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Effect of Rotation and Suspended Particles on the Stability of an Incompressible Walters' (Model B') Fluid Heated from Below Under a Variable Gravity Field in a Porous Medium

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The effect of rotation and suspended particles on the stability of an incompressible Walters' (model B') fluid heated from below under a variable gravity field in a porous medium is considered. By applying a normal mode analysis method, the dispersion relation has been derived and solved numerically. It is observed that the rotation, gravity field, suspended particles, and viscoelasticity introduce oscillatory modes. For stationary convection, the rotation has a stabilizing effect and suspended particles are found to have a destabilizing effect on the system, whereas the medium permeability has a stabilizing or destabilizing effect on the system under certain conditions. The effect of rotation, suspended particles, and medium permeability has also been shown graphically.

Key words: Walters' (model B') fluid, rotation, suspended particles, variable gravity field, porous medium.

MSC 2010: 76A05; 76A10; 76E06; 76E07; 76E19; 76E25; 76S05.

Notations

$\mathbf{q}$ – velocity of fluid,
p – pressure,
$\mathbf{g}$ – gravitational acceleration vector,
g – gravitational acceleration,
k – wave number of disturbance,
p_1 – thermal Prandtl number,
P_l – dimensionless medium permeability.

Greek symbols

ϵ – medium porosity,
$\boldsymbol{\rho}$ – fluid density,
μ – fluid viscosity,
μ' – fluid viscoelasticity,
υ – kinematic viscosity,
υ' – kinematic viscoelasticity,
κ – thermal diffusitivity,
α – thermal coefficient of expansion,
β – adverse temperature gradient,
θ – perturbation in temperature,
δ – perturbation in respective physical quantity,
ζ – Z-component of vorticity,
$\mathbf{\Omega}$ – rotation vector having components $(0, 0, \Omega)$.

1. Introduction

A detailed account of the thermal instability of a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using the linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [15], whereas Scanlon and Segel [7] have considered the effect of suspended particles on the onset of Be'nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer. Sharma [8] has studied thermal instability of a viscoelastic fluid in hydromagnetics.

Sharma and Sunil [9] have studied thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many viscoelastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of fluids is Walters' (model B') viscoelastic fluid having relevance both in the chemical technology and industry. Walters' [14] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters (model B') viscoelastic fluid. Walters' (model B') viscoelastic fluid forms the basis for the manufacture of many important polymers and useful products.

Stommel and Fedorov [13] and Linden [3] have remarked that the length scalar characteristic of double diffusive convecting layers in the ocean may be sufficiently large, so that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium, and the distortion plays an important role in extraction of energy in geothermal regions. The problem of thermal instability of a fluid in a porous medium is of importance in geophysics, soil sciences, ground water hydrology, and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of a young oceanic crust (Lister, [4]).

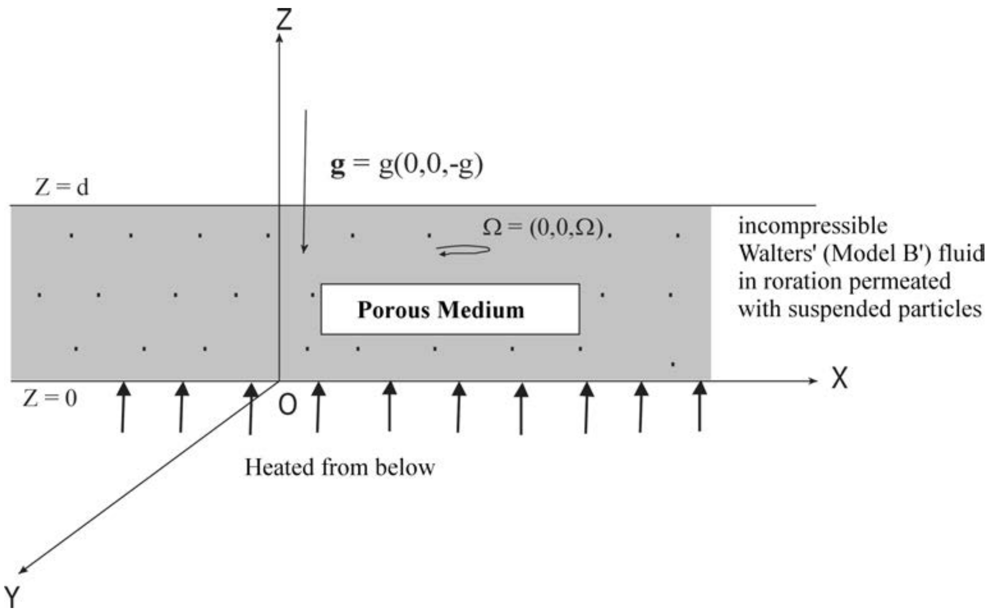
Thermal instability of a fluid layer under a variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal [5]. Although the gravity field of the Earth is varying with the height from its surface, we usually neglect this variation for laboratory purposes and treat the field as constant. However, this may not be the case for large scale flows in the ocean, atmosphere, or mantle. It can become imperative to consider gravity as a quantity varying with distance from the centre of the Earth.

