\right) r^2 + \left(\frac{s}{(2\mu+2)}\right) r^4\right) G_2(x). \tag{3.8}$$

In addition, when $\mu \to \infty$, we get external velocity for a solid sphere with the no-slip condition as:

$$\psi_{in} = \left(-\frac{1}{2} r^2 + \frac{1}{2} r^4\right) G_2(x), \tag{3.9}$$

which matches the results in HAPPEL and BRENNER [2].

(b) For the cap region, internal velocity does not exist.

4. Drag calculation

We know that the drag force for a micropolar fluid in the limiting form is given by Ramkissoon and Majumdar [7] as:

$$F_z = 4\pi(2\mu + k) \lim_{r\to\infty} \left(\frac{\psi_e - \psi_\infty}{R \sin^2\theta} \right). \tag{4.1}$$

Substituting (2.11), (2.12) and simplifying, we get:

$$F_z = 2\pi\,(2\mu + k)\, aU_\infty (B_1 + B_2).$$

Substituting B_1, B_2 from (3.1), we get:

$$F_z = -3\pi(2\mu + k)aU_\infty. \tag{4.2}$$

As $k \rightarrow$, we get $F_z = -6\pi\mu aU_\infty$, which matches the results in Happel and Brenner [2]. The drag coefficient is $Cd = \frac{F_z}{-6\pi\mu U_\infty a}$:

$$Cd = \frac{2\pi(2\mu + k)aU_\infty(B_1 + B_2)}{-6\pi\mu U_\infty a}. \tag{4.3}$$

5. Numerical results

The drag coefficient (Cd), for the micropolar contaminated fluid sphere over which a micropolar fluid passes, is evaluated analytically. The variations of drag coefficient for varying slip parameter (s) for different cross-viscosity parameters (c) are computed numerically. The numerical values are presented in Table 1,

Table 1. Drag coefficient values for varying slip parameter values (s) and different cross-viscosity parameters (c).

Sl. No.	$s\backslash c$	$c = 0.6$	$c = 0.8$	$c = 1.0$	$c = 1.2$
1	2	0.7961	0.7961	0.7961	0.7961
2	4	0.8490	0.8490	0.8490	0.8490
3	6	0.8780	0.8780	0.8780	0.8780
4	8	0.8965	0.8965	0.8965	0.8965
5	10	0.9094	0.9094	0.9094	0.9094
6	12	0.9189	0.9189	0.9189	0.9189
7	14	0.9264	0.9264	0.9264	0.9264
8	16	0.9324	0.9324	0.9324	0.9324

and a graph presenting drag coefficient vs. slip parameter is presented in Fig. 2. It is observed that with an increase in slip parameter values, there is an increase in drag coefficient values. Also, there is no change in drag coefficient values with a change in the cross-viscosity parameter (c).

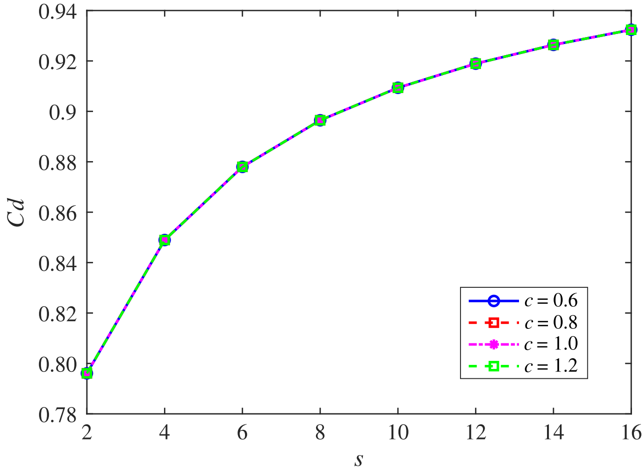

FIG. 2. Drag coefficient versus slip parameter.

6. Conclusion

We have considered external and internal flow to be micropolar fluid, past a contaminated fluid sphere with slip condition over the boundary, regularity condition far away from the body, and continuity of shear stress over the clear part. The velocity field is expressed in terms of stream function and their values over the no-cap region and cap region are evaluated. Drag over the body is also evaluated. The results in special cases are derived, i.e.:

- when $\lambda_{in} \to \infty$, $\lambda_{en} \to \infty$, $\lambda_{ic} \to \infty$, $\lambda_{ec} \to \infty$, i.e., when the micropolar fluid reduces to the viscous fluid,
- the viscous fluid sphere with the no-slip condition when $s \to \infty$,
- fluid sphere changes to solid sphere when $\mu \to \infty$,

which are in good agreement with results available in the literature. In our investigation, we observed that with a rise in the slip parameter values, there is an increase in the drag coefficient values.

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