Sharma and Rana [10] have studied thermal instability of a Walters' (model B') viscoelastic fluid in the presence of a variable gravity field and rotation in a porous medium. Sharma and Rana [11] have also studied the thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of a magnetic field and variable gravity field in a porous medium. Recently, Sharma and Gupta [12] have studied the effect of rotation on thermal convection of micropolar fluids in the presence of suspended particles, whereas Rana and Kango [6] have studied the effect of rotation on thermal instability of a compressible Walters' (model B') viscoelastic fluid in a porous medium.

Keeping in mind the importance in various applications mentioned above, our interest in the present paper is to study the effect of rotation and suspended particles on the stability of an incompressible Walters' (model B') fluid heated from below under a variable gravity field in a porous medium.

2. Formulation of the problem and perturbation equations

Here we consider an infinite, horizontal, incompressible Walters' (model B') viscoelastic fluid of the depth d, bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ϵ and permeability k_1, which is acted upon by a uniform rotation $\mathbf{\Omega}(0, 0, \Omega)$ and variable gravity $\mathbf{g}(0, 0, -g)$, where $g = \lambda g_0$, $g_0(> 0)$ is the value of g at $z = 0$, and λ can be positive or negative as the gravity increases or decreases upwards from its value g_0. This layer is heated from below such that a steady adverse temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.



Schematic Sketch of Physical Situation

Let ρ, υ, υ', p, ϵ, T, α, and $\mathbf{q}(0,0,0)$, denote, respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion, and velocity of the fluid. The equations expressing the conservation of momentum, mass, temperature, and equation of state for the Walters' (model B') viscoelastic fluid are as follows:

$$(2.1)\quad \frac{1}{\epsilon}\left[\frac{\partial q}{\partial t}+\frac{1}{\epsilon}\left(q.\nabla\right)q\right]=-\frac{1}{\rho_0}\nabla p+g\left(1+\frac{\delta\rho}{\rho_0}\right) -\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)q+\frac{2}{\epsilon}\left(q\times\Omega\right)+\frac{K'N}{\rho_0\epsilon}(q_d-q),$$

$$(2.2)\qquad \nabla.q=0,$$

$$(2.3)\qquad E\frac{\partial T}{\partial t}+\left(q.\nabla\right)T+\frac{mNC_{pt}}{\rho_0C_f}\left[\epsilon\frac{\partial}{\partial t}+q_d.\nabla\right]T=\kappa\,\nabla^2\mathrm{T},$$

and

$$(2.4)\qquad \rho=\rho_0\left[1-\alpha(T-T_0)\right],$$

where the suffix zero refers to values at the reference level $z=0$.

Here $q_d(\overline{x}, t)$ and $N(\overline{x}, t)$ denote the velocity and number density of the particles, respectively, $q_d = (l, r, s)$, $\overline{x} = (xyz)$, $K = 6\pi\eta\rho v$, where η is the Stokes drag coefficient, where η is the particle radius and,

$$E = \epsilon + (1 - \epsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$$

is constant, κ is the thermal diffusivity, and ρ_s, c_s, ρ_0, c_f denote the density and heat capacity of the solid (porous) matrix and fluid, respectively.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are as follows:

$$mN\left[\frac{\partial q_d}{\partial t} + \frac{1}{\epsilon}\left(q_d.\nabla\right) q_d\right] = K' N\left(q - q_d\right), \tag{2.5}$$

$$\epsilon \frac{\partial N}{\partial t} + \nabla.\left(N q_d\right) = 0. \tag{2.6}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid, and appears in the equation of motion (2.1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (2.6). The buoyancy force on the particles is neglected. Interparticles' reactions are not considered either since we assume that the distance between the particles is quite large as compared to their diameters. These assumptions have been used in writing the equations of motion (2.6) for the particles.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is as follows:

$$\begin{gathered} q = (0,0,0), \qquad q_d = (0,0,0), \\ T = -\beta z + T_0, \qquad \rho = \rho_0(1 + \alpha\beta z), \qquad N_0 = \text{ constant} \end{gathered} \tag{2.7}$$

and is an exact solution to the governing equations.

Let $q(u, v, w)$, $q_d(l, r, s)$, θ, δp, and $\delta\rho$ denote the perturbations, respectively, in fluid velocity $q(0, 0, 0)$, particle velocity $q_d(0, 0, 0)$, temperature T, pressure p, and density ρ.

The change in density $\delta\rho$ caused by the perturbation θ in temperature is given by:

$$\delta\rho = -\alpha\rho_0\theta. \tag{2.8}$$

The linearized perturbation equations governing the motion of fluids are as follows:

$$\frac{1}{\epsilon}\frac{\partial q}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - g\frac{\delta\rho}{\rho_0} - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)q + \frac{K'N}{\epsilon}\left(q_d - q\right) + \frac{2}{\epsilon}\left(q\times\Omega\right), \tag{2.9}$$

$$\nabla q = 0, \tag{2.10}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right) q_d = q, \tag{2.11}$$

$$\left(E + b\epsilon\right)\frac{\partial\theta}{\partial t} = \beta\left(w + bs\right) + \kappa\nabla^2\theta, \tag{2.12}$$

where $b = \dfrac{mNC_{pt}}{\rho_0 C_f}$, and w, s are the vertical fluid and particles velocity.

In the Cartesian form, Eqs. (2.9)–(2.12) with the help of Eq. (2.8) can be expressed as follows:

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial x}\left(\delta p\right) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)u - \frac{mN}{\epsilon\rho_0}\frac{\partial u}{\partial t} + \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega v, \tag{2.13}$$

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial y}\left(\delta p\right) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)v - \frac{mN}{\epsilon\rho_0}\frac{\partial v}{\partial t} + \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega u, \tag{2.14}$$

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial z}\left(\delta p\right) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)w - \frac{mN}{\epsilon\rho_0}\frac{\partial w}{\partial t} + g\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\alpha\theta, \tag{2.15}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2.16}$$

$$\left(E + b\epsilon\right)\frac{\partial\theta}{\partial t} = \beta\left(w + bs\right) + \kappa\nabla^2\theta. \tag{2.17}$$

Operating Eqs. (2.13) and (2.14) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$, respectively, adding them and using Eq. (2.16), we get:

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial w}{\partial z}\right)-\frac{mN}{\epsilon\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) - \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\zeta, \tag{2.18}$$

where $\zeta = \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ is the z-component of vorticity.

Operating Eqs. (2.15) and (2.18) by $\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$, respectively, and adding them to eliminate δp between these equations, we get:

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}(\nabla^2 w)=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\nabla^2 w + g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta-\frac{mN}{\epsilon\rho_0}\frac{\partial}{\partial t}(\nabla^2 w) - \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial\zeta}{\partial z}, \tag{2.19}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

Operating Eqs. (2.13) and (2.14) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$, respectively, and adding them, we get

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\zeta - \frac{mN}{\epsilon\rho_0}\frac{\partial\zeta}{\partial t}+\frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial w}{\partial z}. \tag{2.20}$$

3. The dispersion relation

Following the normal mode analyses, we assume that the perturbation quantities have x, y, and t dependence of the form:

$$[w,s,\theta,\zeta]=[W(z),S(z),\Theta(z),Z(z)]\exp\left(ik_x x+ik_y y+nt\right), \tag{3.1}$$

where k_x and k_y are the wave numbers in the x and y directions, $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave number, and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (3.1) Eqs. (2.19), (2.20), and (2.17) become:

$$\frac{n}{\epsilon}\left[\frac{d^2}{dz^2} - k^2\right] W = -gk^2\alpha\Theta - \frac{1}{k_1}\left(\upsilon - \upsilon' n\right)\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{mNn}{\epsilon\rho_0\left(\frac{m}{K'}n + 1\right)}\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{2\Omega}{\epsilon}\frac{dZ}{dz}, \tag{3.2}$$

$$\frac{n}{\epsilon} Z = -\frac{1}{k_1}\left(\upsilon - \upsilon' n\right) - \frac{mNn}{\epsilon\rho_0\left(\frac{m}{K'}n + 1\right)} Z + \frac{2\Omega}{\epsilon}\frac{dW}{dz}, \tag{3.3}$$

$$\left(E + b\epsilon\right) n\Theta = \beta\left(W + bS\right) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta. \tag{3.4}$$

Equations (3.2)–(3.4) in a non-dimensional form become:

$$\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right]\left(D^2 - a^2\right) W + \frac{ga^2d^2\alpha\Theta}{\upsilon} + \frac{2\Omega d^3}{\epsilon\upsilon} DZ = 0, \tag{3.5}$$

$$\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right] Z = \left(\frac{2\Omega d}{\epsilon\upsilon}\right) DW, \tag{3.6}$$

$$\left[\left(D^2 - a^2\right) - E_1 p_1\sigma\right]\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right) W, \tag{3.7}$$

where we have put:

$$a = kd, \qquad \sigma = \frac{nd^2}{\upsilon}, \qquad \tau = \frac{m}{K'}, \qquad \tau_1 = \frac{\tau\upsilon}{d^2},$$
$$M = \frac{mN}{\rho_0}, \qquad E_1 = E + b\epsilon, \qquad B = b + 1, \qquad F = \frac{\upsilon'}{d^2},$$

and $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number.

Eliminating Θ and Z from Eqs. (3.5)–(3.7), we obtain:

$$\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right]\left(D^2 - a^2\right)\left(D^2 - a^2 - E_1p_1\sigma\right) W - Ra^2\lambda\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right) W + \left[\frac{\frac{T_A}{\epsilon^2}\left(D^2 - a^2 - E_1p_1\sigma\right)}{\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}}\right] D^2W = 0, \tag{3.8}$$

where $R = \dfrac{g_0 \alpha \beta d^4}{\upsilon \kappa}$, is the thermal Rayleigh number and $T_A = \left(\dfrac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries, and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, [1]):

$$W = D^2 W = DZ = \Theta = 0 \qquad \text{at} \qquad z = 0 \qquad \text{and} \qquad z = 1. \tag{3.9}$$

The case of two free boundaries, though a little artificial, is the most appropriate for stellar atmospheres. Using the boundary conditions (3.9), we can show that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence the proper solution of W characterizing the lowest mode is:

$$W = W_0 \sin \pi z, \tag{3.10}$$

where W_0 is a constant.

Substituting Eq. (3.10) in (3.8), we obtain the following dispersion relation:

$$\begin{aligned} R_1 x \lambda = {} & \left[\frac{i\sigma_1}{\epsilon}\left(1 + \frac{M}{1 + \tau_1 \pi^2 i\sigma_1}\right) + \frac{1 - F\pi^2 i\sigma_1}{P}\right] \\ & \cdot (1 + x)(1 + x + E_1 p_1 i\sigma_1)\left(\frac{1 + \tau_1 \pi^2 i\sigma_1}{B + \tau_1 \pi^2 i\sigma_1}\right) \\ & + \frac{\dfrac{T_{A_1}}{\epsilon^2}(1 + x + E_1 p_1 i\sigma_1)}{\dfrac{i\sigma_1}{\epsilon}\left(1 + \dfrac{M}{1 + \tau_1 \pi^2 i\sigma_1}\right) + \dfrac{1 - F\pi^2 i\sigma_1}{P}}\left(\frac{1 + \tau_1 \pi^2 i\sigma_1}{B + \tau_1 \pi^2 i\sigma_1}\right), \end{aligned} \tag{3.11}$$

where

$$R_1 = \frac{R}{\pi^4}, \qquad T_{A_1} = \frac{T_A}{\pi^4}, \qquad x = \frac{a^2}{\pi^2}, \qquad i\sigma_1 = \frac{\sigma}{\pi^2}, \qquad P = \pi^2 P_l.$$

Equation (3.11) is a required dispersion relation accounting for the effect of suspended particles, medium permeability, variable gravity field, rotation on the stability of a Walters' (model B') viscoelastic fluid heated from below in a porous medium.

4. Stability of the system and oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in a Walters' (model B') viscoelastic fluid due to the presence of suspended particles, rotation, viscoelasticity, and variable gravity field. Multiplying Eq. (3.5) by W^*, the complex conjugate of W, integrating over the range of z, and making use of Eqs. (3.6)–(3.7) with the help of the boundary conditions (3.9), we obtain:

$$\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1-\frac{\alpha a^2\lambda g_0\kappa}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right) \times\left(I_2+E_1p_1\sigma^*I_3\right)+d^2\left[\frac{\sigma^*}{\epsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma^*}{P_l}\right]I_4=0, \tag{4.1}$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz, \qquad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_3=\int_0^1|\Theta|^2\,dz, \qquad I_4=\int_0^1|Z|^2\,dz.$$

The integral parts I_1–I_4 are all positive definite. Putting $\sigma=i\sigma_i$ in Eq. (4.1), where σ_i is real and equating the imaginary parts, we obtain:

$$\sigma_i\left[\frac{1}{\epsilon}\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)-\frac{F}{P_l}\right]\left(I_1+d^2I_4\right)+\frac{\alpha a^2\lambda g_0\kappa}{\upsilon\beta} \cdot\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_2+\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}E_1p_1I_3\right]=0. \tag{4.2}$$

Equation (4.2) implies that $\sigma_i=0$ or $\sigma_i\neq 0$, which means that modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation, gravity field, suspended particles, and viscoelasticity.

5. The stationary convection

For the stationary convection, putting $\sigma=0$ in Eq. (3.11) reduces it to:

$$R_1=\frac{1+x}{\lambda xB}\left[\frac{1+x}{P}+\frac{T_{A_1}}{\epsilon^2}P\right], \tag{5.1}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P; and then the Walters' (model B') viscoelastic fluid behaves like an ordinary Newtonian fluid, since the viscoelastic parameter F vanishes with σ.

To study the effects of suspended particles, rotation, and medium permeability, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dT_{A_1}}$, and $\frac{dR_1}{dP}$ analytically.

Equation (5.1) yields:

$$\frac{dR_1}{dB} = -\frac{1+x}{\lambda x B^2}\left[\frac{1+x}{P} + \frac{T_{A_1}}{\epsilon^2}P\right], \tag{5.2}$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system when the gravity increases upwards from its value g_0 (i.e., $\lambda > 0$).

From Eq. (5.1), we also get:

$$\frac{dR_1}{dT_{A_1}} = \frac{1+x}{\lambda x B \epsilon^2}P, \tag{5.3}$$

which shows that rotation has a stabilizing effect on the system when the gravity increases upwards from its value g_0 (i.e., $\lambda > 0$).

It is evident from Eq. (5.1) that:

$$\frac{dR_1}{dP} = -\frac{1+x}{\lambda x B}\left[\frac{1+x}{P^2} - \frac{T_{A_1}}{\epsilon^2}\right]. \tag{5.4}$$

From Eq. (5.4), we observe that the medium permeability has a destabilizing effect when $\frac{1+x}{P^2} > \frac{T_{A_1}}{\epsilon^2}$, and it has a stabilizing effect when $\frac{1+x}{P^2} < \frac{T_{A_1}}{\epsilon^2}$, when the gravity increases upwards from its value g$_0$ (i.e., $\lambda > 0$).

In the absence of rotation and for a constant gravity field, $\frac{dR_1}{dP}$ is always negative implying thereby the destabilizing effect of the medium permeability.

The dispersion relation (5.1) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

In Fig. 1, Rayleigh number R_1 is plotted against suspended particles B for $\lambda = 2$, $T_{A_1} = 5$, $\epsilon = 0.5$, $P = 0.2$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$. For the wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$, suspended particles have a destabilizing effect.

In Fig. 2, Rayleigh number R_1 is plotted against rotation T_{A_1} for $B = 3$, $\lambda = 2$, $\epsilon = 0.5$, $P = 0.2$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$. This shows that rotation has a stabilizing effect for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$.

In Fig. 3, Rayleigh number R_1 is plotted against the medium permeability P for $B = 3$, $\lambda = 2$, $\epsilon = 0.5$, $T_{A_1} = 5$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$. This shows that the medium permeability has a destabilizing effect for $P = 0.1$ to 0.3, and has a stabilizing effect for $P = 0.3$ to 1.0.

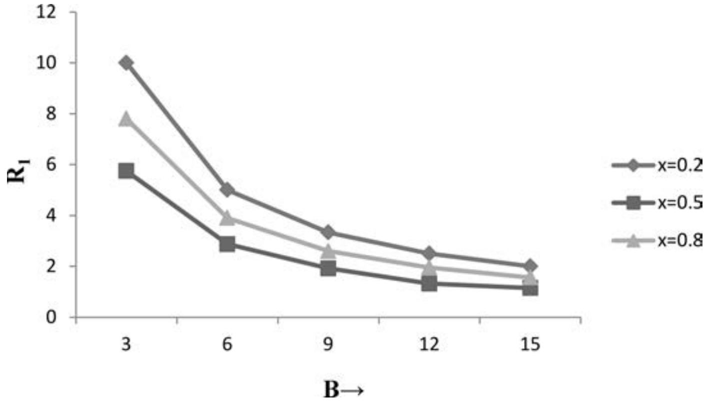


FIG. 1. Variation of Rayleigh number R_1 with suspended particles B for $\lambda = 2$, $T_{A_1} = 5$, $\epsilon = 0.5$, $P = 0.2$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$.

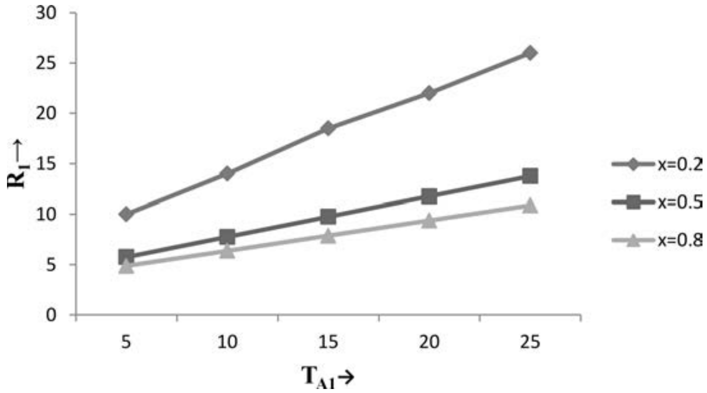


FIG. 2. Variation of Rayleigh number R_1 with rotation T_{A_1} for $B = 3$, $\lambda = 2$, $\epsilon = 0.5$, $P = 0.2$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$.

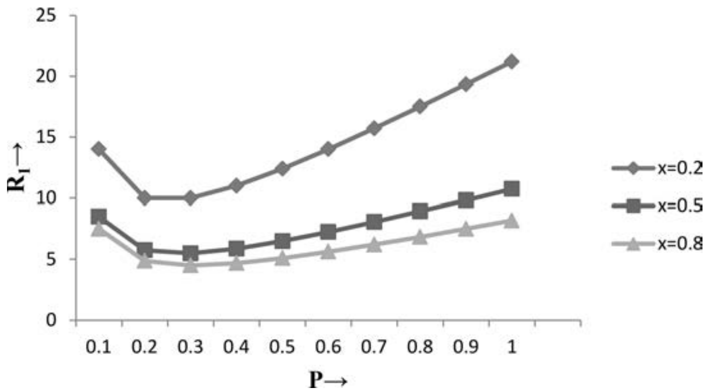


FIG. 3. Variation of Rayleigh number R_1 with the medium permeability P for $B = 3$, $\lambda = 2$, $\epsilon = 0.5$, $T_{A_1} = 5$ for fixed wave numbers $x = 0.2$, $x = 0.5$, and $x = 0.8$.

6. Conclusions

The effect of rotation and suspended particles on the stability of an incompressible Walters' (model B') fluid heated from below under a variable gravity field in a porous medium has been investigated. For the stationary convection, it has been found that the rotation has a stabilizing effect for $\lambda > 0$ and destabilizing effect for $\lambda < 0$, opposite to the Newtonian fluids. Suspended particles are found to have a destabilizing effect on the system as the gravity increases upwards from its value g_0 (i.e., for $\lambda > 0$) and a stabilizing effect as the gravity decreases upwards from its value g_0 (i.e., for $\lambda < 0$), whereas the medium permeability has a destabilizing/stabilizing effect on the system for $\frac{1+x}{P^2} > \frac{T_{A_1}}{\epsilon^2} \Big/ \frac{1+x}{P^2} < \frac{T_{A_1}}{\epsilon^2}$ as the gravity increases upwards from its value g_0 (i.e., for $\lambda > 0$). The presence of rotation, gravity field, suspended particles, and viscoelasticity introduces oscillatory modes. The effects of rotation, suspended particles, and medium permeability on thermal instability have also been shown graphically.

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